A Basic Foundation for Unravelling Quantity Discounts: Gaining more Insight into Supplier Cost Mechanisms

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Summary

Selling organizations often offer quantity discounts schedules, but do not provide the underlying Quantity Discount Price Functions (QDPF). In literature an analysis on how QDPF could be derived from discount schedules is lacking. This is remarkable as QDPF contain useful information for buying organizations. QDPF give more insight into the fixed and variable costs of selling organizations and can be a useful tool for buying organizations in selecting and negotiating processes. Furthermore, QDPF can be used for calculating and allocating price savings in group purchasing and multiple sourcing decisions. In this paper we develop one general QDPF and two related measures for negotiating spaces. We prove that our QDPF gives a highly reliable approximation of 66 quantity discount schedules of different selling organizations. Finally, we compare the QDPF parameters of the 66 schedules and discuss their basic properties.

Educator and practitioner summary

In this paper we develop a general quantity discount price function and two indicators for negotiating spaces. These instruments provide more insight into the fixed and variable costs of selling organizations and can be used **(1)** as a tool in selecting and negotiating processes, and **(2)** to calculate and allocate price savings in multiple sourcing decisions and group purchasing.

Keywords

Quantity Discounts Price Function; Measures for Negotiating Spaces; Group Purchasing

Introduction of the topic

Quantity discounts have been used widespread and discussed for centuries (Elmaghraby and Keskinocak, 2002). Selling organizations often offer quantity discounts schedules. Buying organizations often expect a price break for purchasing large amounts (Nason and Della Bitta, 1983). Specific motivations for quantity discount schedules are:

- Achieving perfect price discrimination against a single customer or a set of homogenous customers (e.g. Buchanen, 1953);
- Achieving partial price discrimination against heterogeneous customers (e.g. Oi, 1971);
- Charging a higher price for the first items sold allows covering fixed costs, while discounts increase efficiency as larger customers are priced closer to marginal costs (Miravete, 1999);
- Influencing the buying organization's ordering pattern to increase the logistics system efficiency and/or to coordinate and lower costs between levels in a distribution channel (e.g.

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Crowther, 1967). It is claimed by Munson and Rosenblatt (1998) that this type of discount is intended to influence the quantity per order, but not the total quantity demanded over the long run. We claim however that a lower price could increase demand given a high price elasticity of demand. Marketing literature also suggests that the optimal reaction of sellers facing a decrease in costs is to pass some of the savings to their customers by lowering prices (Chan, and Qiang Wang, 2003), what could increase demand.

Practitioners encounter different types of discount schedules which can be determined by:

- The number of price breaks of a discount schedule may be one, two (e.g. a price per item of 10 applies to an interval of 1 – 10 items; a price per item of 9 applies to an interval of 11 and more items), multiple, or infinite (Dolan, 1987; Munson and Rosenblatt, 1998). It is claimed by Munson and Rosenblatt (1998) that an infinite number of price breaks represents a continuous discount schedule. We claim however that this is only true if all the price breaks have an interval of only 1 (e.g. a price per item of 11 applies to 1 item; a price per item of 9 applies to 2 items; a price per item of 8 applies to 3 items; et cetera);
- The price breaks may be based on quantities (e.g. a price per item of 10,2 applies to 1 – 10 items; a price per item of 9,6 applies to 11 – 20 items) or prices (e.g. a price per item of 20 applies to 1 – 43 items; a price per item of 15 applies to 44 – 117 items);
- The form may be all-items (the discount applies to all items) or incremental (only items within a price break interval receive that interval's discount) (Munson and Rosenblatt, 1998);
- Item aggregation describes if the discount applies to one or multiple items (bundling items). A business volume discount represents item aggregation where the price breaks are based on the total monetary volume for all products purchased (Munson and Rosenblatt, 1998);
- Finally, time aggregation describes if the discount applies to individual or multiple purchases over a certain time period (Munson and Rosenblatt, 1998).

The body of knowledge on quantity discounts is large, both from the point of view of selling and buying organizations. From the selling organization point of view a great deal has been written about whether or not selling organizations should offer quantity discounts, and if so, what type of quantity schedule they should offer to maximize profits (e.g. Lee and Rosenblatt, 1987). From the buying organization point of view the application of quantity discounts in e.g. EOQ-models (e.g. Viswanthan and Wang, 2003) or inventory ownership problems (e.g. Boyaci and Gallego 2002) has been discussed to a large extent. In addition, several authors have written about among other things the characteristics of quantity discount schedules (e.g. Dolan, 1987).

Research relevance

Academic relevance

Literature is extensive thus; nevertheless, what we do not know is how buying organizations can derive Quantity Discount Price Functions (QDPF) from discount schedules. As mentioned earlier selling organizations often offer quantity discount schedules. However, mostly they do not provide the underlying function used. If they do provide the underlying function this often does not give direct and comparable information about e.g. fixed and variable costs. In literature a thorough analysis on how buying organizations could derive QDPF from discount schedules is lacking. This is remarkable as QDPF contain useful information for buying organizations.

