

Short Papers

A Robust Adaptive Robot Controller

Harry Berghuis, Romeo Ortega, and Henk Nijmeijer

Abstract—A globally convergent adaptive control scheme for robot motion control with the following features is proposed. First, the adaptation law possesses enhanced robustness with respect to noisy velocity measurements. Second, the controller does not require the inclusion of high gain loops that may excite the unmodeled dynamics and amplify the noise level. Third, we derive for the unknown parameter design a relationship between compensator gains and closed-loop convergence rates that is independent of the robot task. A simulation example of a two-DOF manipulator features some aspects of the control scheme.

I. INTRODUCTION

The path tracking control problem of rigid robots with uncertain parameters that received the attention of robot control theorists in the last few years has matured to a stage where theoretically satisfactory asymptotic results are now well established, see, e.g., [1]. In order for these results to penetrate the realm of applications there are at least three basic requirements that should be satisfied. First, the adaptation law should not be sensitive to (unavoidable) velocity measurement noise. Second, high gain designs that excite the unmodeled torsional modes and aggravate the noise sensitivity problem (cf. [2]), should be avoided. Third, nonconservative measures to carry out the gain tuning taking into account the closed-loop robustness-performance tradeoff should be provided to the designer. In particular, it is desirable to have available relationships between controller gain ranges and convergence rate bounds, which to some extent are independent of the specific task. To the best of our knowledge, all existing adaptive controllers for which global stability of the closed loop can rigorously be proven fail to satisfy all of the requirements mentioned previously. Some representative examples are briefly discussed below.

Probably the most elegant solutions to the adaptive motion control problem are provided by the so-called *passivity-based methods*, e.g., [3], [4]. An important drawback of these schemes is that they are not robust to velocity measurement noise. Specifically, in underexcited operation, e.g., when performing a regulation task, the well-known phenomenon of parameter drift [5] in the adaption law is prone to occur due to the presence of quadratic terms in the

Manuscript received December 14, 1992. H. Berghuis was supported by the Netherlands Technology Foundation (STW). R. Ortega was supported at the University of Twente by the Dutch Network on Systems and Control.

H. Berghuis is with the Control Group, Department of Electrical Engineering, University of Twente, 7500 AE Enschede, The Netherlands.

R. Ortega was on leave with the Department of Applied Mathematics, University of Twente, 7500 AE Enschede, The Netherlands. He is now with the Université de Technologie de Compiègne, Heudiasyc UR C.N.R.S. 817, Centre de Recherches de Royallieu, 60206 Compiègne cedex, France.

H. Nijmeijer is with the Systems and Control Group, Department of Applied Mathematics, University of Twente, 7500 AE Enschede, The Netherlands.

IEEE Log Number 9212603.

measured velocity. This phenomenon has been illustrated in simulations in [6]–[8] and actual experimentation in [9], [10], [21].

Recently, a number of adaptive schemes that do not suffer from this velocity measurement problem have been proposed by Bayard and Wen [11]. However, a drawback of the Bayard and Wen schemes is that they require high controller gains in order to both overcome the uncertainty in the initial parameter errors and compensate for the dependency on the magnitude of the desired trajectory velocity.

An alternative way to reduce the sensitivity to velocity measurement noise has been proposed by Sadegh and Horowitz [6]. Their idea is to replace the actual position and velocity in the regressor by the desired trajectory values. This modification brings along two new difficulties: the inclusion of an additional feedback proportional to the square of the tracking error that may induce a high gain loop during the transients, and also a lower bound on the compensator gains that is dependent on the magnitude of the desired trajectory velocity. This bound again translates into a high gain requirement when tracking fast reference signals. In [8] the latter restriction on the controller gains is removed, but still a nonlinear feedback is required in order to be able to show global convergence.

The clever inclusion of a normalization term in the parameter adaptation law (as well as the Lyapunov function) allows Whitcomb *et al.* [12] to establish global stability for an adaptive scheme without the parameter drift problem nor the need for the nonlinear proportional feedback term, but still requiring the controller gains to satisfy an inequality that depends on the desired trajectory velocity. As we will show below, this condition translates into a task-dependent upper bound on the attainable convergence rates.

The main contribution of this paper (see also [13] containing part of the theoretical results) is to combine ideas of [8] and [12] to come up with an adaptive controller that has enhanced robustness with respect to velocity measurement noise, does not require high gain loops, and to provide a relationship between convergence rates and compensator gains that is independent of the desired trajectory velocity magnitude. Furthermore, the required additional computations are basically negligible.

The remaining part of the paper is organized as follows. For clarity we have treated the known and the unknown cases separately. Our main results concerning the nonadaptive controller are presented in Section II, whereas the adaptive case is presented in Section III. The robustness of the proposed adaptive control scheme is illustrated in a simulation study of a two-DOF manipulator in Section IV. We will give some conclusions in Section V.

