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ON THE USE OF HIGHER ORDER AND IRREGULARLY SHAPED BOUNDARY ELEMENTS IN NEARFIELD ACOUSTICAL HOLOGRAPHY

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Abstract

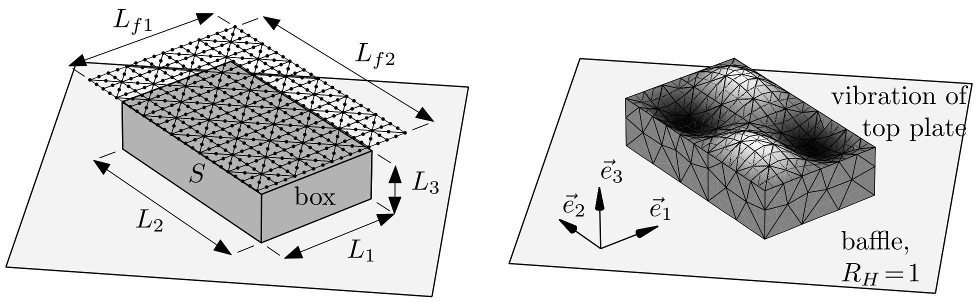
Nearfield acoustical holography (NAH) is used to find the regions of acoustic activity on the surface of a sound radiating object. One of the most general NAH approaches is the inverse frequency response function (IFRF) technique, since it imposes no limitations on the geometry of the radiating boundary. In the IFRF method acoustic measurements on a grid in the nearfield of the object are used to determine the corresponding normal velocity distribution on the surface of the object. The measured sound field is related to the surface vibrations via a transfer matrix, which is calculated using a boundary element method. The required inversion of the transfer matrix is not that simple, because the system is ill-conditioned. Hence, a physically meaningful solution can only be obtained by applying regularization techniques.

In the literature, the boundary surface of the source is usually meshed with constant or linear elements of equal shape and size. Most likely, this is done to avoid the effect that the regularized inversion process favors nodes that are associated with a high mean square surface normal velocity. This effect is due to the fact that such nodes have a more effective contribution to the sound field. In this paper it is demonstrated that the problem in which the inverse solution is affected by the topology of the mesh gets even worse when quadratic elements are applied. A new technique will be described that circumvents this problem completely by the introduction of an appropriate smoothing operator. As a result all boundary nodes are treated in an equal way, irrespective of their associated area or type of shape function. With the presented approach, irregular meshes and/or higher order boundary elements can be successfully used in NAH applications.

INTRODUCTION

Exterior sound radiation caused by structural vibrations is described by the Helmholtz Integral Equation (HIE). The integral equation relates the surface normal velocities on the boundary of a vibrating object to the acoustic pressure fluctuations in the surrounding fluid domain.

This type of problems is efficiently solved with the direct boundary element method (BEM), as this approach reduces the spatial dimensionality of the problem by one. Moreover, the Sommerfeld radiation condition is automatically satisfied in the integral formulation. Furthermore, BEM not only predicts the acoustic quantities itself but additionally yields the so-called acoustic transfer matrices (ATM), which are essential for nearfield acoustic source identification techniques (see Marki [3], Visser [7]). Such techniques estimate the unknown surface vibrations on the boundary of a radiating object based on measurements of the radiated sound field. Most publications on this topic seem to be limited to uniform meshes containing constant or linear elements. In the present paper it is shown that in order to apply quadratic boundary elements (or nonuniform meshes), it is required to incorporate an appropriate smoothing operator in the solution scheme. The influence of such a smoothing operator on the reconstructed solution will be demonstrated by means of an example concerning radiation from a box mounted on a baffle as depicted in figure 1.



Parameter	Value	Units	Description
$L_1 \times L_2 \times L_3$	$0.3 \times 0.5 \times 0.12$	[m]	Size of the box
$L_{f1} \times L_{f2}$	0.4×0.6	[m]	Size of the field grid
d	0.05	[m]	Distance between the field grid and top plate
h	0.05	[m]	Characteristic element size on the top plate
ρ_0	1.22	[kg/m^3]	Fluid density
c_0	343	[m/s]	Propagation speed in fluid
R_H	1	[$-$]	Reflection coefficient of baffle

Figure 1: Numerical example of a box mounted on a baffle. The simply supported top plate of the box is vibrating in a (2,2) mode shape. Simulated measurements are performed in a planar field grid at distance d above the top plate.