Especially concerning the structure of prices, and prices often are the main basis for purchasing decisions (e.g. Lehmann and O'Shaughnessy 1974).
Our next point of interest concerns the huge gap of research between price elasticity and demand elasticity (Ramsay, 1981). Price elasticity of demand is a concept that is used throughout economics and is based on people doing less of what they want to do as the cost of doing it rises. Price elasticity is defined as $E_d = \frac{\%\ change\ in\ quantity\ demanded}{\%\ change\ in\ price}$ (e.g. Case and Fair, 2003) and has been studied in great detail. Almost all textbooks discussing the principles of economy or marketing include the topic. Several detailed papers describe the price elasticity of different commodities (e.g. Babbel, 1985; Loderer et al., 1991). And a Google search on "price elasticity of demand" even gives 198.000 hits (February 2006), what our statements concerning the popularity of price elasticity confirms.
However, a Google search on "demand elasticity of price" gives just 37 hits. And most of these hits refer to pages that actually give information about price elasticity. Even some academic papers use the definition demand elasticity while they mean price elasticity (e.g. Song and Sumner, 1999; Yan et al., 2001). Demand elasticity is based on quantity discounts and indicates the sensitivity of product and service prices to demanded quantities. A measure for demand elasticity is still lacking, let alone the demand elasticity of different commodities.

Practical relevance

In general, research results to QDPF will be applicable to models incorporating quantity discounts. More specific, QDPF give more insight into the supplier cost mechanism, i.e. its fixed and variable costs. Therefore, it can be a useful tool in the supplier selection and negotiation process for buying organizations. Different discount schedules of different suppliers can be easily compared in the selection process while using one QDPF. In addition, QDPF can be used in the negotiation process for determining negotiating spaces and for calculating discounts for deviant quantities. The latter accounts to multiple sourcing decisions as well. One of the issues in multiple sourcing is how to divide the total quantity between different selling organizations. For all of the deviant quantities of the possible divisions the quantity discount can be calculated and weighed up against spreading risks and opportunities.
The usage and calculation of price savings due to the bundling of needs within organizations can be improved by QDPF as well. The same accounts to the bundling of needs with other organizations, i.e. group purchasing. In group purchasing, QDPF can be used to improve calculating and allocating purchasing price savings. Here the total cooperative price paid to the selling organization is known. However, the prices for the buying organizations if they had purchased alone is often unknown. This information is necessary for calculating and allocating price savings for certain allocation methods. Finally, some of the properties of quantity discount schedules play an important role in the fairness of allocation methods for allocating cooperative gains in e.g. purchasing consortia and business unit group purchasing (Schotanus, 2005).

Research objectives

Given the research relevance our main analytical objective is to describe one general QDPF defined by a limited number of parameters. In addition, we aim to analyze its basic properties and to develop related indicators for negotiating spaces. It should be uncomplicated for buying

organizations to derive these parameters from all different types of quantity discount schedules they encounter in practice. Intensive instruments as supplier auditing could provide the same information, but are usually very labour-intensive, both for the selling and the buying organization.
Our main empirical objective is to test the reliability of this general QDPF for different types of quantity discount schedules. In addition, we aim to develop and test several hypotheses.
Our third and final objective is the most ambitious one. It is to build a basic foundation for more research to unravelling quantity discounts. If the applications of our research results are proven to be useful to practitioners a new research line could be set up. This research line could unravel typical QDPF parameter and indicator values for typical product and service groups with typical procurement strategies. These values could serve as guidelines for buying organizations in selecting and negotiating processes as we aim to discuss shortly in our findings section. A similar line of research already exists in the price elasticity literature as mentioned in the research relevance section. To build a basic foundation we aim to develop several propositions related to the general QDPF. In addition, we aim to describe some general estimates of QDPF parameter and indicator values for several commodities.

Analytical findings

In this section we aim to achieve our first objective and partly our third objective. First, we substantiate and develop a general QDPF and its related total cost function. A QDPF gives the price per item for a certain quantity. The total costs function multiplies the price with the quantity. Second and third, we develop two indicators for measuring negotiating spaces. Finally, we discuss the demand elasticity of price.