II. KNOWN PARAMETER CASE

A. Main Result

Consider a standard n -degrees of freedom rigid robot model of the form [14]:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau, \quad q \in \mathbb{R}^n \quad (1)$$

where q is the vector of the generalized coordinates, τ is the input

torque vector, and $M(q)$, $C(q, \dot{q})\dot{q}$, and $G(q)$ represent the inertia matrix, the vector of Coriolis and centrifugal forces, and the gravitation vector, respectively. We assume that $C(q, \dot{q})$ is defined using the Christoffel symbols, see, e.g., [1]. Let the control torque τ be given as

$$\tau = M(q)\ddot{q}_d + C(q, \dot{q} - \lambda e)\dot{q}_d + G(q) - K_d\dot{e} - K_p e \quad (2a)$$

with

$$e \equiv q - q_d \quad (2b)$$

and where $q_d \in \mathbb{R}^n$ is the desired trajectory, $K_d = K_d^T > 0$, $K_p = K_p^T > 0$,

$$\lambda \equiv \frac{\lambda_0}{1 + \|e\|} \quad (3)$$

with λ_0 a positive constant, and $\|\cdot\|$ is defined as the Euclidean norm.

Assume the controller gains are chosen such that

$$\lambda_0 < \min \left\{ \frac{K_{d,m}}{3M_M + 2C_M}, \frac{4K_{p,m}}{K_{d,M} + K_{d,m}} \right\} \quad (4)$$

where

$$K_{d,m} \equiv \sigma_{\min}(K_d), \quad K_{d,M} \equiv \sigma_{\max}(K_d), \quad K_{p,m} \equiv \sigma_{\min}(K_p) \quad (5)$$

with $\sigma_{\max}(\cdot)$, $\sigma_{\min}(\cdot)$ the maximum and minimum singular value, respectively, and M_m , M_M , and C_M satisfy (cf. [15]):

$$0 < M_m \leq \|M(q)\| \leq M_M \quad (6a)$$

$$\|C(q, x)\| \leq C_M \|x\| \quad \text{for all } x \quad (6b)$$

Then we can prove the following proposition.

Proposition 2.1: Under the condition (4), the closed-loop system is globally convergent, that is, e and \dot{e} asymptotically converge to zero and all internal signals are bounded. If besides (4) the condition

$$\lambda_0^2 < \frac{4M_m K_{p,m}}{M_M^2} \quad (7)$$

holds, then the closed-loop system is exponentially stable, that is, there exist $m > 0$, $\rho > 0$, independent of the desired trajectory velocity, such that

$$\|x(t)\|^2 \leq m e^{-\rho t} \|x(0)\|^2 \quad \text{for all } t \geq 0 \quad (8)$$

where $x^T = (e^T \dot{e}^T)$. \square

Proof: We will strongly rely on the following well-known properties of $C(q, \cdot)$

$$C(q, x)y = C(q, y)x \quad (9a)$$

$$C(q, x + \alpha y) = C(q, x) + \alpha C(q, y) \quad (9b)$$

for all $x, y, q \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$.

Combining (1) and (2) and using (9b) we get

$$M(q)\ddot{e} + C(q, \dot{q})\dot{e} + \lambda C(q, e)\dot{q}_d + K_d\dot{e} + K_p e = 0. \quad (10)$$

Consider the positive definite Lyapunov function candidate

$$V(e, \dot{e}) = \frac{1}{2} s^T M(q) s + \frac{1}{2} e^T K_p e. \quad (11)$$

where

$$s = \dot{e} + \lambda e. \quad (12)$$

With abuse of notation we will write $V(e, \dot{e})$ everywhere, although we will freely change the coordinates (e, \dot{e}) into other coordinates.

Taking the time derivative of (11) along the trajectory of (10)

yields

$$\begin{aligned} \dot{V}(e, \dot{e}) &= s^T [\lambda M(q)\dot{e} + \dot{\lambda} M(q)e + \lambda C(q, \dot{q})e - \lambda C(q, e)\dot{q}_d \\ &\quad - K_d\dot{e} - K_p e] + \dot{e}^T K_p e \end{aligned} \quad (13)$$

where we have used (9b) and the skew symmetry of $\dot{M}(q) - 2C(q, \dot{q})$, see, e.g., [1]. Now, (9a) allows us to rewrite (13) as

$$\begin{aligned} \dot{V}(e, \dot{e}) &= -s^T [K_d - \lambda M(q)]\dot{e} + \dot{\lambda} s^T M(q)e + \lambda s^T C(q, \dot{e})e \\ &\quad - \lambda e^T K_p e. \end{aligned} \quad (14)$$

At this moment we introduce a new variable that will simplify our further developments, namely

$$s_1 \equiv \dot{e} + \frac{\lambda}{2} e. \quad (15)$$

In terms of s_1 we can rewrite (14) as

$$\begin{aligned} \dot{V}(e, \dot{e}) &= -s_1^T [K_d - \lambda M(q)] s_1 - \left(\frac{\lambda}{2} e\right)^T \\ &\quad \cdot [4\lambda^{-1} K_p - (K_d - \lambda M(q))] \left(\frac{\lambda}{2} e\right) \\ &\quad + \dot{\lambda} s^T M(q)e + \lambda s^T C(q, \dot{e})e. \end{aligned} \quad (16)$$

