Viscoplastic Regularization of Local Damage Models: A Latent Solution

M. S. Niazi[1, a], H. H. Wisselink[1, b] and T. Meinders[2, c]

[1]Materials innovation institute, P.O. Box. 5008, 2600 GA Delft, The Netherlands

[2]Faculty of Engg. Tech., University of Twente, 7500 AE Enschede, The Netherlands

[a]M.Niazi@m2i.nl, [b]H.Wisselink@m2i.nl, [c]V.T.Meinders@utwente.nl

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Abstract. Local damage models are known to produce pathological mesh dependence in finite element simulations. The solution is to either use a regularization technique or to adopt a non-local damage model. Viscoplasticity is one technique which can regularize the mesh dependence of local damage model by incorporating a physical phenomenon in the constitutive model i.e. rate effects. A detailed numerical study of viscoplastic regularization is carried out in this work. Two case studies were considered i.e. a bar with shear loading and a sheet metal under tensile loading. The influence of hardening / softening parameters, prescribed deformation rate and mesh size on the regularization was studied. It was found that the primary viscoplastic length scale is a function of hardening and softening parameters but does not depend upon the deformation rate. Mesh dependency appeared at higher damage values. This mesh dependence can be reduced by mesh refinement in the localized region and also by increasing the deformation rates. The viscoplastic regularization was successfully used with a local anisotropic damage model to predict failure in a cross die drawing process with the actual physical process parameters.

Introduction

Local continuum damage models are strain softening models which obey the principle of local action i.e. the constitutive behavior at a local point does not depend upon any action/variables at a distance (neighboring points). The problem with the local behavior of a softening model is that as soon as a material point starts to soften, it takes up all the deformation. Since all deformation occurs at the material point, damage only grows in that point, resulting in further softening. If damage models are included in the constitutive behavior in a finite element analysis, then after localization all the deformation accumulates in one element (or one row of elements). This makes the analysis mesh size dependent. If the mesh size is selected to be infinitely small then the energy dissipated in the localized band will vanish [1]. This is purely a numerical artifact. In reality, damage does spread over a finite length scale.

One method, to overcome the problem of mesh dependency in local damage models, is to include rate effects in the constitutive model (i.e. viscoplasticity). The basic viscoplastic regularization mechanism can be explained as follows; when the deformation rate starts to increase in the softer element (the element with damage), the increase in strain rate makes the element stiffer again. This phenomenon prohibits the deformation to accumulate in one element. Needleman [2], first showed with a simple one dimensional case study that rate dependency can remove the pathological mesh dependency in strain softening boundary value problems by making the governing equations well posed. Viscoplastic regularization was also used by Sluys et al. specifically for dynamic problems [3,4]. Since there is not much literature available for viscoplastic regularization of static problems, this writing is dedicated to static problems i.e. the inertial effects are not considered. Two length scales are identified in this study; primary and secondary length scale. The primary length scale is the length over which damage spreads out whereas the secondary length scale is a measure of the width of the peak of the damage distribution. Fig. 1 shows the schematic of the two length scales. Viscoplasticity can be introduced in the constitutive model either in the form of an over-stress viscoplasticity model (i.e. Perzyna and Duvaut-Lions models) or by using a consistency model. Wang showed that the results obtained from the consistency model are as good as the results from the over-stress models [5]. The consistency model is used in this study. Two entirely different types of strain rate hardening models are selected in this study; the Power hardening model and the Extended Bergström hardening model. The Power hardening law is a phenomenological model and multiplies the strain hardening part with a strain rate hardening factor. The Extended Bergström hardening law is a physically based model and adds a strain rate hardening part to the strain hardening part [6]. The strain softening / damage model plays also an important role in the regularization effect. Actually it is the combined hardening /softening parameters that set the viscoplastic length scale. The modified Lemaitre's anisotropic local damage model is used in this study. The major modification to Lemaitre's damage model is that the damage parameters are made a function of strain rate. This makes the softening due to damage also rate dependent. The details of the damage model can be found in [7].