The QDPF defined

To achieve our first objective our main choice is to use a continuous QDPF to approximate (gradual) discount schedules. A continuous type can approximate all different types of discount schedules mentioned in the introduction. However, in practice usually graduated prices are used to establish quantity discounts (Munson and Rosenblatt, 1998). For instance, a price of 400 applies to 50–99 items, and a price of 390 applies to 100–199 items. Nonetheless, when a buying organization needs 95 items it could order 100 items, or otherwise could negotiate a lower price than 400. The selling organization could be able to offer a lower price due to economies of scale, which may affect fixed and variable (transaction) costs. A continuous price function represents this process better than graduated functions.
Prices and lot sizes are mostly determined through negotiations (e.g. Munson and Rosenblatt, 1998). Graduated price functions do not incorporate this flexibility. Note that most of the EOQ-models consider quantity discount schedules as a given and do not consider negotiating possibilities. In some situations this is actually the case as logistical aspects like truck capacities may be a limiting condition. Still, due to negotiations most discount schedules are not rigid in practice. In addition, note that using graduated QDPF could lead to an anomaly (e.g. Arcelus and Rowcroft, 1992; Sethi, 1984). This anomaly concerns the possibility that it can save money by purchasing more items than needed at a discount and throwing the surplus away. Several other researchers proposed and applied continuous price functions in their studies as well (e.g. Spence 1977; Hahn 2003). For all of the reasons above, we use a continuous price function in stead of a graduated price function to make a better approximation of reality.

Our QDPF is based upon the function described by Heijboer (2003) as $p(q) = p_0 \cdot \left(c_1 + \frac{c_2}{\sqrt{q}} \right)$, which can be rewritten as $p(q) = p_0 c_1 + \frac{p_0 c_2}{\sqrt{q}}$. Here $p_0 \cdot c_1$ represents the minimum price and c_2 is used to further scale the price function *p(q)* for quantity *q*. In our QDPF we introduce one new parameter η which represents the theoretical steepness. We introduce this parameter to be able to make better estimates of different types of quantity discount schedules with different types of steepness, as are shown in figure 1.

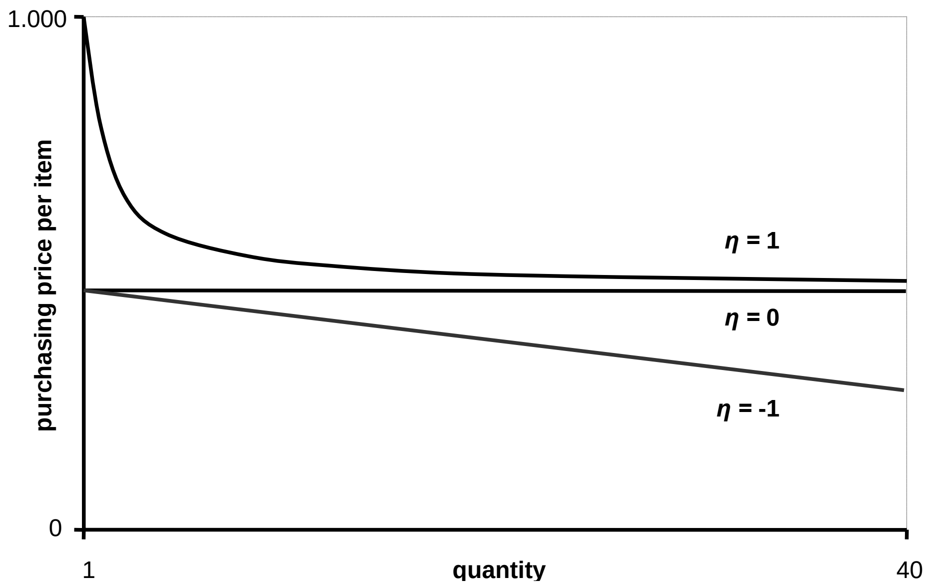

Figure 1. Examples of Different Quantity Discount Schedules and Related Purchasing Prices

The schedules shown in figure 1 are deduced from schedules found in practice by our QDPF:

$$\text{QDPF} = \text{fixed amount} \pm \text{variable amount} = p(q) = p_m + \frac{S}{q^{\eta}} \quad \textbf{(1)}$$

Here p_m represents the theoretical minimum price (with $S \geq 0$) or maximum price (with $S < 0$) and *S* represents the theoretical spread of the function. For instance, a positive steepness and $S = 100$ means that the difference between the price per item for purchasing 1 item and the price per item for purchasing an infinite number of items equals 100.
With the three parameters QDPF can be shaped and scaled into two main categories: positive steepness (and $S \geq 0$) and negative steepness (and $S < 0$). For both categories the following restrictions apply in theory:

- $\eta \geq -1$
- $q > 0$
- $p_m > 0$

We apply the first restriction as $\eta < -1$ would lead to a price function that decreases increasingly. The corresponding total costs function $q \cdot \text{QDPF}$ of **(1)** gives information about the ratio between fixed and variable costs of a selling organization. We define the total costs function as:

$$\text{TC} = q \cdot p_m + \frac{S}{q^{\eta-1}} \quad \textbf{(2)}$$

The f-factor defined for positive steepness

Equation **(1)** can be used in an indicator in the negotiation process for quantity discount schedules with a positive price steepness. We name this indicator the f-factor and it can be used as one indicator for the space in which to negotiate. For instance, a f-factor score near 0% means that almost all costs per item are fixed. This could indicate a low negotiating space concerning the purchasing price. We define the f-factor for a certain quantity as:

$$\text{f-factor}(q) = \left(1 - \frac{p_m}{p_m + \frac{S}{q^\eta}}\right) \cdot 100\% \quad \textbf{(3)}$$

