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Numerical Evaluation of Acoustic Power Radiation and Radiation Efficiencies of Baffled Plates

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Abstract

In this paper expressions are given for the numerical evaluation of radiation efficiencies and power radiation of baffled plates. The expressions can be used as a postprocessing tool in the Finite Element Method. Numerical results for simply supported plates are presented and compared with results obtained by a statistical technique. There is a good agreement between the results of the two techniques.

1 Introduction

Acoustic design and control of baffled thin plates, which are often sources of noise, is an important subject in structural acoustics. Acoustic power radiated by these plates can be studied by equivalent sources methods. The most important of these methods are: the equivalent surface method and the radiation efficiency method. In both methods the energy of the equivalent source and the energy of the original source are equal.

The equivalent surface method is described in [7] for rib-stiffened plates with various boundary conditions.

Application of the radiation efficiency method requires evaluation of the radiation efficiency, which relates the radiation of the original plate to the radiation of a piston with equal surface area and mean velocity. When the radiation efficiency and energy of vibration of a plate are known, the acoustic power radiation can be determined. In this paper expressions for the radiation efficiency and acoustic

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power radiation of baffled plates are presented. The main objective of this work is the numerical evaluation of these expressions.

In the low frequency region numerical evaluation of the radiated power of individual modes may be of interrest in order to control the low frequency radiation of a structure. For higher frequencies, techniques as Statistical Energy Analysis are more suitable [5]. Numerical results for modal radiation properties of simply supported plates are presented and compared with results obtained by a statistical technique.

2 Equations for radiated power and radiation efficiency

For a baffled plate vibrating in out of plane motion and placed in a light fluid such as air, the Rayleigh integral gives a relation between the normal velocity distribution $v(\vec{r_s})$ on the plate and the pressure p in an arbitrary point $\vec{r}_p$:

$$p(\vec{r_p}) = \frac{i\omega\rho}{2\pi}\int_S v(\vec{r_s})\frac{e^{-ikR}}{R}dS \tag{1}$$

where $k = \omega/c$ and $R = \mid \vec{r_s} - \vec{r_p} \mid$. Singularities in this integral expression occur when $R = 0$, thus for points $\vec{r_p}$ on the plate's surface. The acoustic intensity of any point $\vec{r_p}$ is defined as

$$I(\vec{r_p}) = \frac{1}{2}Re\{p(\vec{r_p})v^*(\vec{r_p})\} \tag{2}$$

where v^* is the complex conjugate of v. The acoustic power W radiated from the surface is given by

$$W = \int_S I(\vec{r_s})dS \tag{3}$$

Combination of equations (1),(2) and (3) results in an integral expression over the plate's surface for the radiated power

$$W = \frac{\omega\rho}{4\pi}\int_{S'}\int_S v(\vec{r_s})\frac{sinkR}{R}v^*(\vec{r_s'})dSdS' \tag{4}$$

The singularities in the integral expression have disappeared since $sinkR/R \rightarrow k$ for $R \rightarrow 0$. Once the radiated power is known, the radiation efficiency can be calculated from

$$\sigma = \frac{W}{\rho c S < v^2 >} = \frac{W}{\frac{1}{2}\rho c \int_S Re\{v(\vec{r_s})v^*(\vec{r_s})\}dS} \tag{5}$$

3 Numerical implementation

The radiated power can be calculated by dividing the plate's surface in a number (N) of quadrilateral 4-noded elements. Assuming a linear velocity field within the elements, the double integral can be carried out with 1-point Gauss quadrature, which is sufficient for a linear velocity field. Equation (4) can then be written as

$$W = \frac{4\omega\rho}{\pi}\sum_{i=1}^{N}\sum_{j=1}^{N} v_i(0,0)|D_i(0,0)||D_j(0,0)|\frac{sinkR_{ij}}{R_{ij}}v_j^*(0,0) \tag{6}$$

where $v_i(0,0)$ is the velocity and $D_i(0,0)$ is the Jacobian in the center of the ith element. For right-sided elements the Jabobians $D_i(0,0)$ are equal to a quarter of the element's surface, thus $D_i = \frac{1}{4}A_i$. The velocities at the element centers are determined by the corner nodes. For a linear velocity distribution

$$v_i(0,0) = \frac{1}{4}\sum_{k=1}^{4} v_k \tag{7}$$

After the radiated power has been evaluated the radiation efficiency can be calculated from equation (5). For the 4-noded quadrilateral elements the radiation efficiency can be approximated by

$$\sigma = \frac{W}{\frac{1}{2}\rho c\int_S Re\{vv^*\}dS} = \frac{W}{\frac{1}{8}\rho c\sum_{i=1}^{N}|D_i(0,0)|Re\{v_i(0,0)v_i^*(0,0)\}} \tag{8}$$