In Appendix I we establish the following bounds for last two right-hand-side terms

$$\dot{\lambda} s^T M(q)e \leq 2\lambda_0 M_M \left(\|s_1\|^2 + \left\| \frac{\lambda}{2} e \right\|^2 \right) \quad (17a)$$

$$\lambda s^T C(q, \dot{e})e \leq 2\lambda_0 C_M \left(\|s_1\|^2 + \left\| \frac{\lambda}{2} e \right\|^2 \right). \quad (17b)$$

Replacing these bounds in (2.16) and rearranging terms we obtain

$$\dot{V}(e, \dot{e}) \leq -\kappa_1 \|s_1\|^2 - \kappa_2 \left\| \frac{\lambda}{2} e \right\|^2 \quad (18)$$

where

$$\kappa_1 \equiv K_{d,m} - 3\lambda_0 M_M - 2\lambda_0 C_M,$$

$$\kappa_2 \equiv \frac{4K_{p,m}}{\lambda_0} - K_{d,M} - 2\lambda_0 M_M - 2\lambda_0 C_M. \quad (19)$$

It is easy to see that (4) ensures that $\kappa_1, \kappa_2 > 0$. Thus $V(e, \dot{e})$ is a nonincreasing function bounded from below. This implies from (11) that $s, e \in L_\infty^n$, and consequently $\dot{e}, s_1 \in L_\infty^n$. Now, because $\lambda \in L_\infty$ we conclude from (18) that $s_1, e \in L_2^n$. From square integrability and uniform continuity of e we conclude that it converges to zero. To complete the first part of the proof notice that we also have $\dot{e} \in L_2^n$, thus it suffices to establish that $\dot{e} \in L_\infty^n$, which follows from the error dynamics (10).

To prove exponential stability let us write $V(e, \dot{e})$ in terms of the coordinates $(s_1, (\lambda/2)e)$:

$$\begin{aligned} V(e, \dot{e}) &= \frac{1}{2} s_1^T M(q) s_1 + s_1^T M(q) \left(\frac{\lambda}{2} e\right) + \frac{1}{2} \left(\frac{\lambda}{2} e\right)^T M(q) \left(\frac{\lambda}{2} e\right) \\ &\quad + \frac{1}{2} e^T K_p e \end{aligned} \quad (20)$$

which can be bound as

$$\frac{1}{2} \xi_1 \|s_1\|^2 + \frac{1}{2} \xi_2 \left\| \frac{\lambda}{2} e \right\|^2 \leq V(e, \dot{e}) \leq M_M \|s_1\|^2 + \frac{1}{2} \xi_3 \left\| \frac{\lambda}{2} e \right\|^2 \quad (21)$$

where

$$\begin{aligned}\xi_1 &\equiv M_m - \left(\frac{M_M}{\alpha}\right)^2, & \xi_2 &\equiv \frac{4K_{p,m}}{\lambda_0^2} + M_m - \alpha^2, \\ \xi_3 &\equiv \frac{4K_{p,m}}{\lambda^2} + 2M_M\end{aligned}\quad (22)$$

and α is any positive number.

Under assumption (7) we can find $\alpha > 0$ such that $\xi_1, \xi_2 > 0$. On the other hand, boundedness of e ensures that λ is bounded away from zero, and consequently $\xi_3 < \infty$. From (21) and (18) we conclude that there exist $m_1, \rho_1 > 0$ such that

$$\|y(t)\|^2 \leq m_1 e^{-\rho_1 t} \|y(0)\|^2 \quad \text{for all } t \geq 0 \quad (23)$$

where $y^T = ((\lambda/2)e^T s^T)$. Now we observe that

$$x = T(\lambda)y \quad (24)$$

where

$$T(\lambda) = \begin{bmatrix} \frac{2}{\lambda} I & 0 \\ -I & I \end{bmatrix}. \quad (25)$$

The proof is completed by noting that

$$\lambda_0 > \lambda \geq \lambda_0 \left(1 + \left(\frac{2V(e(0), \dot{e}(0))}{K_{p,m}}\right)^{1/2}\right)^{-1} \quad (26)$$

and consequently T and T^{-1} are bounded matrices. ■

B. Discussion

1) Notice that in contrast to [6], [11], and [12], the conditions (4) and (7) on the controller gains λ_0, K_p , and K_d are independent of the desired trajectory velocity. Consequently the convergence rate is also independent of \dot{q}_d . This makes the tuning process *task independent*.

2) It is worth remarking that in the stability proof of the scheme proposed by Whitcomb *et al.* [12] a term λ (3) (denoted ϵ in their paper) is introduced in the Lyapunov function. The conditions for stability invoke an upper bound on λ_0 (denoted ϵ_0 in their paper) that depends on $\|\dot{q}_d\|$. Even though λ is not used in the (known parameter) control implementation, λ_0 upper bounds the schemes convergence rate, see $\bar{\nu}$ in [12], and makes it dependent of the desired trajectory velocity.