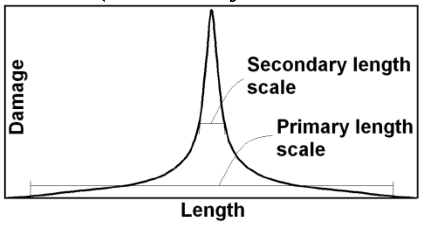


Fig. 1: Schematic for the two viscoplastic length scales

Material Models

The modified Lemaitre's anisotropic damage model [7] was used in this study. The evolution law for the modified Lemaitre's anisotropic damage model is given by Eq. 1.

$$\dot{D}_{ij} = \left(\frac{\tilde{Y}}{S}\right)^{s} \left|\dot{\varepsilon}^{p}\right|_{ij} \text{ from } \varepsilon_I^p > \varepsilon_D^p \text{ up to } D_I \le D_c \tag{1}$$

where $\boldsymbol{D}$ is the second order damage tensor, ε^p is the plastic strain tensor, |..| applied to a tensor means the absolute value in terms of the principal components, S and s are phenomenological damage parameters and $\tilde{Y}$ is the effective damage energy release rate. Damage begins when the maximum principal plastic strain ε_I^p reaches a threshold value ε_D^p. A meso crack is initiated when the maximum principal damage component D_I reaches a critical damage value D_c. The damage parameters s and D_c are made a function of the strain rate. The parameter S is defined as a function of the triaxiality. Further details of the model can be found in [7].

The Power hardening law is given by Eq. 2.

$$\sigma_f = C\left(\varepsilon_{eq}^p + \varepsilon_0\right)^n \left(\frac{\dot{\varepsilon}_{eq}^p}{\varepsilon_0'}\right)^m \tag{2}$$

where σ_f is the flow stress, C, ε_0, n, ε_0' and m are the model's phenomenological parameters. By defining the value $m = 0$, Swift law for strain hardening is obtained.

The Extended Bergström model [6,7] is given in Eq. 3.

$$\sigma_f = \sigma_{f0} + d\sigma_m \left(\beta_v\left(\varepsilon_{eq}^p + \varepsilon_0\right) + \left[1 - \mathrm{e}^{-\omega\left(\varepsilon_{eq}^p + \varepsilon_0\right)}\right]^n\right) + \sigma_{v0}\left(1 + \frac{kT}{\Delta G_0} \ln \frac{\dot{\varepsilon}_{eq}^p}{\varepsilon_0'}\right)^m \tag{3}$$

where σ_f is the flow stress, σ_{f0}, $d\sigma_m$, β_v, ε_0, ω, and n are the strain hardening parameters whereas σ_{v0}, ΔG_0, ε'_0 and m are the strain rate hardening parameters. The Extended Bergström model degenerates to a linear hardening model by setting some parameters to 0 and others to 1.

Case Studies

Two numerical tests for regularization are studied. The first case is a simple one dimensional shear test. This example was extensively used by Wang and Sluys, [3,4,5]. It is interesting to study this case because in this problem localization will occur only due to damage development. The second case is a tensile test with significant diffuse necking before failure. This example is also selected from the work of Wang and Sluys [8]. They showed that viscoplasticity was not effective in this case, rather it only delays the onset of localization. The conclusion from their work was the motivation to choose this case study. Wang and Sluys considered these examples as dynamic problems, on contrary, in this study both the cases are taken as static problems.

One Dimensional Shear Test. Fig. 2 shows the dimensions of the 1D bar used in this study. The bar is fixed at one end and the vertical (y) displacement is prescribed at the other end. All nodes are fixed in horizontal (x) direction. The thickness of the bar varies linearly from 1mm at the free end to 0.95mm at the fixed end. The non-uniform thickness will act as an imperfection and the bar will localize at the fixed end. Three mesh sizes were used in this study i.e. 20, 40 and 80 elements over the length. Plane strain elements with 4 integration points were selected. The (y) velocity was set to 1mm/sec at the free end. The material parameters for Eq. 1, 2 and 3 are selected in such a way to get localization at the early stage of the deformation process. Simulations without strain rate hardening were also carried out along with the simulations using Power and Extended Bergström strain rate hardening. The value of '*m*' in the Power hardening law (Eq. 2) was set to 0 to eliminate the strain rate hardening effect. Fig. 3 shows the mesh shape for different mesh sizes after a certain amount of deformation. The simulation without any strain rate sensitivity clearly shows a mesh dependent shape. On the other hand for Power hardening and Extended Bergström hardening, the mesh shapes are size independent, at least up to the corresponding displacement mentioned in the Figure. Fig. 4 shows the evolution of damage for the simulation with 80 elements and Power strain

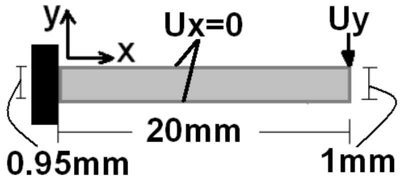


Fig. 2: 1D shear test model.

