Implementation of an Anisotropic Damage Material Model using a General Second Order Damage Tensor

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Damage in metals is mainly the process of the initiation and growth of voids. With the growing complexity in materials and forming processes, it becomes inevitable to include anisotropy in damage (tensorial damage variable). Most of the anisotropic damage models define the damage tensor in the principal damage direction, with the assumption that the principal damage direction coincides with that of principal plastic strain direction. This assumption limits the applicability of the model to proportional loads. This research is an effort towards implementing an anisotropic damage model for non-proportional loads. The implementation of an anisotropic damage model in an implicit FEA code is presented. The model is based on the hypothesis of strain equivalence. A second order general damage tensor is used as an internal variable to represent the damage at macro scale. Two simulations were carried out to check the implementation of the model; a single element orthogonal load change simulation and a rectangular cup deep drawing simulation. Promising simulation results are obtained at acceptable CPU costs.

Keywords: anisotropic damage, continuum damage mechanics, non-proportional loading

Introduction

Damage is the process of nucleation, growth and coalescence of voids in ductile materials. These processes take place at the micro-scale. Damage develops under plastic deformation and results in the formation of a meso-crack therefore it can be recognized as a pre-fracture mechanism. Due to the phenomenon of damage, the material looses its load carrying capacity. In Continuum Damage Mechanics (CDM), damage is introduced as a state variable to model the effect of damage while the material is considered to be continuous. The damage variable is used to map the nominal state variables in the damaged material configuration to effective state variables in a fictitious undamaged material configuration. The pioneers [1] of CDM assumed damage to be isotropic. In the late eighties it was realized that damage is anisotropic in nature [2, 3]. Anisotropy in damage can be classified into two categories; Material Induced Anisotropy in Damage (MIAD) and Load Induced Anisotropy in Damage (LIAD). MIAD is related to the anisotropy in distribution and shape of second phase particles and is governed by void nucleation. It can be observed from **Figure 1(a)** that initially the mechanical response of the material is the same in rolling and transverse directions but the response starts to deviate after some amount of straining. This deviation is attributed to the anisotropy of second phase particles in the rolling and transverse directions. An example of MIAD can be found in [4]. LIAD is related to the stress state and is governed by void growth. The schematic in Figure 1(b) shows how the stress state induces anisotropy in void growth.

To model anisotropic damage, a damage tensor instead of a scalar damage parameter is required. Some well known anisotropic damage models are developed by Murakami, Chow and Lemaitre. Murakami [5] introduced anisotropic damage in the framework of large deformations. Hypothesis of elastic strain energy equivalence was utilized by Chow for the implementation of anisotropic damage [6, 7]. The use of the energy equivalence principle requires the definition of effective strain along with the effective stress. On the other hand, Lemaitre opted for the hypothesis of strain equivalence [8] which requires only the definition of effective stress. This makes the formulation relatively simpler but is not a true representation of the physical process. Recently, Lemaitre's anisotropic damage model coupled with a necking criterion was implemented [9] for the prediction of FLC.

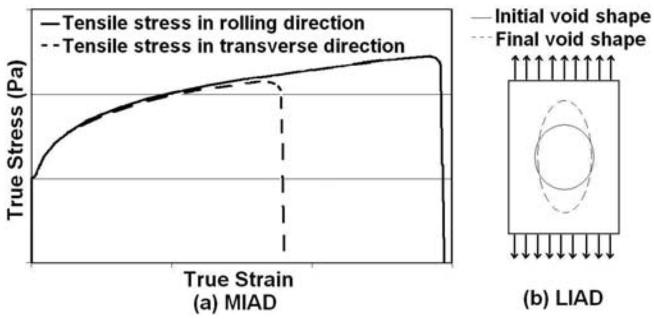

Figure 1. Two types of induced anisotropy in damage.

In this article a phenomenological based anisotropic damage model for non-proportional load paths is presented. The damage model in this work is based on the Lemaitre's work [8]. The 3D-implementation of this anisotropic damage model in an in-house implicit finite element code "DiekA" is presented. Results for a one-element simulation with orthogonal load change and the deep drawing of a rectangular cup simulation are given.