The q-factor defined for negative steepness

If for a price function with $-1 \le \eta < 0$ the quantity range given by the selling organization would get extrapolated with our QDPF, then this would finally lead to a negative price. The extrapolated point where $p(q)$ and $q \cdot p(q)$ would become 0 concerns $\text{QDPF} = p_m + \frac{S}{q^\eta} = 0$. Rewriting this equation leads to (see also figure 2):

$$q^* = \left(-\frac{S}{p_m}\right)^{\frac{1}{\eta}} \quad \textbf{(4)}$$

The point after which the total costs $q \cdot p(q)$ would decrease can also be defined. This point q^{**} occurs always before q^* and can be calculated by differentiating $\text{TC} = q \cdot \left(p_m + \frac{S}{q^\eta}\right)$. This gives $\text{TC}' = \frac{-\eta \cdot S}{q^\eta} + \frac{S}{q^\eta} + p_m = q^\eta + (1-\eta) \cdot \frac{p_m}{S} = 0$. Rewriting this equation leads to:

$$q^{**} = \left((-1+\eta) \cdot \frac{S}{p_m}\right)^{\frac{1}{\eta}} \quad \textbf{(5)}$$

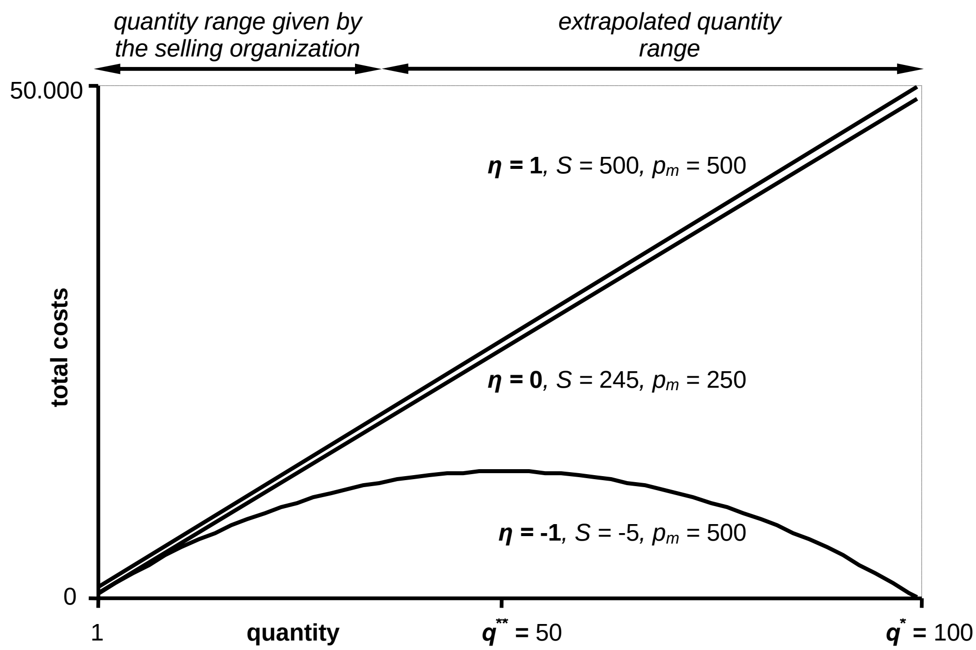

Figure 2. Examples of Different Quantity Discount Schedules and Related Total Costs

Equation **(5)** can be used in an indicator for the negotiation process for discount schedules with a negative steepness. We name this indicator the q-factor and it can be used as one indicator for the space in which to negotiate. For instance, a q-factor score near 1 could indicate a low negotiating space concerning the purchasing price. We define the q-factor for a certain quantity as:

$$\text{q-factor}\left(q\right)=\frac{q^{**}}{q}=\frac{\left(\left(-1+\eta\right)\cdot\frac{S}{p_m}\right)^{\frac{1}{\eta}}}{q} \qquad \textbf{(6)}$$

Demand elasticity of price defined

A measurement for demand elasticity of price as $E_p=\frac{\%\ change\ in\ price}{\%\ change\ in\ quantity\ purchased}$ is of little use, as such a measure could vary at every point along a price curve. To find an alternative measure we could use one or more of the three parameters in our QDPF. The spread and the minimum or maximum price however only scale quantity discounts. The steepness is the main characterizing parameter of quantity discounts. We therefore propose that the best measure for E_p is the steepness η of quantity discounts.