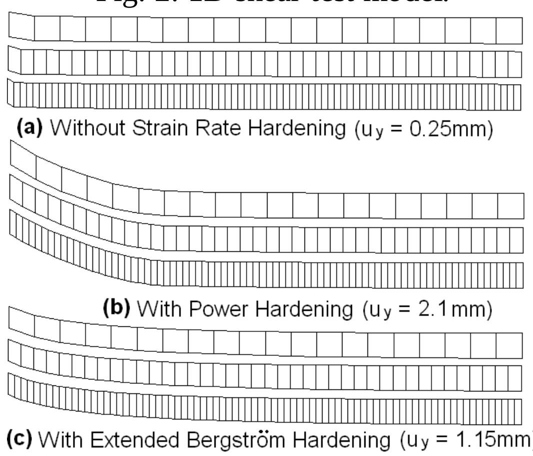


Fig. 3: Mesh comparison for 20, 40 and 80 elements with different hardening laws.

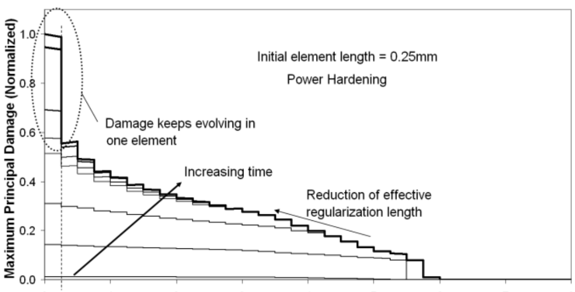
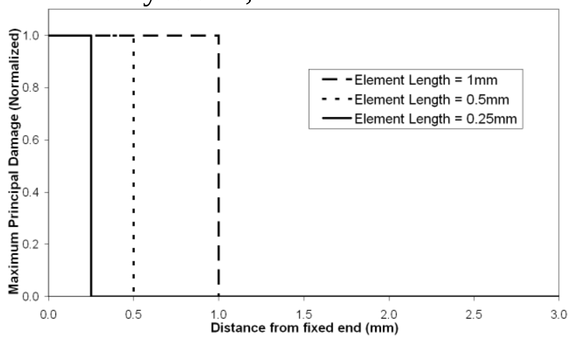
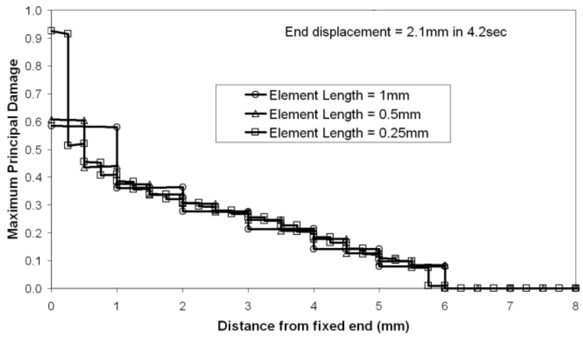


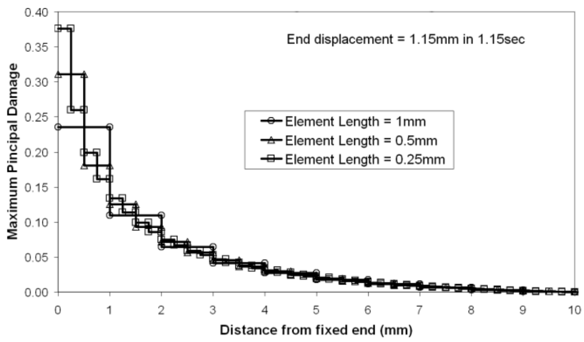
Fig. 4: Damage evolution in the 80 element mesh model with Power hardening.