BASIC EQUATIONS

Assuming a stationary sound source in a homogeneous fluid, the corresponding acoustic wave propagation is governed by the Helmholtz differential equation

$$\nabla^2 p(\vec{x}) + k^2 p(\vec{x}) = 0, \tag{1}$$

with $k = \omega/c_0$ the wave number, c_0 the speed of sound, ω the angular frequency, and $p(\vec{x})$ representing the complex amplitude of the harmonic pressure perturbation. Using Green's second identity the Helmholtz differential equation (1) can be rewritten in the HIE, which forms the basis of direct BEM [1, 5, 8]. This integral equation relates the surface pressure $p(\vec{y})$ and normal velocity $v_n(\vec{y})$ on a vibrating closed boundary S to the radiated pressure $p(\vec{x})$ in field point $\vec{x}$ (see figure 2):

$$\alpha(\vec{x})p(\vec{x}) = \oint_S \left\{ \frac{\partial G_H(r,r')}{\partial n_y} p(\vec{y}) - G_H(r,r') \frac{\partial p(\vec{y})}{\partial n_y} \right\} \mathrm{d}S\,, \tag{2}$$

with distances $r = \|\vec{r}\|$ and $r' = \|\vec{r}\,'\|$. Note that the present example concerns radiation into half space instead free space, hence the Green's function reads

$$G_H(r,r') = \frac{e^{-\mathrm{i}kr}}{4\pi r} + R_H \frac{e^{-\mathrm{i}kr'}}{4\pi r'}\,. \tag{3}$$

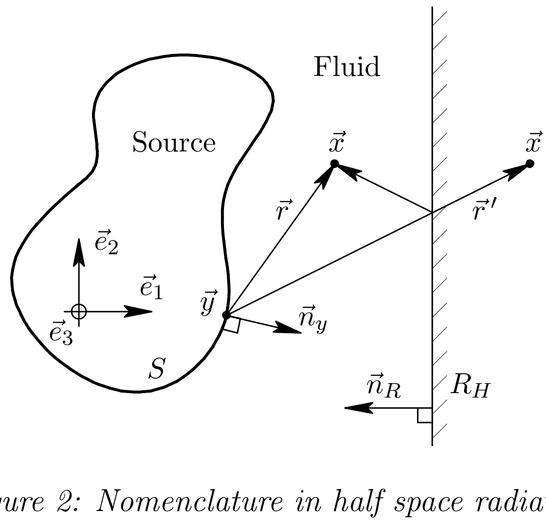


Figure 2: Nomenclature in half space radiation.

NUMERICAL IMPLEMENTATION

For the numerical evaluation of integral equation (2), a discretization of the boundary surface S into so-called boundary elements is made. In the sample problem, the surface of the box has been divided into 222 quadratic triangular elements (TRIA6) with a total of 461 nodes. After a piecewise polynomial interpolation of the acoustic quantities, the numerical integration is performed for each element separately and the contributions of each of these elements to their nodes are assembled in a matrix. In this way the continuous integral equation is approximated by the following algebraic system of equations

$$\mathbf{p}_f = \mathbf{H} \cdot \mathbf{v}_n \,, \tag{4}$$

with vector $\mathbf{p}_f$ representing the pressures in a discrete set of field points. Vector $\mathbf{v}_n$ represents the prescribed surface normal velocity on the nodes of the source mesh. Note that the acoustic transfer matrix $\mathbf{H}$ is a function of the wave number.

ACOUSTIC SOURCE IDENTIFICATION

Problem formulation

In BEM based acoustic source identification methods [3, 4, 7] the objective is to obtain the unknown surface velocities in normal direction ($\mathbf{v}_n$) from the acoustic pressures ($\mathbf{p}_f$) measured in the field grid.
In inverse theory a problem like $\mathbf{p}_f = \mathbf{H} \cdot \mathbf{v}_n$ is commonly written as

$$\mathbf{H} \cdot \mathbf{x} = \mathbf{b} \,, \quad \text{or as} \quad \min_{\mathbf{x}} \|\mathbf{H} \cdot \mathbf{x} - \mathbf{b}\| \quad \text{with} \quad \mathbf{H} \in \mathbb{C}^{m \times n} \,, \tag{5}$$