Empirical methodology

We are aware that some researchers proposed more complex quantity discount functions than our QDPF (e.g. Lee and Rosenblatt, 1986; Dada and Srikanth, 1987). However, more complex does not always mean better. To prove that our QDPF is reliable we test if it is able to give a reliable approximation of different discount schedules found in academic papers (Dolan, 1987; Lal and Staelin; 1984), actual offers provided to purchasing groups, and internet stores. The internet price schedules were picked by Google searches on the keyword “quantity discounts” for different products. None of the products had exceptional discounts for e.g. marketing reasons. Some product groups occur more often in our selection than others. As the schedule properties within groups can differ significantly we used all products in our analysis.

All of the different gradual types of discount schedules described in the introduction of this paper are incorporated in the discount schedules analyzed. After we had found all different gradual types we stopped collecting and analyzing new discount schedules, leaving a total number of 66 discount schedules.

When we approximate gradual discount schedules with a continuous QDPF we assume that the price given by the supplier for a certain range applies to the lowest quantity in this range. For instance, when a price of 400 applies to 50-99 items, and a price of 300 applies to 100-199 items, we assume that a price of 400 applies to 50 items and a price of 300 applies to 100 items. As mentioned before, the selling organization does not give the exact prices for 51-99 items, but we assume that while negotiating in most cases a lower price than 400 can be obtained. We approximated the three parameters in our QDPF in MatLab with an exact algorithm and several nonlinear least squares algorithms:

- Gauss-Newton (Dennis and Schnabel, 1983; Foresee and Hagan, 1997);
- Levenberg-Marquardt (Levenberg, 1944; Marquardt, 1963; Moré and Sorensen, 1983);
- Trusted region (Branch et al., 1999; Byrd et al., 1988; Coleman and Verma, 2001; Steihaug, 1983; Sorensen, 1994);

in combination with no method or with the following robust fitting methods:

- Bisquare (DuMouchel and O'Brien, 1989);
- Least absolute residuals (Meyer and Glauber, 1964).

As an exact algorithm exceeds an acceptable calculation time, we used the Levenberg-Marquardt algorithm in combination with the least absolute residuals method. Without going into detail we claim that this combination most frequently gives the most reliable results for our QDPF. Five exceptions occurred where the Levenberg-Marquardt algorithm created a minimum price somewhat smaller than zero for an infinite quantity. Here we applied the trusted region algorithm with lower and upper bounds for the minimum price.

See table 1 and 2 for an example of how we approximate the QDPF parameters from a quantity discount schedule with six price breaks based on one described by Dolan (1987). The discount schedule on its own in table 1 does not provide underlying information. The QDPF values in table 2 do provide useful information for buying organizations.

Quantity Given by Supplier	Price Given by Supplier	Quantity Used for our Approximation	Approximated Price with our QDPF
1.000 – 5.000	$ 50,00	1.000	€ 50,03
5.000 – 10.000	45,40	5.000	43,92
10.000 – 30.000	40,90	10.000	41,62
30.000 – 50.000	38,10	30.000	38,32
50.000 – 200.000	37,05	50.000	36,91
200.000 and more	33,50	200.000	33,49

Note: The R^2 of our approximation equals 0,991, the adjusted R^2 = 0,986, and the Root Mean Squared Error (RMSE) = 0,718. With all prices being multiplied with x, p_m will also be multiplied with x. S and η will stay the same. Multiplying all quantities with x affects only S.

Table 1. Example of an Approximation of a Quantity Discount Schedule

Parameter	Description	Value	Application
$p(1)$ $p(7.500)$	• Theoretical Maximum Price (q = 1) • Deviant Quantity (q = 4.000)	$ 93,50 $ 42,83	• Calculate Deviant Quantity Prices for Supplier Comparisons, Negotiating Spaces, Purchasing Groups, and Multiple Sourcing Decisions
$p_m = p(\infty)$	Theoretical Minimum Price ($q = \infty$)	$ 13,37	• Compare Supplier Offers • Indicate Negotiating Spaces
S	Theoretical Spread	$ 80,13	• Compare Supplier Offers • Indicate Negotiating Spaces
η	Theoretical Steepness	0,113	• Characterization of Quantity Discounts • Indicate Fairness of Certain Allocation Method
f-factor (200.000)	f-factor (q =200.000)	% 60	• Compare Supplier Offers • Indicate Negotiating Spaces
Application for a Purchasing Group in which 2 Partners Pay an Equal Price of $ 50,00 to 1 Supplier: • Savings Partner A with q = 200: 200 · ($p(200)$ – 50,00) • Savings Partner B with q = 800: 800 · ($p(800)$ – 50,00)		 $ 1.460 $ 748	• Allocate Price Savings for Deviant Quantity Prices

Table 2. Examples and Applications of the QDPF Parameters

Empirical findings

In this section we aim to achieve our second objective and partly our third objective. First, we give a description of the data set analyzed. Second, we test and discuss the reliability of our general QDPF for all different types of gradual discount schedules. Third, we describe several QDPF parameter and indicator values of the discount schedules analyzed.