The numerical evaluation of modal power radiation and modal radiation efficiencies can thus be acheived in two steps for each mode using FEM:

- Solving: calculate the required modeshape (nodal velocities) and natural frequency
- Postprocessing: calculate the radiated power and radiation efficiency with equations (6)-(8)

Equations (6)-(8) for linear elements are quite simple, however the double summation can be rather expensive for a high number of elements (proportional to N^2). In the literature [9] it can be found that 6 linear elements per wavelength are required to obtain accurate results. When the modeshapes of a plate are described in terms of modenumber indices m and n in resp. x- and y-direction, the number of required elements can be expressed as $N = 9mn$.

4 Simply supported baffled plates

In this section the equations for radiated power and radiation efficiency for simply supported plates will be derived. These equations can be used for a semi-analytical evaluation of the radiation efficiency. The modeshapes of a simply supported plate can be written as

$$\psi_{mn} = v_{mn} sin\frac{m\pi x}{L_x} sin\frac{n\pi y}{L_y} \tag{9}$$

The radiation efficiency of a mode with modenumbers m and n can be obtained by substitution of (9) into (5)

$$\sigma_{mn} = \frac{W_{mn}}{\frac{1}{8}\rho c L_x L_y |v_{mn}|^2} \tag{10}$$

Combining equations (4), (9) and (10)

$$\sigma_{mn} = \frac{\omega_{mn}\rho}{\pi c L_x L_y} \int_{S'}\int_{S} sin\frac{m\pi x}{L_x} sin\frac{n\pi y}{L_y} \frac{sinkR}{R} sin\frac{m\pi x'}{L_x} sin\frac{n\pi y'}{L_y} dxdydx'dy' \tag{11}$$

where $R = \{(x-x')^2 + (y-y')^2\}^{\frac{1}{2}}$ and $k = \omega_{mn}/c$ with ω_{mn} the eigenfrequency of mode (m,n). From this equation we may conclude that the modal radiation efficiency of a simply supported plate is dependent of the plate's

- modeshapes (m,n)
- natural frequencies corresponding to each modeshape (m,n)
- area

These conclusions are valid for lightly damped structures. When modes do not radiate independently, the modal radiation efficiency is also dependent of the coupling between modes [2]. Then, equation (8) is more appropriate, using e.g. FEM to determine the modeshapes. Assuming modeshapes of form (9) the radiation efficiency can be determined once the natural frequencies have been determined. For various configurations such as isotropic, sandwich and rib-stiffened plates [1] these natural frequencies can be derived analytically. Semi-analytical evaluation of the radiation efficiency can then be performed in 2 steps for each modeshape:

- analytical: determine the natural frequency of modeshape (m,n)
- numerical: calculate the radiation efficiency (11) with 1-point Gauss quadrature

When the structure is excited by a harmonic point force with amplitude $|F|$ at location $\vec{\xi}$, the energy of vibration can be determined as [4]

$$E(\omega) = \frac{1}{2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{|F|^2 \phi_{mn}^2(\vec{\xi})\omega_{mn}^2}{(\omega_{mn}^2 - \omega^2)^2 + \omega_{mn}^4 \eta^2} \tag{12}$$

where ω is the excitation frequency, η is the structural damping and $\phi_{mn} = \sqrt{4/M} sin(m\pi x/L_x) sin(n\pi y/L_y)$ in case of a plate. When the structure is lightly damped, the energy of vibration of a modeshape (m,n) can be approximated by

$$E_{mn} = \frac{1}{2}\frac{|F|^2 \phi_{mn}^2(\vec{\xi})}{\omega_{mn}^2 \eta^2} \tag{13}$$