3) In [16] an upper and lower bound has been determined on m and ρ , respectively. These bounds depend on the initial tracking error $x(0)$, which is due to the normalization of λ . For global exponential stability of a differential equation it is in the mathematics literature normally understood that (8) holds for some m and ρ independent of the initial state. As a consequence, the exponential stability result (8) is not global in a strict mathematical sense.

4) The proposed control law does not contain a nonlinear PD term as in [6] and [8], which injects into the loop a gain proportional to the square of the tracking error.

5) Two key modifications are introduced in the controller (2). The inclusion of an additional term $-\lambda C(q, e)\dot{q}_d$ and the use of the normalization factor λ . The first idea exploits the structural property (9) of $C(q, \cdot)$ and was introduced in [8], while the normalization factor is being used in [12]. The λ factor is needed in the controller to be able to bound the cubic term $s^T C(q, \dot{e})e$ by quadratic terms, as done in (17b). Furthermore, the additional term that appears in $\dot{V}(e, \dot{e})$ due to λ can be upper bounded by quadratic terms in s_1 and $(\lambda/2)e$, as shown in Appendix I.

6) To motivate our choice of the Lyapunov function (11), which was inspired by [17], let us consider the one proposed in [12]:

$$V_w(e, \dot{e}) = \frac{1}{2} \dot{e}^T M(q) \dot{e} + \lambda e^T M(q) \dot{e} + \frac{1}{2} e^T K_p e. \quad (27)$$

This function is related to (11) by

$$V_w(e, \dot{e}) = V(e, \dot{e}) - \frac{1}{2} \lambda^2 e^T M(q) e. \quad (28)$$

If we evaluate $\dot{V}_w(e, \dot{e})$ we obtain an additional term in $e^T \dot{M}(q) e$. Using the skew-symmetry property this amounts to an extra term in $e^T C(q, \dot{q}) e$. This term cannot be compensated by the control and can only be bounded, in terms of e and \dot{e} , with a bound on \dot{q}_d .

III. UNKNOWN PARAMETER CASE

A. Main Result

In order to extend the foregoing result to the unknown parameter case, we use the linear in the parameters property of robot dynamics, see, e.g., [1]. That is, we can write (1) as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \ddot{q}) \theta \quad (29)$$

where $Y(\cdot)$ is a regressor matrix, which is linear in the second, third, and fourth argument and $\theta \in \mathbb{R}^p$ represents a vector of unknown parameters. Now, consider (1) in closed loop with

$$\begin{aligned}\tau &= Y(q, \dot{q} - \lambda e, \dot{q}_d, \ddot{q}_d) \hat{\theta} - K_d \dot{e} - K_p e \\ &= \hat{M}(q)\ddot{q}_d + \hat{C}(q, \dot{q} - \lambda e)\dot{q}_d + \hat{G}(q) - K_d \dot{e} - K_p e\end{aligned}\quad (30)$$

where λ is as in (2.3) and $\hat{\theta}$ adjusted by

$$\frac{d}{dt} \{\hat{\theta}\} = -\Gamma Y^T(q, \dot{q} - \lambda e, \dot{q}_d, \ddot{q}_d) s \quad (31)$$

where s is given by (2.12). Then we have:

Proposition 3.1: Assume that (2.4) holds. Then the adaptive system (1), (30)–(31) is globally convergent, that is e and \dot{e} asymptotically converge to zero and all internal signals are bounded. □

Proof: Putting (30) into (1) we obtain

$$\begin{aligned}M(q)\ddot{e} + C(q, \dot{q})\dot{e} + \lambda C(q, e)\dot{q}_d + K_d \dot{e} + K_p e \\ = Y(q, \dot{q} - \lambda e, \dot{q}_d, \ddot{q}_d) \bar{\theta}\end{aligned}\quad (32)$$

where

$$\bar{\theta} = \hat{\theta} - \theta. \quad (33)$$

Consider the Lyapunov function candidate

$$V_A(e, \dot{e}, \bar{\theta}) = V(e, \dot{e}) + \frac{1}{2} \bar{\theta}^T \Gamma^{-1} \bar{\theta} \quad (34)$$

with $V(e, \dot{e})$ as in (11). The time derivative of $V_A(e, \dot{e}, \bar{\theta})$ along the error dynamics (32) with the choice of the adaptation law (31) yields (18). Global convergence then follows from the arguments used in the proof of Proposition 2.1. ■

B. Discussion

1) The remarks as given in Section II-B also hold for the adaptive case.