(a) Without strain rate hardening.



(b) Power strain rate hardening.



(c) Extended Bergström strain rate hardening.

Fig. 5: Damage distribution along the bar length at failure.

rate hardening. It can be observed that as the accumulated damage value increases, the length, up to which regularization is effective, decreases. This makes the damage softening more dominant in comparison to strain rate hardening. Thus the length over which the viscoplastic regularization is effective, keeps reducing with increasing amount of damage. This effective regularization length can not get smaller than the element length in the localization region. Therefore at a certain damage level the deformation /damage starts to accumulate in one element. This makes the results in the end, mesh dependent. This is a normal finite element mesh dependency. This problem can only be avoided if an infinitely small mesh size is used. It is worth mentioning that the phenomenon of reduction of effective regularization length is closely related to reality (the formation of a meso-crack). At the instant just before a meso-crack is initiated, it is expected that all the deformation will occur in an infinitely small length. Fig. 5 represents the damage distribution along the bar length for the three mesh sizes with the three hardening models. Fig. 5(a) shows the distribution of the normalized damage value (D_I/D_{Imax}) along the bar length for the simulations without strain rate hardening. When damage begins in the element at the fixed end, it becomes softer in comparison to the other elements in the bar. All of the deformation accumulates in this element while the rest of the elements unload elastically. Reducing the element size reduces the length in which damage grows. A length scale, independent of the mesh size, can be observed in both Fig. 5(b) and (c). A primary viscoplastic length scale of approximately 6mm was observed for Power hardening and 9mm for Extended Bergström hardening. The length scale itself can be increased or decreased by modifying the hardening parameters. Despite of the fact that the primary length scale is the same for all three mesh sizes, the damage level at the very fixed end is highly dependent upon the mesh size. This is clearly evident in both Fig. 5(b) and (c). High damage values are obtained for shorter element lengths. As mentioned earlier, this is the normal, and expected, finite element mesh dependency. In the coarse mesh the damage is averaged over 1mm (element length) whereas in the fine mesh it is averaged over 0.25mm (element length). The damage gradient within the element is almost zero. This is due to the fact that x-displacement is fixed in the model, which gives almost a constant shear strain increment in one element. The shape of the damage distribution is dependent on the strain rate hardening model.

Tensile Loading Test. Wang and Sluys [8] showed that viscoplastic regularization is not very effective for a tensile loading case on sheet metal, especially if there is a considerable post localization strain before failure. The aim of this case study is to check if the mesh dependency, which Wang observed [8], is the pathological mesh dependence of local damage models or if it is the normal finite element mesh dependency (as observed in the Section 'One Dimensional Shear Test'). If the latter is true, then refinement of the mesh must be a solution to the problem. The dimensions of the model are shown in Fig. 6. Refinement was carried out only in the central 12mm region. Four mesh sizes were used. In each refinement level, the element length is reduced to half. Four elements were taken through thickness in all cases. Only the Extended Bergström strain rate hardening model was used in this case study. Simulations with four different viscosities at two different strain rates were performed. The strain rate hardening parameters were adjusted to keep the initial yield stress the same for all cases. Fig. 7 shows the maximum principal damage distribution along the central axis of the tensile sheet for different mesh sizes.