The average response $< v^2 >$ is related to this energy by $< v^2 >_{mn} = E_{mn}/M$ with M the mass of the plate. Then from equation (5) the radiated power of a modeshape can be determined as

$$W_{mn} = \rho c L_x L_y \sigma_{mn} \frac{E_{mn}}{M} \tag{14}$$

5 Statistical methods

Previous kind of analysis is known as deterministic vibration analysis: the analysis determines radiation efficiencies and power radiation of modeshapes at specific frequencies. When we are interested in vibrational behaviour at higher frequencies, it is more appropriate to use statistical methods such as Statistical Energy Analysis (SEA) [5] or Structural Acoustic Optimization (SAO) [3]. These methods describe vibrational behaviour in terms of space and frequency averaged variables (energy). Instead of determining a radiation efficiency at a specific frequency, SEA and SAO determine the average radiation efficiency in a frequency band. As an illustration we consider the case of point excitated plates. The average response of a structure within a frequency band can be determined by averaging equation (12) over a frequency band. The average response can then be expressed as [4]

$$< v^2 > = \frac{|F|^2 \pi}{4M\eta\omega}\frac{\Delta N}{\Delta\omega} \tag{15}$$

where $\Delta N/\Delta\omega$ is the modal density of the structure. For a plate

$$\frac{\Delta N}{\Delta\omega} = \frac{L_x L_y}{4\pi}\sqrt{\frac{12\rho_p(1-\nu)}{Eh^2}} \tag{16}$$

where L_x and L_y are the plate's lengths in x- and y-direction, ρ_p the density, ν Poisson's constant, E Young's modulus and h the thickness.

An expression for the frequency averaged radiation efficiency has been determined by Maidanik [6]. This result has generally been accepted and has its application in various SEA software packages. The radiation efficiency of a flat simply supported plate can be summarized by

$$\sigma_{rad} = \frac{2}{\pi}\frac{L_s}{A_p}\frac{k_0^2}{k_p^3\left(1+\frac{\pi}{2}\frac{k_0^2}{k_p^2}\right)} + \frac{1}{\sqrt{\left(\frac{k_p^2}{k_0^2}-1\right)^2\left(\frac{\pi k_p^4}{k_0^2}+1\right)^2+\frac{2\pi}{k_p\sqrt{A_p}}}} \tag{17}$$

where L_s is the total length of boundaries and stiffeners, A_p is the area of the plate, k_p the plate's bending wavenumber and k_0 the wavenumber of the air.

After $< v^2 >$ and σ are known for a frequency range, the radiated power can be calculated from equation (5). In the next section, the numerical method will be compared with frequency averaged results.

6 Numerical Results

6.1 Radiation efficiency of an aluminium plate

For an aluminium plate with $L_x = L_y = 1m$ the radiation efficiencies have been calculated for thicknesses $h = 0.01m$ and $h = 0.005m$, for the first 49 modes. The plate was divided into 25*25 linear elements. The results are shown in figure 1a and b. For the plate of thickness $h = 0.01m$ the first 49 modes have radiation efficiencies from 0.03 below coincidence upto 1.8 at coincidence. At coincidence the bending wavenumber is equal to the acoustic wavenumber. For modes above coincidence the radiation efficiency converges to unity. The computational results agree very well with the frequency averaged result of Maidanik. The coincidence effect is always exaggerated by the curve of Maidanik. The plate of thickness $h = 0.005m$ shows a great variance in radiation efficiencies. This can be explained by the fact that the first 49 modes are all below coincidence. Below coincidence odd-odd modes are most efficient. These modes are the points around the curve of Maidanik. Modes with even modenumbers do not contribute to the frequency averaged radiation efficiency.

6.2 Steel plate excitated by a harmonic point force

For a steel plate with $L_x = 0.7m$, $L_y = 0.5m$ and thickness $h = 0.0022m$ excitated by a harmonic point force of amplitude $|F| = 1$ at location $x = 0.34m$, $y = 0.24m$ the modal response, radiation efficiency and radiated power have been calculated. Results are shown in figure 2a,b and c. The modal response shows a great variance for different modes. Not all modes are equally excited by the point force near the middle of the plate. Odd modes are most energetic. The frequency averaged result is a good average of the modal energies. At low frequency the variance is large. For higher frequencies the modal energies converge to the frequency averaged curve. After 1/3th octave band averaging this effect is more obvious. The radiation efficiency also shows a great variance. In general only odd-odd modes contribute to the radiation of power below coincidence. This is very clear in the figure of the radiated power. All points above the frequency averaged curve are odd-odd modes.