Description of the data set

The basic properties of our data set are described in table 3. The first two columns of table 3 give the properties related to the number of price breaks in the gradual discount schedules analyzed. The table shows that there is quite some variety in the number of price breaks. Note that we did not take schedules into account with only two price breaks.

Number of Price Breaks	Value	$p(q_{min})$ and $p(q_{max})$ Difference	Value
Mean Number	4,91*	Mean Difference	% 24,5*
Median Number	5*	Minimum Difference	% 1,82
Minimum Number	3	Maximum Difference	% 90,1
Maximum Number	11	Standard Deviation	% 21,0*
Standard Deviation	1,65*	Skewness of Distribution	0,82*
Skewness of Distribution	1,18*		
Total Number of Prices	320		

Note: $n = 65$
The price break measures are corrected by removing 2 schedules with an infinite number of breaks
* corrected by removing double products from the same product group

Table 3. Number of Price Breaks and Differences between Maximum and Minimum Prices

The last two columns of table 3 give the properties of our data set regarding the difference between the maximum price $p(q_{\max})$ and minimum price $p(q_{\min})$ in the discount schedules given by the supplier. The difference is formulated as $\frac{p(q_{\max}) - p(q_{\min})}{p(q_{\max})}$. The minimum difference that we found applies to notebooks; the maximum differences apply to printed matter and mobile phoning costs. The table clearly shows that quantity discounts can have a major impact on the total purchasing costs for some product groups.

One could argue that the higher the number of price breaks is, the larger the difference between minimum and maximum prices is. A higher number of price breaks could mean that selling organizations provide prices for a wider range. Thus, we hypothesize:

H_1 *The higher the number of price breaks is, the larger the difference between the minimum and maximum price in a quantity discounts schedule is.*

To test H_1 we carried out a one-way ANOVA test. We found however no significant correlation for out data set ($F = 2{,}114$, $p = 0{,}057$).

Reliability of the QDPF

Using our continuous QDPF we approximated the 66 gradual discount schedules described in the previous section. We aim to achieve our second main objective here, thus we test the following hypothesis:

H_2 *The general QDPF gives reliable approximations for different types of quantity discount schedules.*

Table 4 shows that our price function provides highly reliable approximations for most of the data set. Both the minimum and average R^2 and the adjusted R^2 are very high. The adjusted R^2 is not significantly lower than the R^2. This normally means that no explanatory variable(s) are missing. Three modelled schedules had one outlying point explaining for those cases a larger difference between the R^2 and the adjusted R^2. On first sight H_2 seems to be supported by our data. However, we have to give some marginal comments.

Not all different quantity schedules can be approximated by our continuous price function though. One modelled schedule has a q-factor < 1 for several quantities within the quantity range given by the supplier. We assume that such a modelled schedule is not reliable. Here a q-factor < 1 would mean that the total costs of a tender decrease after a certain point within the quantity range of the supplier. Therefore we removed this schedule from our analysis.

In addition, for five modelled schedules we found a minimum price somewhat smaller than zero for an infinite quantity when we did not use lower bounds for the minimum price. Despite the fact that the six divergent schedules have a high R^2 and adjusted R^2, not all the parameter values found will be useful to buying organizations. Calculating deviant quantities is still possible though.

Measure	Value
Average R^2	0,995
Average Adjusted R^2	0,988
Average RMSE	0,642
Minimum R^2	0,961
Minimum Adjusted R^2	0,913
Maximum RMSE	4,628
Number of Schedules with a $P_m \leq 0$ without Boundaries	5*
Number of Schedules with a q-factor < 1	1**

Note: $n = 65$
* applies to schedules with a positive steepness
** applies to a schedule with a negative steepness

Table 4. Reliability of the QDPF

Note that the remarkably high values for R^2 can partly be explained because there were several discount schedules with just three price breaks. Nevertheless, for the other price breaks we found a very good match as well. Still, one could argue that the higher the number of price breaks is, the lower the value of R^2 will be. This is because it might become more difficult to approximate the prices of all of the different price break points. Therefore we hypothesize:

H_3 *The higher the number of price breaks in a quantity discount schedule is, the lower the reliability of the general QDPF is.*

To test whether or not the number of price breaks significantly influences R^2 we carried out a one-way ANOVA test. We found however no significant correlation for our data set ($F = 1{,}286$, $p = 0{,}270$). Our explanation for the high R^2 of all discount schedules is that most of the discount schedules given by the selling organizations in our sample seem to have a fairly simple underlying basis. In addition, the discount schedules usually have no outliers, but follow a more or less logical line. Regardless of the simple basis of the discount schedules there are numerous differences between its QDPF parameters. We show this in the next section.