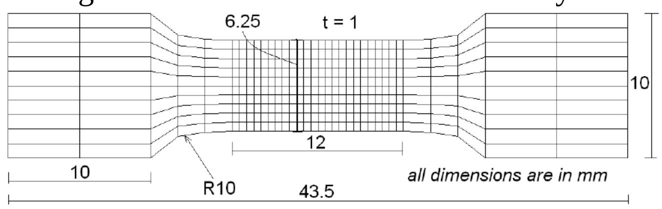
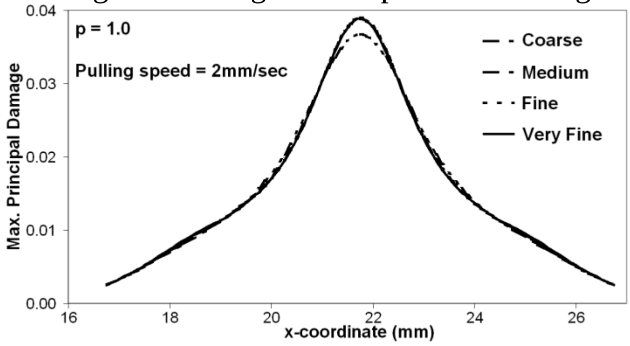
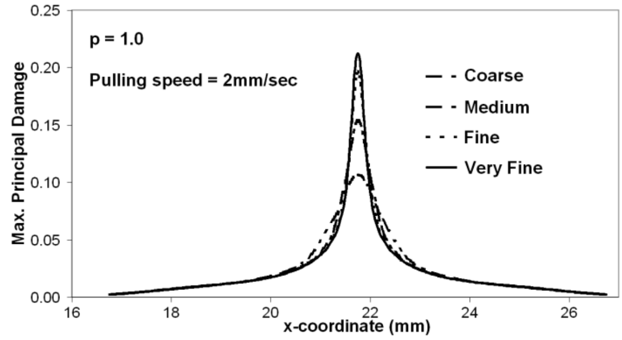


Fig. 6: Finite element model for the tensile loading case.

At lower damage values i.e. Fig. 7(a), the distributions are completely mesh independent. In Fig. 7(b), the primary viscoplastic length scale is the same for all mesh sizes, only the maximum damage values in the peaks are different. This difference is inevitable due to the finite size of the elements. It can only be reduced by further mesh refinement. The distribution converges with mesh refinement. This shows that the mesh dependency in this case is not the pathological mesh dependence. It can be observed from Fig. 8 that the onset of localization is delayed by increasing the viscosity. It can also be observed that the force displacement curves converge upon refinement except for the one without strain rate hardening. Fig. 9 shows that increasing the viscosity increases the length scale. Fig. 10 compares the damage distributions obtained at the same deformation level



(a) Total Displacement = 3.0mm



(b) Total displacement = 4.0mm

Fig. 7: Damage distribution at a fixed strain rate and viscosity.

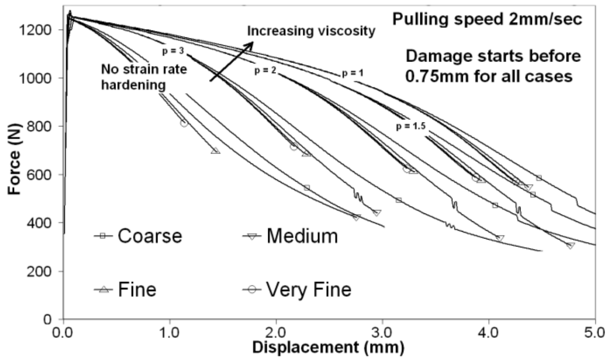


Fig. 8: Force displacement curves with varying viscosities and mesh sizes.

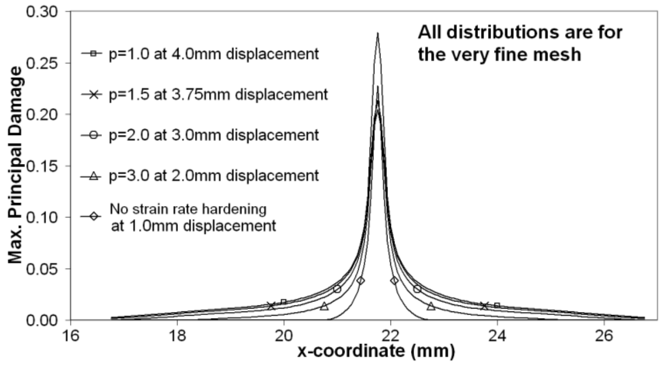
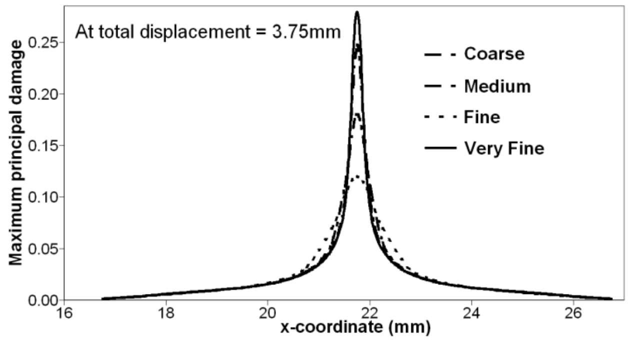
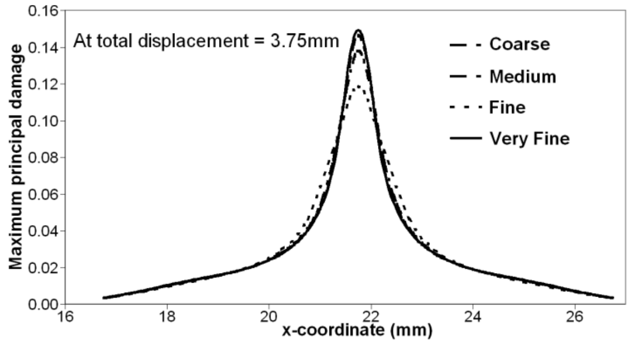