6.3 Stiffening of the steel plate

The steel plate of the previous section is stiffened by one stiffener in x- and y-direction with rectangular cross-section: $b = 0.004$ and $h = 0.020$. The radiation efficiency and radiated power as calculated are shown in figure 3a and b. The radiation efficiency of the plate increases by stiffening. The modal response however, decreases because of the higher stiffness and mass of the plate. Therefore the radiated power decreases in a frequency band, as shown in figure 3b: the radiated power of the modes decreases and the eigenfrequencies increase. In a band from 0 to 300 Hz where the first four modes of the non-stiffened plate occure, the stiffened plate has only two modes. As shown in [7] the radiated power does not neccesarily decrease for all combinations of stiffeners but has for specific combinations a minimum. Therefore the stiffening parameter can be a good parameter for optimization of acoustic properties of plates.

7 Conclusions

The expressions for the numerical evaluation of radiation efficiencies and power radiation of baffled plates can be used as a postprocessing tool in the Finite Element Method. Radiation efficiencies of complex plate shapes and materials can be determined using this approach in the low frequency region.

The semi-analytical technique can be used as a test for a FEM implementation of the method.

Numerical results for simply supported plates are presented and compared with results obtained by a statistical technique. There is a good agreement between the results of the two techniques.

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Figures

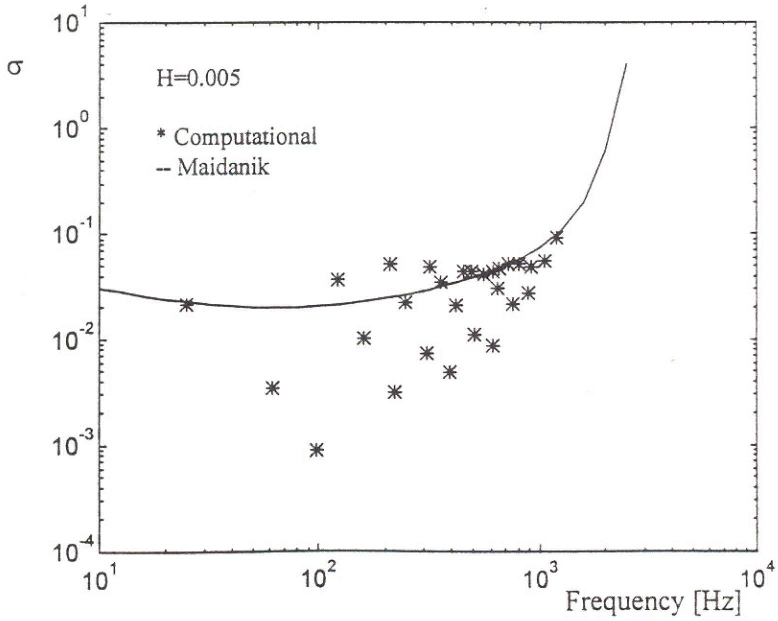

Figure 1a

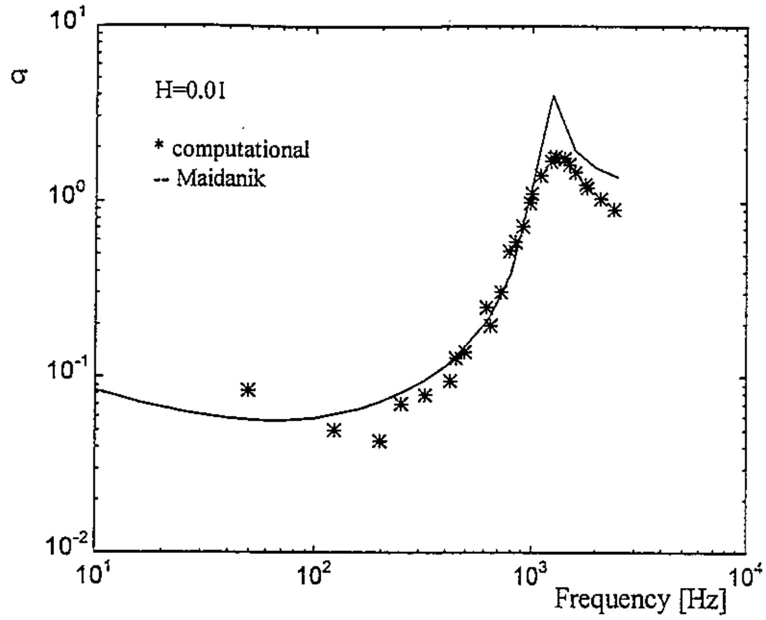

Figure 1b

Figure 1: radiation efficiencies for an aluminium plate of thickness (a) h=0.01 m and (b) h=0.005 m