The Anisotropic Damage Model

As starting point the anisotropic damage model of [8] is used in this research. A brief description of the model is given here. partial Legendre transform of the Helmholtz free energy was performed to get the Gibbs specific free enthalpy. This is done to select stress as an independent variable, as it will be more convenient while dealing with the state law of elasticity. The Gibbs specific free enthalpy is given by

$$\psi^{*}=\psi_{e}^{*}+\frac{1}{\rho}\sigma_{ij}\varepsilon_{ij}^{p}-\psi_{p}-\psi_{T} \tag{1}$$

Where Ψ^* is the Gibbs specific free enthalpy, Ψ_e^* is the elastic Gibbs specific free enthalpy, Ψ_p is the plastic state potential, Ψ_T is the thermal state potential, ρ is the density, $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\boldsymbol{\varepsilon}^p$ is the plastic strain tensor.

Several assumptions are made to simplify the implementation of the model. The process is assumed to be isothermal. A small strain formulation with an additive split of the strain tensor is taken. Elasticity is assumed to be isotropic. Only isotropic hardening will be considered. The elastic potential is defined based on the hypothesis of strain equivalence [8] and is given by:

$$\psi_{e}^{*}=\frac{1}{\rho}\left[\frac{1+\nu}{2E}H_{ij}\sigma_{jk}^{D}H_{kl}\sigma_{li}^{D}+\frac{3(1-2\nu)}{2E}\frac{\sigma_{H}^{2}}{1-\eta D_{H}}\right] \tag{2}$$

Where v is the poisson ratio, E is the Young's modulus, $\mathbf{H}$ is the second order damage effect tensor, η is the hydrostatic sensitivity parameter and D_H is the hydrostatic part of the damage tensor. The superscript '^{D}' and the subscript '$_H$' represent the deviatoric and hydrostatic part of a tensor resectively. The observable state variable ε_{ij} and its associated variable σ_{ij} are defined using the state law i.e. using the state potential 'Ψ^*', whereas the internal state variables ε_{ij}^p, r and D_{ij} and their associated state variables σ_{ij}, R and Y_{ij} are defined by a dissipation potential 'F'.

$$F=F(\boldsymbol{\sigma},R,\mathbf{D})=f+F_{D} \tag{3}$$

Where r and R are the isotropic hardening variable and modulus respectively, D_{ij} and Y_{ij} are the second order damage and damage energy release tensors respectively, f is the plastic dissipation potential and F_D is the damage dissipation potential, given by

$$f=\tilde{\sigma}_{eq}-\sigma_{f} \quad \text{and} \quad F_{D}=\left(\frac{\bar{Y}}{S}\right)^{s}Y_{ij}\left|\frac{d\boldsymbol{\varepsilon}^{p}}{dr}\right|_{ij} \tag{4}$$

Where $\tilde{\sigma}_{eq}$ is the equivalent stress based on the effective stress '$\tilde{\boldsymbol{\sigma}}$', σ_f is the flow stress, S and s are material dependent damage parameters, $\bar{Y}$ is the effective damage energy release rate. The effective damage energy release rate and the effective stress are given by:

$$\bar{Y}=\frac{\tilde{\sigma}_{eq}^{2}\bar{R}_{v}}{2E} \quad \text{and} \quad \tilde{\boldsymbol{\sigma}}=\left(\mathbf{H}\boldsymbol{\sigma}^{D}\mathbf{H}\right)^{D}+\frac{\sigma^{H}}{1-\eta D^{H}}\mathbf{I}_{2} \tag{5}$$

Where. $\bar{R}_v$ is a triaxiality factor. Using the dissipation potential, the evolution laws for the state variables are defined as

$$\dot{\varepsilon}_{ij}^{p}=-\dot{\lambda}\frac{\partial F}{\partial(-\sigma_{ij})}=\dot{\lambda}\frac{\partial f}{\partial\sigma_{ij}} \tag{6}$$

$$\dot{r}=-\dot{\lambda}\frac{\partial F}{\partial R}=\dot{\lambda} \tag{7}$$

$$\dot{D}_{ij}=-\dot{\lambda}\frac{\partial F}{\partial(-Y_{ij})}=\dot{\lambda}\frac{\partial F_{D}}{\partial Y_{ij}}=\left(\frac{\bar{Y}}{S}\right)^{s}\left|\dot{\lambda}\frac{\partial f}{\partial\sigma_{ij}}\right| \tag{8}$$

Equation (8) is valid when the damage threshold is reached i.e. $\varepsilon_{eq}^p > \varepsilon_D^p$. A meso-crack is initiated when damage reaches a critical value i.e. $\max(D_{ij})=D_c$. The initiation of meso-crack is taken as the point of failure as the scope of this article is limited to pre-crack formation.