The QDPF parameters

In this section we describe some general estimates of QDPF parameter and indicator values for several commodities (see table 5 and 6). Note that all of the values in table 5 and 6 still heavily depend on our relatively small data set. More specific research will be necessary to find more reliable values. Our only objective here is to build a basic foundation for more research.
In table 5 we show some overall estimates to provide an initial indication of the QDPF parameters and indicators and their behaviour. For instance, the steepness of the discount schedules analyzed ranges from -1,00 to 1,60. The schedules analyzed with a positive steepness (40% of the total number of observations) have a mean of 0,58. Schedules with a negative steepness (60% of the observations) have a mean of -0,50. We found five linear schedules (8%) with a steepness of exactly -1,00 and one schedule (2%) with a steepness of exactly 1,00.

Positive Steepness	Value	Negative Steepness	Value
Mean Positive Steepness	0,58	Mean Negative Steepness	-0,50
Minimum Positive Steepness	1,60	Minimum Negative Steepness	-1,00
Maximum Positive Steepness	0,04	Maximum Negative Steepness	-0,03
Standard Deviation	0,43	Standard Deviation	0,30
Skewness of Distribution	0,64	Skewness of Distribution	-0,35
Mean Minimum Price	246	Mean Maximum Price	121
Minimum Minimum Price	0,00	Minimum Maximum Price	0,25
Maximum Minimum Price	2.731	Maximum Maximum Price	1.585
Standard Deviation	710	Standard Deviation	269
Skewness of Distribution	3,31	Skewness of Distribution	4,66
Mean Spread	13.315	Mean Spread	-8,06
Minimum Spread	2,51	Minimum Spread	-63,1
Maximum Spread	121.400	Maximum Spread	-0,00
Standard Deviation	37.360	Standard Deviation	13,8
Skewness of Distribution	2,60	Skewness of Distribution	-2,45
Mean Min/Max Difference	% 38,0	Mean Min/Max Difference	% 25,8
Minimum Min/Max Difference	% 5,92%	Minimum Min/Max Difference	% 1,82
Maximum Min/Max Difference	% 90,1	Maximum Min/Max Difference	% 50,0
Standard Deviation	% 26,7	Standard Deviation	% 15,6
Skewness of Distribution	0,46	Skewness of Distribution	0,34
Mean f-factor	% 35,5	Mean q-factor	109
Minimum f-factor	% 0,00	Minimum f-factor	1,20
Maximum f-factor	% 100	Maximum q-factor	1.537
Standard Deviation	% 36,8	Standard Deviation	288
Skewness of Distribution	0,68	Skewness of Distribution	3,89

Note: n = 26 for positive steepness, n = 39 for negative steepness
The price break measures are corrected by removing 2 schedules with an infinite number of breaks

Tables 5. Positive and Negative Steepness and Related QDPF Parameter and Indicator Values

Quantity discount schedules with a negative steepness for the given range by selling organizations are somewhat peculiar, because extrapolating those leads eventually to negative purchasing prices. To try to explain the existence of negative steepness one could argue that there is a relationship between steepness and the difference between minimum and maximum prices in discount schedules. It could be that negative steepness only exists in discount schedules with a small range concerning the minimum and maximum price in its schedule. This is because if the range would be larger, the q-factor would approach one and eventually negative prices occur. So, for larger ranges concerning the minimum and maximum prices a positive steepness could be found. Therefore, we hypothesize:

H_4 *Quantity discount schedules with a positive steepness have a higher difference between minimum and maximum prices than schedules with a negative steepness.*

With an independent samples t-test we analysed the relationship between the difference between minimum and maximum prices and negative or positive steepness. We assumed the variances of both groups being unequal (Levene's test p = 0,002) and found a significant correlation (t = -2,173, df = 37,060, p = 0,036, 2-tailed) supporting H_4. Price schedules with a positive steepness

(mean = 38,0 %) have a significantly higher difference between minimum and maximum prices than schedules with a negative steepness (mean = 25,8 %). Therefore we assume that discount schedules with a negative steepness only provide prices for relatively low quantities. For a large enough quantity all discount schedules will eventually lead to schedules with a positive steepness. So, discount schedules with a negative steepness cannot reliably be used for extrapolating and calculating prices for larger quantities than given by the supplier's schedule.
By removing one or more price breaks from the positive discount schedules we tested if these would become negative. This happened in a few cases (20%), but most schedules are based for the whole range on a positive steepness.