Fig. 9: Damage distribution along the loading direction with varying viscosities.



(a) Pulling speed = 2mm/sec



(b) Pulling speed = 16mm/sec

Fig.10: Damage distribution for two different strain rates.

in the tensile simulations at two different pulling speeds. It can be observed from Fig. 10(a) and (b) that the primary length scale is not affected by the change in the strain rate. Lower pulling speed produces a larger spread in the damage peak values, Fig. 10(a), whereas the damage distribution almost converges upon refinement with the higher pulling speed, Fig. 10(b). It shall be noticed that the damage levels are not the same in Fig. 10(a) and (b). This is due to the fact that the damage evolution is also strain rate dependent [7]. Nevertheless increasing the velocity (strain rate) does help reduce the mesh dependence.

Application to Cross-die Deep Drawing Test [7]

It is important to check if the viscoplastic regularization works with actual process conditions i.e. using the measured strain rate hardening parameters and prescribing the actual process velocity. An industrially relevant [6] process i.e.

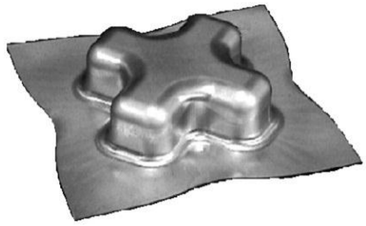


Fig. 11: Final cross die product

cross die drawing process was used to study the viscoplastic regularization effect. The final cross die product is shown in Fig. 11. The extended Bergström hardening parameters were determined from tensile tests carried out at different strain rates. The actual punch velocity was prescribed in the simulations. Fig. 12 shows the damage distribution obtained with three different mesh sizes. The viscoplastic regularization was very effective in this case. The results are almost mesh independent.

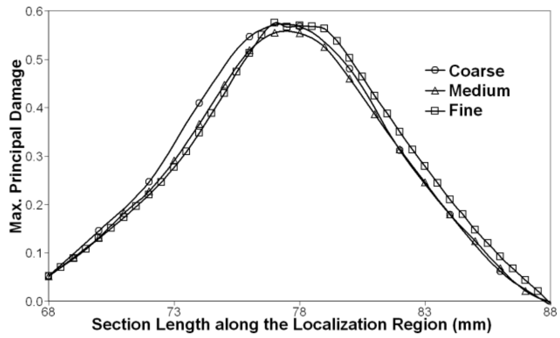


Fig. 12: Damage distributions in the localization region for the cross die test.

Conclusion

Adding strain rate hardening (i.e. viscoplasticity) does regularize the pathological mesh dependency in local damage models. The viscoplastic length scale is a function of the strain hardening parameters, strain rate hardening parameters and damage parameters. The shape of the damage (plastic strain) distribution is dependent upon the strain rate hardening model used. The viscoplastic regularization loses its effect prior to fracture because of the finite size of the elements. This is a normal mesh dependency. Refining the mesh or increasing the deformation rate will reduce this mesh dependency. An increase in viscosity will increase the primary length scale and thus delay the moment at which the regularization is lost. The preferable way to reduce this mesh dependency is by refining the mesh because this will not change the physics of the problem. Viscoplastic regularization was successfully applied on an industrial problem.

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