Adaptation in Lemaitre's Anisotropic Damage Model

From a physical point of view, there is a restriction on the damage tensor defined in CDM. Damage can not be negative because the physical interpretation of negative damage is that the material is getting stronger than the virgin material. This statement can be true for a very porous virgin material, but for engineering metals this phenomenon is not feasible. For this reason all researchers define the anisotropic damage evolution law in the principle damage direction and ensure that the evolution law will not yield negative damage values. In this way the principal damage values remain always positive. But if the principal damage tensor is rotated to an arbitrary direction, it may contain negative components. Lemaitre assumes that the principal damage directions coincide with that of the principal plastic strain directions and thus finds the rotation tensor based on the plastic strain tensor. This assumption is valid only for proportional loads. When the load changes its path the principal damage directions and principal plastic strain directions do not coincide anymore.

The condition that all the components of the damage tensor are positive imposes a complication in finding the principal direction of damage based on the damage tensor. Lets examine two loading conditions, pure shear loading and biaxial loading. The damage tensor does not have a negative component, therefore it can not recognize the difference between these two loading conditions and it will always give the principal direction for the biaxial case. This problem becomes even more complex when non-proportional loads come into play, as the principal damage direction rotates. Due to these reasons, the determination of the principal damage directions using the damage tensors becomes impossible.

Another difficulty with non-proportional loads is that damage tensor and strain tensor do not remain compatible. In Lemaitre's model, the damage evolution is based on the plastic strain (Equation (8)). Use of this evolution law complicates the formulation when the principal directions of damage and plastic strains do not coincide.

The solution to these problems was to eliminate the condition of evolution of damage in its principal direction. Rotation of the state variables to the principal damage direction will not be required if the evolution of damage is defined in the material direction. For this implementation, all the tensor multiplications have to be performed for general tensors. The implementation in a finite element code then becomes very tedious as there are many tensorial operations to be carried out with a general damage tensor rather than the damage tensor in the principal direction.

Implementation

The implementation of a material model in a finite element code is carried out through a stress (state variables) update routine. For a given strain increment, the new stress and damage state is found in a coupled manner.

The evolution Equations (6) to (8) are implicit and can not be solved analytically. Therefore an iterative numerical scheme is required to update the state variables incrementally. In this implementation an Euler Backward numerical scheme is used. Equations (6) to (8) are transformed from differential form to difference form. The solution converges when the residuals of these difference equation are below a user specified value. To update the independent state variables (i.e $\tilde{\boldsymbol{\sigma}}$, $\mathbf{D}$ and λ) in the iterative procedure the following scheme is adopted

$$\begin{Bmatrix} \mathbf{R}_{\tilde{\sigma}}^{i} \\ \mathbf{R}_{D}^{i} \\ R_{\lambda}^{i} \end{Bmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{M}_{23} \\ \mathbf{M}_{31} & \mathbf{M}_{32} & M_{33} \end{bmatrix}^{i} \begin{Bmatrix} \delta\tilde{\boldsymbol{\sigma}}^{i} \\ \delta\mathbf{D}^{i} \\ \delta\lambda^{i} \end{Bmatrix} \tag{9}$$

The vector 'R' contains the residuals of the difference equations in the i^{th} iteration, $\delta\tilde{\boldsymbol{\sigma}}^{i}$, $\delta\mathbf{D}^{\mathrm{i}}$ and $\delta\lambda^{\mathrm{i}}$ are increments to update the variables for next iteration and the matrix 'M' is an iteration matrix. 'M' is developed based on minimization of the residual functions. The derivatives of power 2 and higher are neglected in developing 'M'. A detailed description of this procedure can be found in [10].

Since the material model is implemented in an implicit finite element code, a consistent formulation for the stiffness matrix was required. This stiffness is determined using Equation (9) with the condition that the R_{D} and R_{λ} are set to zero and the $R_{\tilde{\boldsymbol{\sigma}}}$ is set to a small increment in strain $\delta\boldsymbol{\varepsilon}$.

$$\begin{Bmatrix} \delta\boldsymbol{\varepsilon} \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{M}_{23} \\ \mathbf{M}_{31} & \mathbf{M}_{32} & M_{33} \end{bmatrix}^{i} \begin{Bmatrix} \delta\tilde{\boldsymbol{\sigma}}^{i} \\ \delta\mathbf{D}^{i} \\ \delta\lambda^{i} \end{Bmatrix} \tag{10}$$