2) It is well known [5] that the equilibrium set of adaptive systems is unbounded. Therefore, in underexcited conditions and in the presence of noise in the adaptation law, the instability mechanism of parameter drift appears. To exemplify this phenomenon, consider a single link pendulum moving in the horizontal plane,

that is,

$$m_p l^2 \ddot{q} = \tau \quad (35)$$

where only the payload mass m_p is unknown, and take $l = 1$ [m]. One particular situation in which excitation is lost is in the regulation part of the task, so assume the $q_d = \text{constant}$. In these circumstances the adaptation law of [3] with velocity measurement noise $\eta \equiv N(0, \sigma^2)$ looks like

$$\frac{d}{dt} \{\hat{m}_p\} = -\Gamma [-\lambda_0(\dot{q} + \eta)] (\dot{q} + \eta + \lambda_0 e) \quad (36)$$

which has as expectation

$$E \left(\frac{d}{dt} \{\hat{m}_p\} \right) = \Gamma \lambda_0 \dot{q} (\dot{q} + \lambda_0 e) + \Gamma \lambda_0 \sigma^2. \quad (37)$$

On the other hand, in this situation the adaptation law (31) of the proposed controller becomes

$$\frac{d}{dt} \{\hat{m}_p\} = 0. \quad (38)$$

The integral of the second term in (37) introduces a drift proportional to the noise variance σ^2 , whereas (38) is robust for η .

In this illustrative example it was assumed that $q_d = \text{constant}$. We would like to stress, however, that the increased noise robustness feature of the controller ((30) and (31)) will definitely hold in other underexcited situations.

3) The adaptation laws presented in [6], [8], [11], and [12] possess also enhanced robustness with respect to velocity measurement noise, but these control schemes have the drawbacks mentioned in the introduction.

4) The extra computations needed in the implementation of the controller ((30) and (31)) due to the additional term $-\lambda C(q, e)q_d$ are negligible. Since λe is already needed in s we only require an extra addition.

5) For a stable implementation of the controller (3.2)–(3.3) and the ones in [6], [8], [11], [12], the coefficients M_M and C_M are required. Since these coefficients bound the actual system dynamics, one has to assume that the unknown parameters θ_i belong to some interval $[\theta_{i,\min}, \theta_{i,\max}]$, $i = 1, \dots, p$, and take the supremum of M_M , C_M over these intervals. From a practical perspective this is quite a reasonable procedure. Nevertheless, notice that it requires some minor additional information on θ in comparison to the controllers in [3] and [4].

6) As can easily be seen in (30) and (31), for $q_d = \text{constant}$ the controller reduces to PD control with adaptive gravitation compensation. Note also that in this case a PID controller could be employed to overcome steady-state errors due to the uncertainties in the gravitation parameters. In [18] it was shown, however, that the PID controller has a number of drawbacks. First, to ensure stability of the PID controller, the gain matrices must satisfy complicated inequalities that depend on the initial conditions. Second, in the common case in which only the payload mass is unknown, a PID controller requires as many integrators as the number of robot links, whereas for the implementation of the controller (30)–(31) one integrator suffices. Third, and most importantly, the PID controller exhibits worse control performance compared to the PD plus adaptive gravitation compensation, see [18].

IV. SIMULATION STUDY

In order to show the robustness of the proposed adaptive control scheme (30)–(31) for noisy velocity measurements, we consider a relatively simple but illustrative example of a two-DOF robot sys-

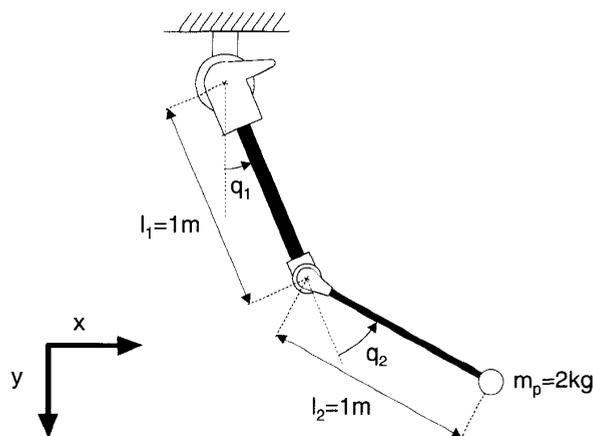


Fig. 1. Two-DOF robot system.

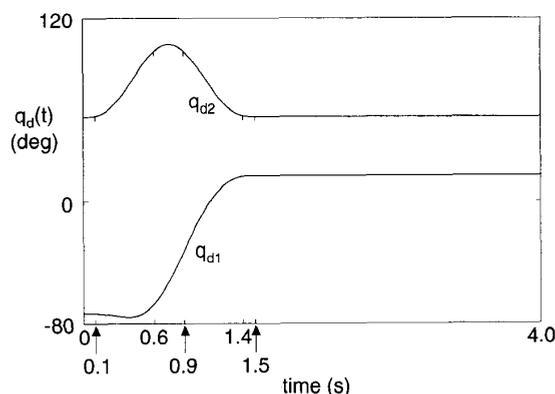


Fig. 2. Desired trajectory.

tem moving in the horizontal plane ([19], see Fig. 1). The dynamic equations describing the robot system are given in Appendix B. In this simulation study it was assumed that the system dynamics are known except for an unknown payload, for which the controller has to adapt. The actual payload m_p the robot manipulator has to transport in the simulations is equal to 2 kg. The robot system has to follow a straight line in the Cartesian space, from the initial position $(x, y) = (-1.25, 1.25)$ to the end position $(1.25, 1.25)$ within 1.5 s, where the origin is located at the joint of link 1. The desired trajectory in joint coordinates is shown in Fig. 2.