Commodity	$p(q_{min})$ and $p(q_{max})$ Difference	Std. Dev.	Freq.	Mean Positive Steepness	Std. Dev.	Freq.	Mean Negative Steepness	Std. Dev.
Printed Matter	% 64	% 22	5	0,93	0,23	2	-0,45	0,59
Paper	26	8	7	0,59	0,36			
Advertisements	29	22	3	0,77	0,81	3	-0,71	0,29
Software	50	15	1	0,16		3	-0,43	0,50
Clothing	42	7				3	-0,48	0,43
Computer Hardware	7	5	1	1,40		2	-0,60	0,57
Subscriptions	34	10	1	0,12		2	-0,41	0,12
Mailing Services	13	12				2	-0,45	0,08
Hosting Services	50	0				2	-0,49	0,29
Temporary Employment	16	4				2	-0,37	0,31
Pharmaceuticals	34	19	1	0,41		1	-0,65	
Semiconductors	9	1	2	0,20	0,15			
Mobile Phoning	67	4	2	0,78	0,12			
Entry Tickets	33					1	-1,00	
Food Products	7					1	-0,59	
DVDs and CDs	14					1	-0,36	
Packaging Materials	33		1	0,11				
Other	17	11	2	0,16		14	-0,46	
Total			26	0,58	0,43	39	-0,50	0,30

Table 6. QDPF Parameter Values for Several Commodities

Note that the concept of group purchasing might be interesting to consider for commodities with a high mean positive steepness and/or a high difference between minimum and maximum prices. For instance, the standardized commodities software, subscriptions, and paper in table 6 could be interesting commodities for group purchasing. In these commodity markets bundled purchasing volumes might have a large impact on the purchasing price. Of course limiting conditions of group purchasing have to be taken into account, like trust, commitment, and similar purchasing needs. More research will be necessary to confirm our statements. Therefore, we propose:

P_1 *Group purchasing is profitable in standardized commodity markets with a high mean positive steepness and/or a high difference between the minimum and maximum price in a quantity discounts schedule.*

Note as well that some markets have similar methods to determine discount schedules. In these markets most of the schedules of different selling organizations are alike. Other markets show a different behaviour. Here selling organizations differentiate by offering schedules different from their competitors. This could for instance be the case for the advertisement commodity where the standard deviations are high (see table 6). In such markets it might be interesting to consider a larger number of suppliers in the supplier selection process. In the negotiating process there might be more negotiating space as well. Hence we propose to verify in further research:

P_{2a} *With the general QDPF commodity markets can be characterized by the extent to which selling organizations differentiate by offering more divergent quantity discount schedules.*

P_{2b} *In commodity markets where selling organizations differentiate by offering more divergent quantity discount schedules it is profitable to consider more suppliers in selection processes.*

P_{2c} *In commodity markets where selling organizations differentiate by offering more divergent quantity discount schedules there is more space in which to negotiate.*

Limitations and further research

Due to the general character of this paper there are some limitations concerning our analytical and empirical results. We already discussed some assumptions and limitations in the paper. Our main assumption already discussed is that a continuous QDPF makes a better approximation of reality than graduated QDPF. Further case study research among selling and buying organizations could be carried out to test whether or not this is actually the case. In such case studies it could also be tested whether or not the minimum price of a discount schedule found by our QDPF equals the minimum price of the selling organization's production process. At a larger scale it could also be tested if certain discount schedule types exist that cannot be approximated by our QDPF. In addition, it could be tested how schedule types influence QDPF parameters and indicators. Finally, propositions P_1 and P_2 could be subject to further research to set up a new research line to demand elasticity of price.
Limitations concerning our analytical findings concern among other things the fact that we only considered one continuous type of QDPF. Other equations could be formulated as $p(q) = p_m + S \cdot e^{-\eta \cdot q}$ or $p(q) = \frac{p_m}{q^\gamma} + \frac{S}{q^\eta}$. Another option is to approximate the total costs of a purchase with a TC function. The total costs approximation will become more reliable; the purchasing prices in this function will however become less reliable. Limitations concerning our empirical findings concern among other things the fact that we can only give estimates for the specific QDPF parameters and indicators (see table 5 and 6).

Conclusions

Our first main conclusion is that our continuous Quantity Discount Price Function (QDPF) gives reliable approximations for most different types of quantity discount schedules. Our QDPF consists of three parameters which can be easily derived from all kind of different types of quantity discounts. A simple QDPF tool is available at our website http://www.bbt.utwente.nl/leerstoelen/bbim.
The QDPF parameters and indicators as the f-factor and the q-factor have several applications for buying organizations (see table 1 and 2 for an example). These parameters and indicators provide

information about the cost mechanism of suppliers and can therefore be used in among other things the supplier selection and negotiation process.

More research to the QDPF parameters could provide useful information for purchasing groups concerning indications for profitable commodities. More general, more research to the QDPF parameters could characterize commodity markets. For some characteristic commodity markets it could be interesting to consider a larger number of suppliers in the supplier selection process. In the negotiating process there could be more space in which to negotiate. The QDPF parameters therefore provide a basic foundation for a new research line to unravel demand elasticity of price.

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