Condensing Equation (10) to form a relation between $\delta\boldsymbol{\varepsilon}$ and $\delta\tilde{\boldsymbol{\sigma}}$ gives the stiffness matrix as

$$\mathbf{K} = \left(\mathbf{M}_{11} + \mathbf{M}_{12} : \mathbf{M}_{22}^{-1} : \left[\frac{\mathbf{M}_{23} \otimes \mathbf{M}_{31} - M_{33}\mathbf{M}_{21}}{M_{33}} \right] - \frac{\mathbf{M}_{13} \otimes \mathbf{M}_{31}}{M_{33}} \right)^{-1} \tag{11}$$

Results

The anisotropic damage material model was tested for nonproportional load cases. Two types of simulations were selected; a single element simulation with an orthogonal load change and a rectangular cup deep drawing simulation. The material parameters in both simulations were taken from literature [8].

A hexahedral element of dimensions 1mm x 1mm x 1mm was first loaded in tension up to 22 percent strain (in the z-direction). Then the element was sheared in the x-z direction without unloading. **Figure 2** shows the evolution of the damage components as a function of equivalent plastic strain. The damage evolution is as expected. A small increase in the x and z components of damage can be observed at the end, which is due to an error in the rotation of stresses to current coordinates. The rotation tensor is taken at the mid of the step which introduces a small error per step. This error in damage can be neglected as its effect will be very small as compared to the effect of damage in the loading direction i.e. x-z direction. Critical damage is reached at an equivalent plastic strain of 41 percent.

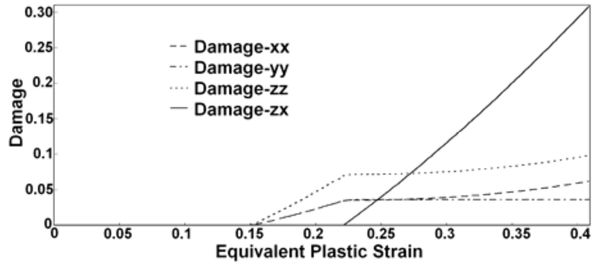

Figure 2. Damage plot for orthogonal load change.

The result obtained from the current anisotropic damage model is compared with that obtained from Lemaitre's model. **Figure 3(a)** shows the stress behavior obtained from the simulation when Lemaitre's model was used. Figure 3(b) shows the stress behavior obtained from the implemented non-proportional anisotropic damage model. For the tensile loading, the stresses are equal for both cases because the load is proportional. After the orthogonal load change occurs, it is expected that the tensile stress will drop to zero and shear stress will build up. It can be observed from Figure 3 that Lemaitre anisotropic damage model does not show this behavior, whereas the non-proportional anisotropic damage model behaves exactly as expected. The small error in the rotation of stresses can again be observed in Figure 3(b).

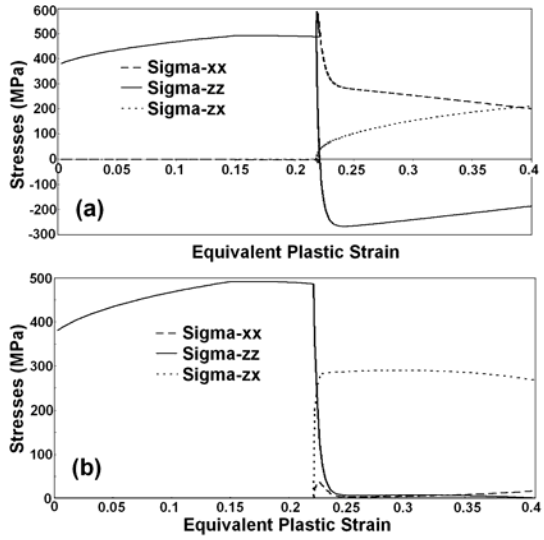

Figure 3. Stress plot for orthogonal load change (a) Lemaitre's model unchanged (b) Non-proportional anisotropic damage model.

The rectangular cup drawing simulation was carried out with three different material models i.e. without damage, with Lemaitre's isotropic damage model and with the non-proportional anisotropic damage model.

Distributions of two components of the damage tensor are plotted in **Figure 4**. Damage is growing only in those regions where the equivalent plastic strain has crossed the damage threshold. It can be observed that damage in yz-direction (Figure 4(a)) is higher than that in the zx-direction (Figure 4(b)). This difference in the damage values is due to the fact that a rectangular cup is drawn. For a square cup these two damage values will be equal. Isotropic damage models can not predict these effects.