First a simulation of the robot system controlled by the adaptive controller of Slotine and Li [3] was performed. This controller is given by

$$\tau = \hat{M}(q) (\dot{q}_d - \lambda_0 \dot{e}) + \hat{C}(q, \dot{q}) (\dot{q}_d - \lambda_0 e) + \hat{G}(q) - K_d \dot{e} - \lambda_0 K_d e$$

$$\frac{d}{dt} \{\hat{\theta}\} = -\Gamma Y^T(q, \dot{q}, \dot{q}_d - \lambda_0 e, \dot{q}_d - \lambda_0 e) (e + \lambda_0 e).$$

(39)

The velocity signal \dot{q} was assumed to be contaminated with zero mean Gaussian noise. The used controller settings are $K_d = 25I$, $\lambda_0 = 1$ and $\Gamma = 15$, which result in a satisfactory performance of

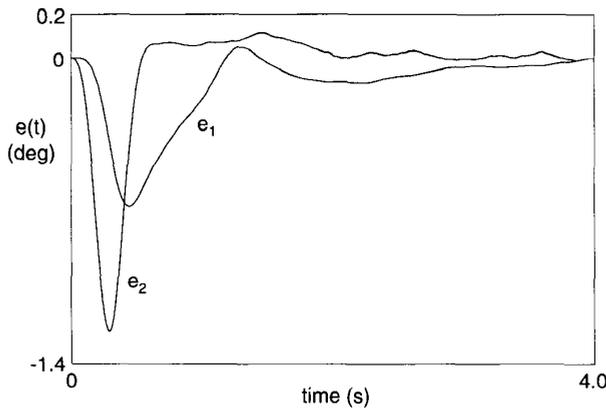


Fig. 3. Angular errors with Slotine and Li controller (4.1).

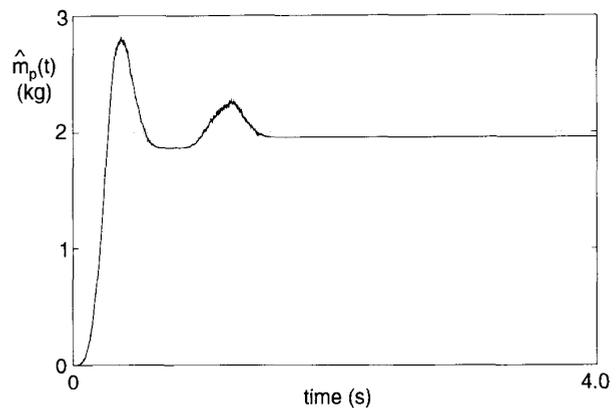


Fig. 6. Estimated mass with proposed controller (3.2), (3.3).

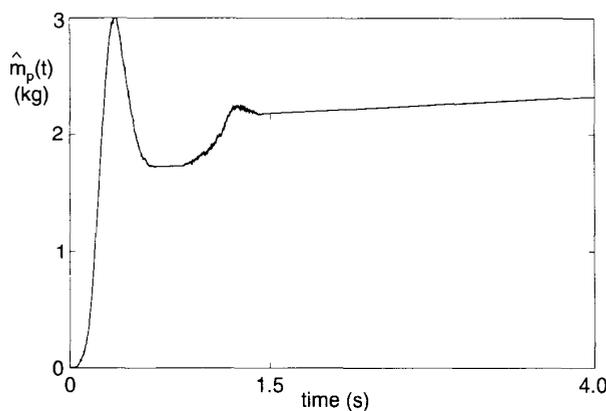


Fig. 4. Estimated mass with Slotine and Li controller (4.1).

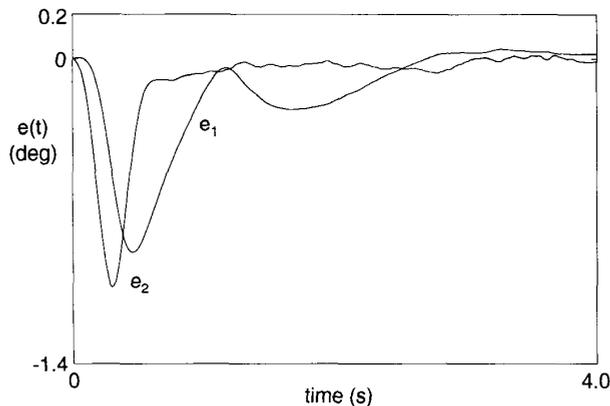


Fig. 5. Angular errors with proposed controller (3.2), (3.3).

the controlled robot system. Fig. 3 shows the angular errors obtained with the Slotine and Li controller.

In Fig. 4 the estimated payload mass $\hat{m}_p(t)$ is shown. Parameter adaptation mainly occurs during the periods that $q_{d2}(t)$ is persistently exciting ("sufficiently rich"), which is the case, see Fig. 2, in the time intervals $0.1 \leq t \leq 0.6$ and $0.9 \leq t \leq 1.4$ s. The

reason that $q_{d2}(t)$ is mainly responsible for the parameter adaptation comes from the fact that link 2 is most sensitive for the payload. Notice the drift in the parameter estimate after the time that the desired trajectory has lost its persistent excitation, $t > 1.5$ s.