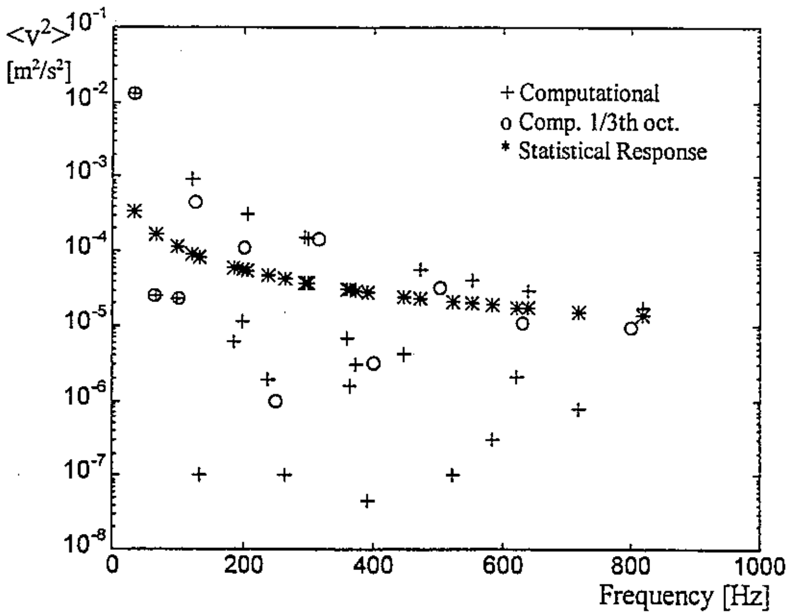

Figure 2a

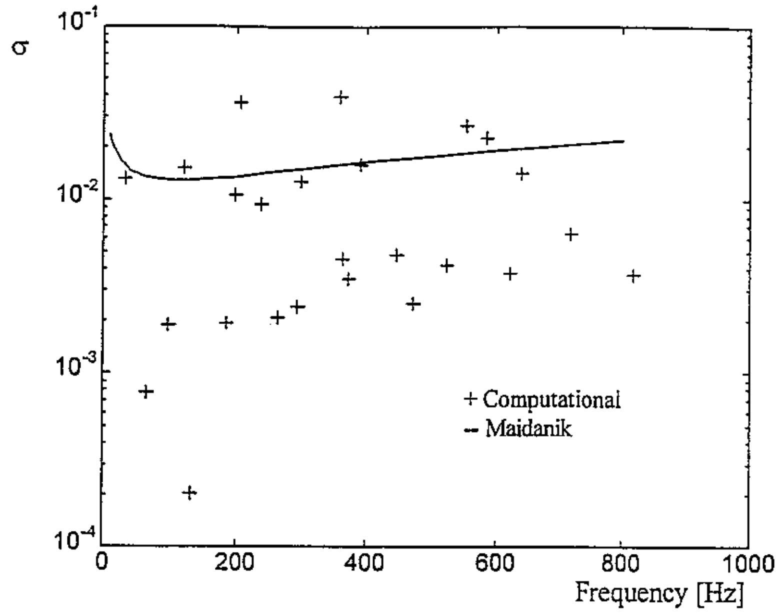

Figure 2b

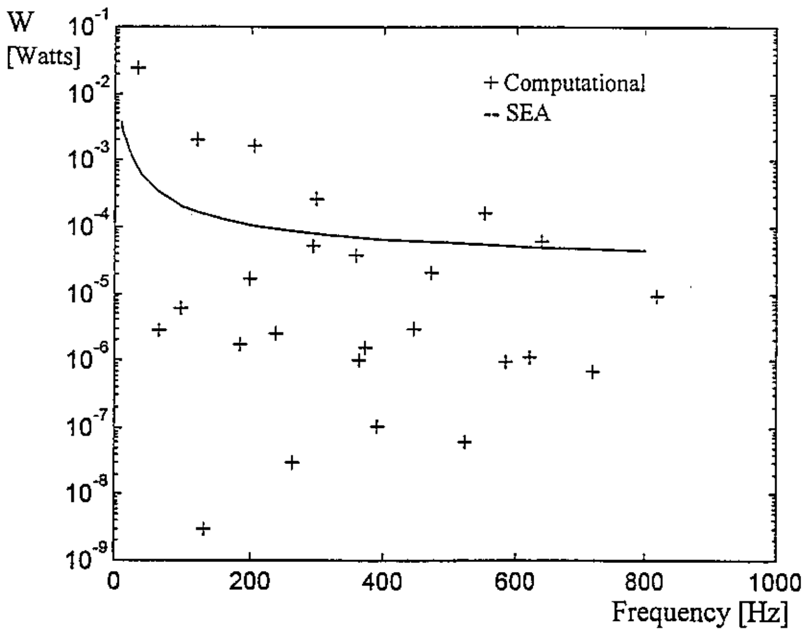

Figure 2c

Figure 2: Steel plate excitated by a point force (a) modal response (b) radiation efficiency (c) radiated power