where $\mathbf{H}$ represents the transfer matrix that relates an input vector $\mathbf{x} = \mathbf{v}_n$ (cause) to an output vector $\mathbf{b} = \mathbf{p}_f$ (effect) of dimensions n and m, respectively. A particulary useful tool for solving such systems is the compact singular value decomposition (SVD), being defined as

$$\mathbf{H} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^{\mathrm{H}} = \sum_i \mathbf{u}_i s_i \mathbf{v}_i^{\mathrm{H}} \,, \tag{6}$$

where in case of an overdetermined system ($m > n$), matrix $\mathbf{U}$ is of dimensions $m \times n$ and $\mathbf{V}$ of $n \times n$ while for the underdetermined system ($m < n$), $\mathbf{U}$ is of dimensions $m \times m$ and $\mathbf{V}$ is of $n \times m$. For both cases $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, $\mathbf{U}^{\mathrm{H}} \cdot \mathbf{U} = \mathbf{I}_m$ and $\mathbf{V}^{\mathrm{H}} \cdot \mathbf{V} = \mathbf{I}_n$, with their columns representing the left and right singular vectors $\mathbf{u}_i$ and $\mathbf{v}_i$, respectively

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{\min(m,n)} \end{bmatrix} \quad \text{and} \quad \mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_{\min(m,n)} \end{bmatrix} . \tag{7}$$

The nonnegative and real singular values are collected on the diagonal of matrix $\mathbf{S} = \operatorname{diag}\begin{pmatrix} s_1 & s_2 & \cdots & s_{\min(m,n)} \end{pmatrix}$ appearing in descending order such that $s_1 \geq s_2 \geq \cdots \geq s_{\min(m,n)} \geq 0$.
In terms of this decomposition the standard least-squares solution to system (5) reads

$$\mathbf{x}_{\mathrm{LS}} = \sum_i \frac{\mathbf{u}_i^{\mathrm{H}} \cdot \mathbf{b}}{s_i} \mathbf{v}_i \,. \tag{8}$$

Unfortunately, the acoustic transfer matrix proves to be ill-conditioned, which implies that arbitrary small perturbations in the measured pressure data $\mathbf{b}$ result in large errors in the solution of the surface velocities $\mathbf{x}$. Nevertheless, a meaningful solution can be found in both physical and mathematical terms with the help of regularization methods.

It is known [2, 3, 7, 8] that standard regularization techniques like truncated singular value decomposition (TSVD), Tikhonov regularization or iterative least-squares QR factorization (LSQR) give adequate solutions in these inverse BEM techniques. In this paper the TSVD technique has been adopted as it provides an excellent physical interpretation of the inverse problem. By imposing TSVD regularization, the original ill-conditioned least-squares problem (5) is replaced by the following generalized problem [2, 9]

$$\min_{\mathbf{x}} \|\mathbf{H}\cdot\mathbf{x}-\mathbf{b}\| \quad \text{subject to} \quad \min_{\mathbf{x}} \|\mathbf{L}\cdot\mathbf{x}\| \,, \tag{9}$$

which shows a trade-off between minimizing the residual and the magnitude of the solution measured according to a discrete smoothing operator $\mathbf{L}$. Basically any arbitrary matrix $\mathbf{L}$ can be applied to constrain the norm of $\mathbf{x}$, but preferably it should reflect some physical property of the solution.

Solving the problem - TSVD

Once the transfer matrix has been computed and a smoothing norm has been chosen, the first step towards solving the generalized problem (9) is to convert it into standard form by introducing the transformation $\hat{\mathbf{x}} = \mathbf{L}\cdot\mathbf{x}$, leaving

$$\min_{\hat{\mathbf{x}}} \left\|\hat{\mathbf{H}}\cdot\hat{\mathbf{x}}-\mathbf{b}\right\| \quad \text{subject to} \quad \min_{\hat{\mathbf{x}}} \|\hat{\mathbf{x}}\| \,, \tag{10}$$

with $\hat{\mathbf{H}} = \mathbf{H}\cdot\mathbf{L}^{-1}$. Now that the problem is in standard form, a TSVD regularized solution similar to that of equation (8) can be found:

