Harmonic Enhancement of Single-Bubble Sonoluminescence

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A systematic method to determine the optimal pressure amplitudes to enhance single-bubble sonoluminecence by using several driving frequencies is described. In this way, for example, the ratio of maximum to minimum radius of a 5 μm-radius bubble driven at 26.5 kHz is increased by one order of magnitude, while maintaining both spherical and diffusive stability.

INTRODUCTION

The remarkable phenomenon of single-bubble sonoluminescence [1] – the periodic light emission from a gas bubble driven into pulsation by a sound field – has found a convincing explanation in the work of Lohse and coworkers [2, 3], which agrees very well with the experiments of Refs. [4]. Briefly, the light is due to a weakly ionized plasma that forms in the bubble due to the intense, nearly adiabatic compression of the gas that takes place during the bubble collapse [5]. A further enhancement of the intensity of these phenomena is difficult to achieve since, at high driving amplitudes, the spherical shape becomes unstable, which leads to the destruction of the bubble.

It is the contention of this paper that this difficulty can be controlled by the use of a multi-frequency drive. In the last few years, investigation of the effect of a multi-frequency acoustic drive has begun [6, 7, 8]. In particular, Ref. [6] reports an observed increase by up to 300% of the emitted light intensity; similar results were reported in Ref. [7]. We describe a systematic approach to optimizing the use of more than one frequency in such a way that spherical stability is preserved while the bubble compression ratio $s = R_{Max}/R_{min}$ is greatly increased; here R_{Max} is the maximum radius reached by the bubble in the course of the expansion phase and R_{min} is the subsequent minimum radius at the end of the first catastrophic collapse. The method uses simulated annealing to identify maxima of s. For additional details see Ref. [9].

METHOD

The mathematical model used is that of Ref. [5] which, while simplified, has been shown to describe the phenomenon with sufficient accuracy. The only difference between our model and that of Ref. [5] consists in the fact that we include a second harmonic in the acoustic pressure $P_A(t)$:

$$P_A = p_1 \cos \omega_1 t + p_2 \cos \omega_2 t + q_2 \sin \omega_2 t\,. \qquad (1)$$

Here the time origin has been chosen in such a way that $q_1 = 0$. Our objective is to choose the individual amplitudes p_1, p_2, q_2 in such a way as to maximize the ratio s under the constraint that the bubble maintains shape and diffusional stability; we take $\omega_2 = 2\omega_1$, with ω_1 prescribed, for simplicity. The special form (1) is chosen as an example: our method is readily extended to more frequencies, of course at the price of a greater computational effort.

An exploratory calculation readily shows that, for a given bubble equilibrium radius R_0, the compression ratio $s(p_1, p_2, q_2)$ possesses a great many points of relative maximum and minimum, which renders the more straightforward optimization algorithms ineffective. We use simulated annealing, as described in Ref. [10], which allows the search process to escape local extrema.

We start with a set of values for the amplitudes p_1, p_2, q_2 in the shape-stable region and calculate the corresponding value of s. A random number generator produces a new neighboring set of amplitudes p_1', p_2', q_2' which is used to calculate a new s'. Stability of the prolate-oblate distortion amplitude a_2 of the bubble [11] is tested by calculating its Floquet multipliers [2]. If the bubble is predicted to be unstable, a new set p_1'', p_2'', q_2'' is generated and shape stability tested again. If instead the original set p_1', p_2', q_2' corresponds to shape-stable conditions, s' is compared with s: if $s' > s$, the set p_1, p_2, q_2 is replaced by p_1', p_2', q_2' and the process repeated. If s' is smaller than s, then the set p_1', p_2', q_2' is accepted with a probability $\exp[(s'-s)/T]$, where T is a pseudo-temperature that is gradually decreased as the iterations converge to the desired maximum. The process is stopped when the last 10-20 values of s tested are very close to each other. While we have no proof that our results correspond to the absolute maximum, we have been able to considerably increase the value of s over what is achievable with a single-frequency drive.

When the maximum s has been found, we check that the higher-order shape modes a_n, with n up to 5, are stable

Table 1. Compression ratio $s = R_{Max}/R_{min}$ and pressure amplitudes p_1, p_2, q_2 for the optimal two-frequency drive; the corresponding values for the single-frequency drive are shown in parentheses.

R_0 (μm)	s	p_1 (kPa)	p_2 (kPa)	q_2 (kPa)
2.5	612	319	-64.9	-24.6
	(359)	(305)	–	–
3.5	321	255	23.9	-225
	(132)	(162)	–	–
4.5	190	185	-9.62	-135
	(10.5)	(107)	–	–
5.5	45.8	43.4	-21.1	-112
	(3.64)	(97.1)	–	–

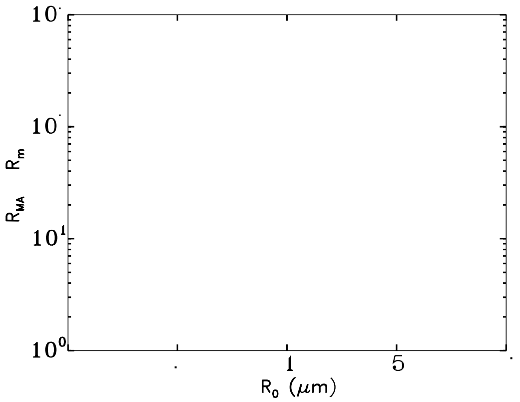

FIGURE 1. Ratio of the maximum bubble radius to the subsequent minimum as a function of the equilibrium radius for single-frequency drive (dashed line) and optimal multi-frequency drive; the corresponding numerical values are given in Table 1.

as well. We also calculate the concentration of dissolved gas with which the bubble would be in diffusional equilibrium [2] and we check that diffusional stability prevails.

RESULTS

In order to demonstrate the results that are obtainable by the present method we consider a specific example in which the fundamental frequency $\omega_1/2\pi$ is taken as 26.5 kHz, which is of the order of the frequency used in much of the experimental work (see e.g. Refs. [1, 4, 13]). The maximum compression ratio achievable with a dual-frequency drive are compared in Table 1 with the single-frequency drive ones (in parentheses) for an argon-water system at standard temperature and pressure. The same information is summarized in graphical form in Fig. 1, in which the dotted line shows the single-frequency results and the solid line the multi-frequency ones. It is apparent that, for the larger bubbles, the optimum compression ratio found with the two-frequency drive can be an order of magnitude higher than with a single frequency. Much smaller bubbles are inherently more shape-stable and, accordingly, the improvement found by the use of additional frequencies is not as dramatic.

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