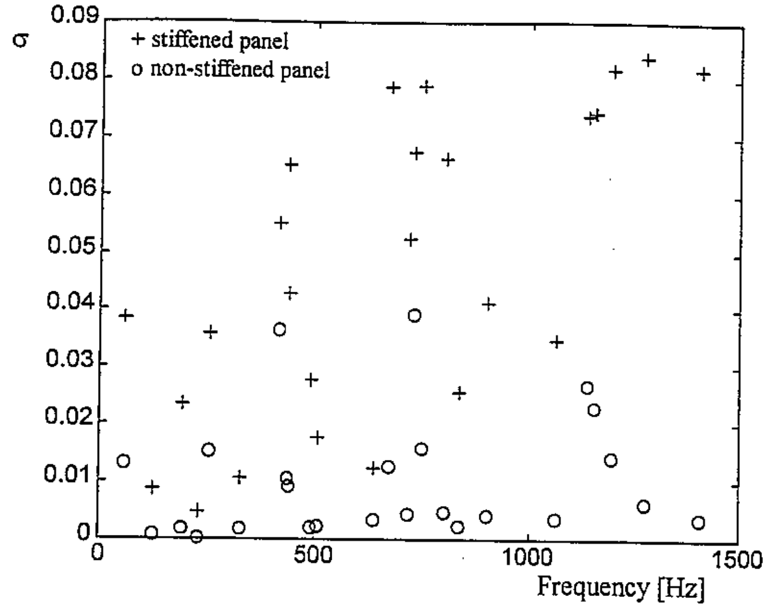

Figure 3a

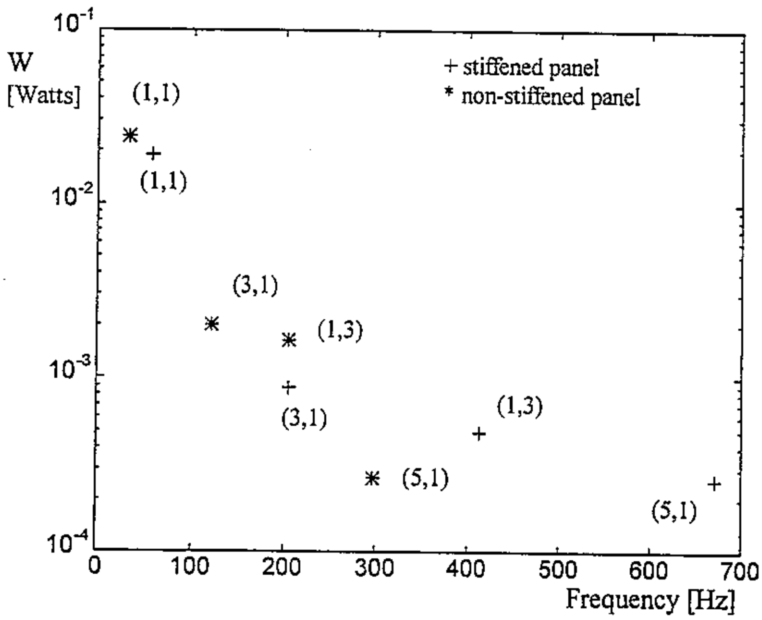

Figure 3b

Figure 3: Acoustic properties of a stiffened and non-stiffened steel plate: (a) radiation efficiency (b) radiated power of the first 4 most radiant modes