$$\hat{\mathbf{x}}_\kappa = \sum_{i=1}^{\kappa} \frac{\hat{\mathbf{u}}_i^{\mathrm{H}}\cdot\mathbf{b}}{\hat{s}_i}\,\hat{\mathbf{v}}_i \,, \quad \text{with} \quad \kappa \le \min\left(m,n\right), \tag{11}$$

where only the first κ singular components are accounted for in the summation. The symbol κ represents the regularization parameter which controls the amount of filtering (regularization) applied to the least-squares problem. To determine the optimal amount of regularization the L-curve has been applied that balances the size of the residual norm $\left\|\hat{\mathbf{H}}\cdot\hat{\mathbf{x}}_\kappa-\mathbf{b}\right\|$ and the solution norm $\|\hat{\mathbf{x}}_\kappa\|$ (e.g. see Hansen [2]). Finally, the regularized solution to the original generalized problem (9) is obtained through the back-transformation $\mathbf{x}_\kappa = \mathbf{L}^{-1}\cdot\hat{\mathbf{x}}_\kappa$.

Choosing a smoothing norm

Most studies in literature are restricted to boundary surface meshes with equally sized and identically shaped constant or linear elements, since choosing such a discretization effectively circumvents the choice of an appropriate smoothing norm.

In fact in those cases the smoothing operator has implicitly been chosen equal to the identity matrix $\mathbf{L} = \mathbf{I}$. As a consequence the inverse problem (9) is automatically in standard form and strictly speaking no transformation is required. In practical situations however, choosing such a regular discretization is generally unwanted or simply not possible and the boundary mesh consists of elements of different sizes or types. Especially when higher order elements are applied, like the quadratic TRIA6 elements of the sample model, it is required to select an appropriate smoothing operator since the boundary nodes have no longer equal contributions in the radiation process. This problem with higher order elements has recently been reported in a paper of Valdivia and Williams [6]. The current study offers a solution to the problem by choosing the operator $\mathbf{L}$ such that the contributions of the boundary nodes in the inverse problem are weighted with respect to the *mean square value* of the surface normal velocity, defined as

$$M = \frac{1}{2S} \int_S v_n^{\mathrm{H}}(\vec{y}) v_n(\vec{y}) \, \mathrm{d}S \,, \tag{12}$$

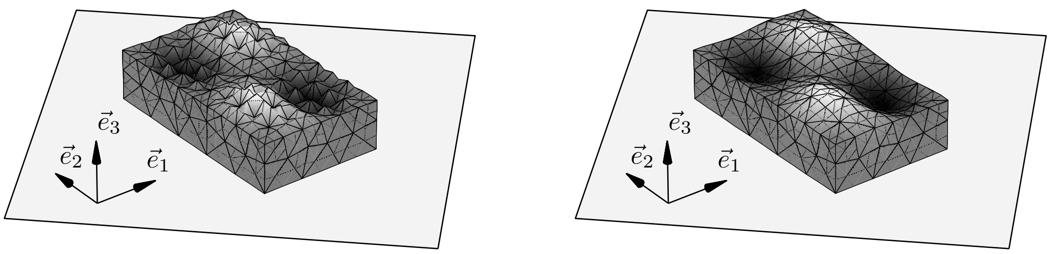
which can be approximated in discrete sense by

$$M = \mathbf{v}_n^{\mathrm{H}} \cdot \mathbf{B} \cdot \mathbf{v}_n \,. \tag{13}$$

where $\mathbf{B}$ contains the integration results over the squared shape functions. As the discrete smoothing operator $\mathbf{L}$ must work on solution vector $\mathbf{v}_n$, it is rather logical to take $\mathbf{L} = \mathrm{chol}(\mathbf{B})$, the Cholesky factor of matrix $\mathbf{B} = \mathbf{L}^{\mathrm{H}} \cdot \mathbf{L}$.

Regularized solutions

Returning to the example of the box with a (2,2) structural vibration as depicted in figure 1, the corresponding inversely obtained TSVD solutions based on a numerical measurement of the acoustic pressures in the field grid are presented in figure 3. In order to make the simulated measurements more realistic, 10% noise has been added to the pressure data.



(a) Reconstruction for $\mathbf{L} = \mathbf{I}$.