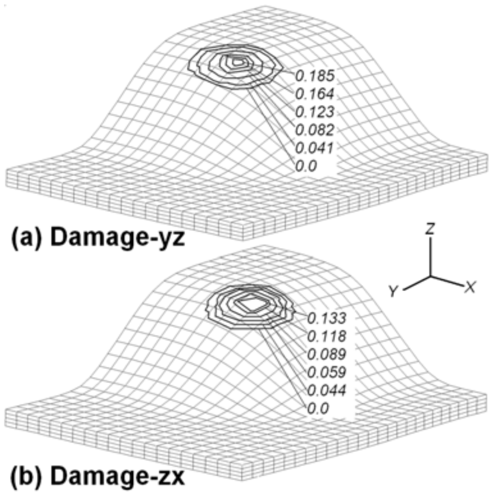

Figure 4. Damage distribution (a) yz component (b) zx component.

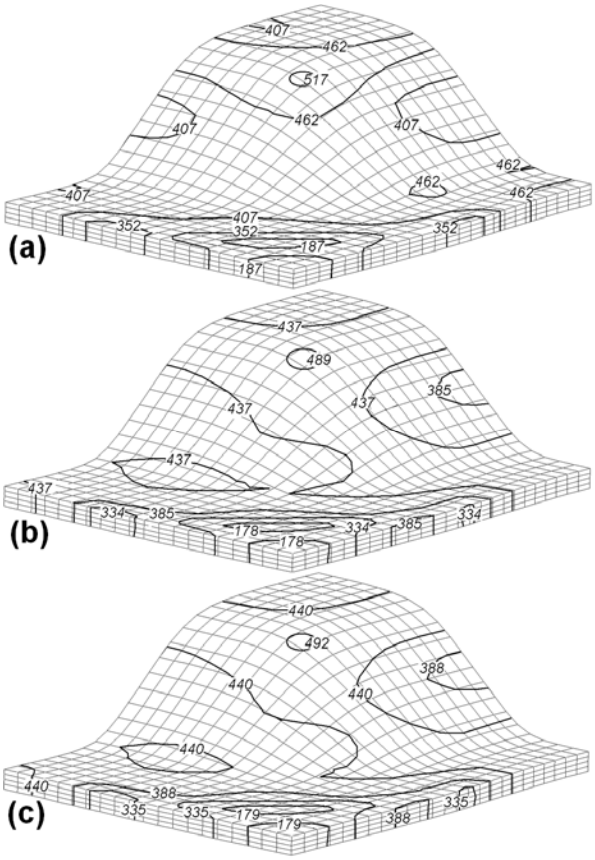

Figure 5. Equivalent stress distribution (MPa) (a) Without damage (b) With isotropic damage (c) With anisotropic damage.

Figure 5 shows the distribution of equivalent stress for the simulations without damage, with isotropic damage and with anisotropic damage. The equivalent stress shows remarkable differences. The stresses obtained from the model without damage and with isotropic damage constitutes the two extremes. The stresses obtained from the nonproportional anisotropic damage model is intermediate between the two extremes. The effect of damage on the state variables is more severe in the case of isotropic damage. The reason is that the isotropic damage model effects all the components of the state variable tensor by the same amount. On the other hand anisotropic damage has a directional effect on the state variables and as a consequence less severity on the degradation and most likely gives a better prediction of stresses.

The simulation time is important for the applicability of the model in industry. **Table 1** shows a comparison of the CPU times of all the three simulations. The simulation time for both the damage models is not very large as compared to the model without damage. The reason for the increase in CPU time is the limitation on the step size to attain convergence. Nevertheless, the simulation time is still acceptable for industrial applications. The benefit achieved from this model in terms of accuracy is much more than what is lost in terms of CPU time.

Table 1: CPU times comparison for cup drawing simulation.

Model	Step Size Limitation (mm)	Time (sec)
No damage	-0.1	170
Isotropic Damage	-0.05[a]	221
Anisotropic Damage	-0.01[a]	1003

a) Applies after the damage threshold is reached

Conclusions

An anisotropic continuum damage model, applicable for non-proportional loads, was successfully implemented in the implicit in-house FEA code DiekA. The performance of the model was tested with a single element orthogonal load change simulation and a deep drawing simulation (non-proportional load cases). The model performed as expected. The implemented anisotropic damage model gives improved prediction as compared to an isotropic damage model. The performance of the model in terms of CPU time was satisfactory.

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