In a second simulation the proposed controller (30) and (31) was applied to the robot system. Under the assumption that the robot has to transport payloads up to 2 kg, i.e., $m_{p,\min} = 0$ and $m_{p,\max} = 2$, the upper bounds in (2.6) have been determined as $M_M = 20$ and $C_M = 5$. The controller settings for this simulation were $K_p = 75I$, $K_d = 40I$, $\lambda_0 = 0.5$, and $\Gamma = 15$, so the condition (4) on λ_0 is satisfied. Fig. 5 shows the angular errors obtained when applying controller (30) and (31) to the robot system.

Comparing these angular errors with the ones in Fig. 3 shows that the performance of the controllers with respect to path tracking is quite similar. Fig. 6 shows the estimated payload mass $\hat{m}_p(t)$ for the controller (30) and (31). As can be seen, there is no drift in the estimate any more.

V. CONCLUSION

We have presented a globally convergent adaptive control algorithm for robot motion control with enhanced noise sensitivity properties. Moreover, the controller does not contain nonlinear proportional compensation gains and the controller gains and the convergence rate are independent of the desired reference velocity.

To attain this objective we propose a new controller structure that incorporates the normalization idea of Whitcomb *et al.* [12] and the additional compensation term of Berghuis *et al.* [8]. From the analysis point of view, a Lyapunov function similar to the one proposed in [17] is used to ensure negative definiteness of its time derivative via a suitable change of coordinates. In the nonadaptive case this Lyapunov function allows us to conclude exponential stability with a convergence rate independent of the robot task.

In a simulation study of a two-DOF robot manipulator the enhanced noise robustness of the proposed adaptive control scheme was illustrated. Nevertheless, the ultimate justification for adaptive control lies in its practical implementation. In relation to this one should realize that due to the availability of fast processing equipment the computational complexity of the model-based algorithms no longer impedes their implementation. This can be concluded from the increasing number of applications, see for instance [9], [10], [12], [20]. Similar experiments need to be done in order to see if the proposed adaptive controller also performs successfully in practice. Currently we are working on this (cf. [21]).

APPENDIX I

Upper bounds for last two right-hand-side terms in (2.16) are given by

$$\begin{aligned} \lambda s^T M(q) e &= -\frac{\lambda e^T \dot{e}}{\|e\|(1 + \|e\|)} s^T M(q) e = -\frac{\lambda e^T \left(s_1 - \frac{\lambda}{2} e \right)}{\|e\|(1 + \|e\|)} \\ &\cdot \left(s_1 + \frac{\lambda}{2} e \right)^T M(q) e \\ &\leq \frac{\lambda M_M \|e\|}{1 + \|e\|} \left(\|s_1\|^2 + 2\|s_1\| \left\| \frac{\lambda}{2} e \right\| + \left\| \frac{\lambda}{2} e \right\|^2 \right) \\ &\leq \frac{2\lambda M_M \|e\|}{1 + \|e\|} \left(\|s_1\|^2 + \left\| \frac{\lambda}{2} e \right\|^2 \right) \\ &\leq 2\lambda_0 M_M \left(\|s_1\|^2 + \left\| \frac{\lambda}{2} e \right\|^2 \right) \end{aligned} \quad (40)$$

$$\begin{aligned} \lambda s^T C(q, \dot{e}) e &= \lambda \left(s_1 + \frac{\lambda}{2} e \right)^T C(q, e) \left(s_1 - \frac{\lambda}{2} e \right) \leq \lambda \|e\| C_M \\ &\cdot \left(\|s_1\|^2 + 2\|s_1\| \left\| \frac{\lambda}{2} e \right\| + \left\| \frac{\lambda}{2} e \right\|^2 \right) \\ &\leq 2\lambda \|e\| C_M \left(\|s_1\|^2 + \left\| \frac{\lambda}{2} e \right\|^2 \right) \leq 2\lambda_0 C_M \\ &\cdot \left(\|s_1\|^2 + \left\| \frac{\lambda}{2} e \right\|^2 \right). \end{aligned} \quad (41)$$

APPENDIX II

The robot system used in the simulations was derived from [19]. The equations of motion are given by (1), with

$$\begin{aligned} M(q) &= \begin{bmatrix} m_{110}(q) & m_{120}(q) \\ m_{210}(q) & m_{220}(q) \end{bmatrix} + m_p \\ &\cdot \begin{bmatrix} 2 + 2 \cos(q_2) & 1 + \cos(q_2) \\ 1 \cos(q_2) & 1 \end{bmatrix} \end{aligned} \quad (42)$$

$$\begin{aligned} C(q, \dot{q}) &= \begin{bmatrix} c_{110}(q, \dot{q}) & c_{120}(q, \dot{q}) \\ c_{210}(q, \dot{q}) & 0 \end{bmatrix} + m_p \\ &\cdot \begin{bmatrix} -\sin(q_2)\dot{q}_2 & -\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ \sin(q_2)\dot{q}_1 & 0 \end{bmatrix}. \end{aligned} \quad (43)$$