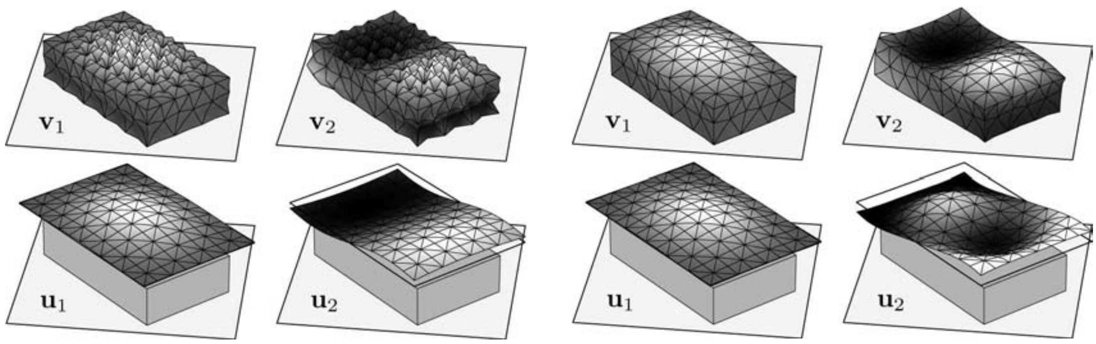
(b) Reconstruction for $\mathbf{L} = \mathrm{chol}(\mathbf{B})$.

Figure 3: Optimal reconstructed surface normal vibrations obtained for different smoothing operators $\mathbf{L}$.

It is obvious that the reconstructed solution obtained through application of the smoothing operator $\mathbf{L} = \text{chol}(\mathbf{B})$ makes sense, whereas the solution obtained with $\mathbf{L} = \mathbf{I}$ is severely influenced by the element characteristics as explained later. Because the actual physical solution is of interest and not a mesh dependent solution, application of $\mathbf{L} = \mathbf{I}$ in combination with quadratic elements is unacceptable. Note that this behavior is also found when other types of regularization are imposed (e.g. Tikhonov, LSQR). The observed behavior can be explained by taking a closer look at the singular vectors from which the regularized solution is composed.

Source modes and field modes

After performing a singular value decomposition of matrix $\hat{\mathbf{H}}$ and application of the back-transformation $\mathbf{v}_i = \mathbf{L}^{-1} \cdot \hat{\mathbf{v}}_i$, the first two singular vectors are plotted in figure 4 for both a smoothing operator of $\mathbf{L} = \mathbf{I}$ and $\mathbf{L} = \text{chol}(\mathbf{B})$. It is obvious that



(a) Singular vectors for $\mathbf{L} = \mathbf{I}$. (b) Singular vectors for $\mathbf{L} = \text{chol}(\mathbf{B})$.

Figure 4: Real parts of first two singular vectors $\mathbf{v}_i$ (source modes) and $\mathbf{u}_i$ (field modes) for different smoothing operators $\mathbf{L}$.

for the latter choice the source modes indeed represent a physically meaningful distribution whereas without an appropriate smoothing norm the source modes are strongly influenced by the element properties of the TRIA6 elements. More specifically, in contrast with the mid-side nodes, the corner nodes of such elements have no net contribution to the specific nodal volume velocity q_i, defined as

$$q_i = \int_{S_m} N_i \, \mathrm{d}S \quad \text{with} \quad q_i = \begin{cases} 0\,, & \text{if node } i \text{ is a corner node,} \\ S_m/3\,, & \text{if node } i \text{ is a mid-side node.} \end{cases} \tag{14}$$

where S_m is the element area and N_i the shape function corresponding to node i. This element behavior is directly observed in the source modes as shown in figure 4(a), where each corner node shows a zero valued contribution to the sound field. In order to avoid this unwanted behavior it is recommended to apply the smoothing operator $\mathbf{L} = \text{chol}(\mathbf{B})$ at all times since this completely avoids the nonphysical effects arising from the element discretization.

CONCLUSIONS

The current paper investigated the application of higher order and irregularly shaped boundary elements in acoustic source identification techniques. It was shown that boundary nodes that are associated with a high mean square surface normal velocity are favored in the inversion process, resulting in solutions that depend on the applied element size and type. To prevent this unwanted nonphysical behavior, a discrete smoothing operator based on a Cholesky decomposition of the spatially averaged square surface normal velocity should be applied.

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