The known system parameters are equal to

$$m_{110}(q) = 8.77 + 1.02 \cos(q_2) \quad (44a)$$

$$m_{120}(q) = 0.76 + 0.51 \cos(q_2) \quad (44b)$$

$$m_{210}(q) = 0.76 + 0.51 \cos(q_2) \quad (44c)$$

$$m_{220}(q) = 0.62 \quad (44d)$$

$$c_{110}(q) = -0.51 \sin(q_2)\dot{q}_2 \quad (44e)$$

$$c_{120}(q) = -0.51 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \quad (44f)$$

$$c_{210}(q) = 0.51 \sin(q_2)\dot{q}_1. \quad (44g)$$

ACKNOWLEDGMENT

The second author would like to thank Dan Koditschek for sending him a preprint of the interesting paper by Whitcomb *et al.* [12].

REFERENCES

- [1] R. Ortega and M. W. Spong, "Adaptive motion control of rigid robots: A tutorial," *Automatica*, vol. 25, pp. 877-888, 1989.
- [2] P. K. Khosla and T. Kanade, "Experimental evaluation of nonlinear feedback and feedforward control schemes for manipulators," *Int. J. Robotics Res.*, vol. 7, pp. 18-28, 1988.
- [3] J.-J. E. Slotine and W. Li, "On the Adaptive Control of Robot Manipulators," *Int. J. Robotics Res.*, vol. 6, pp. 49-59, 1987.
- [4] N. Sadegh and R. Horowitz, "Stability analysis of an adaptive controller for robotic manipulators," in *Proc. IEEE Conf. Robotics Automat.*, 1987, pp. 1223-1229.
- [5] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*. Englewood Cliffs, NJ: Prentice-Hall, 1988.
- [6] N. Sadegh and R. Horowitz, "Stability and robustness analysis of a class of adaptive controllers for robotic manipulators," *Int. J. Robotics Res.*, vol. 9, pp. 74-92, 1990.
- [7] H. M. Schwartz, G. Warshaw, and T. Janabi, "Issues in robot adaptive control," in *Proc. Amer. Control Conf.*, 1990, pp. 2797-2805.
- [8] H. Berghuis, P. Löhnerberg, and H. Nijmeijer, "Adaptive 'PD+' control of robot manipulators," in *Proc. Symp. Robot Control*, Vienna, 1991, pp. 459-464.
- [9] M. B. Leahy and P. V. Whalen, "Direct adaptive control for industrial manipulators," in *Proc. IEEE Conf. Robotics Automation*, 1991, pp. 1666-1672.
- [10] F. Ghorbel, A. Fitzmorris, and M. W. Spong, "Robustness of adaptive control of robots: Theory and experiment," in *Advanced Robot Control, Lecture Notes in Control and Information Sciences*, C. Canudas de Wit, Ed., vol. 162. Berlin: Springer-Verlag, 1991, pp. 1-29.
- [11] D. S. Bayard and J. T. Wen, "New class of control laws for robotic manipulators: Adaptive case," *Int. J. Control*, vol. 47, pp. 1387-1406, 1988.
- [12] L. L. Whitcomb, A. A. Rizzi, and D. E. Koditschek, "Comparative experiments with a new adaptive controller for robot arms," in *IEEE Trans. Robotics Automat.*, vol. 9, pp. 59-70, 1993.
- [13] H. Berghuis, R. Ortega, and H. Nijmeijer, "A robust adaptive controller for robot manipulators," in *Proc. IEEE Conf. Robotics Automation*, 1992, pp. 1876-1881.
- [14] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*. New York: Wiley, 1989.
- [15] J. J. Craig, *Adaptive Control of Robotic Manipulators*. Reading, MA: Addison-Wesley, 1988.
- [16] D. H. de Vries, "Performance evaluation of a new adaptive controller for robot manipulators," Intern. Rep. 92R014, Dep. Electr. Eng., Univ. Twente, Enschede, The Netherlands, 1992.
- [17] M. W. Spong, R. Ortega, and R. Kelly, "Comments on 'Adaptive Manipulator Control: A Case Study,'" *IEEE Trans. Automat. Contr.*, vol. 35, pp. 761-762, 1990.
- [18] P. Tomei, "Adaptive PD Controller for Robot Manipulators," *IEEE Trans. Robotics Automat.*, vol. 7, pp. 565-570, 1991.
- [19] L. Kruise, "Modeling and control of a flexible beam and robot arm," Ph.D. dissertation, Dept. Electr. Eng., University of Twente, Enschede, The Netherlands, 1990.
- [20] N. Sadegh and K. Guglielmo, "Design and implementation of adaptive and repetitive controllers for mechanical manipulators," *IEEE Trans. Robotics Automat.*, vol. 8, pp. 395-400, 1992.
- [21] H. Berghuis, "Model-based robot control: From theory to practice," Ph.D. dissertation, Dept. Electr. Eng., University of Twente, The Netherlands, 1993.