

## IDENTIFICATION OF ELECTRICITY SPOT MODELS BY USING CONVOLUTION PARTICLE FILTER

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**ABSTRACT.** *We consider a slight perturbation of the Schwartz-Smith model for the electricity futures prices and the resulting modified spot model. Using the martingale property of the modified price under the risk neutral measure, we derive the arbitrage free model for the spot and futures prices. As the futures price formula is based on the arithmetic average of the unobservable spot prices, it is highly nonlinear. We use the particle filtering methodology for estimating the model parameters. The main advantage of the new model is that it avoids the inclusion of artificial noise to the observation equation for the implementation of the particle filter. The extra noise is built within the model in an arbitrage free setting.*

**Keywords:** Parameter estimation, Finance, Kalman filter, Maximum likelihood estimators, Particle filter, Electricity spot model

1. **Introduction.** Since the recent deregulation of power markets, large volumes of electricity contracts are frequently traded. Noting that electricity is a non-storable commodity, spot trading of electricity is not defined in the usual sense. Moreover, unlike other commodities such as oil or gas, electricity futures and forwards are based on the arithmetic average of the spot prices over a delivery period. During the delivery period the contract is settled in cash against the spot price. Hence, it resembles a swap contract, exchanging a floating spot price against a fixed price, see [1]. More specifically, a futures contract is a contract that obligates the seller of the contract to deliver and the buyer to receive a given quantity of electricity or gas over a fixed period  $[T_0, T]$  at a price  $K$  specified in advance. The payoff of these futures are based on the arithmetic average of the spot price  $\frac{1}{n} \sum_{t=T_0}^T S(t)$  and not one fixed spot price  $S(T)$  as in most financial and commodity futures markets. Here  $n$  is the number of days during the delivery period  $[T_0, T]$ . This makes the problem highly nonlinear and we use particle filter for estimating model parameters. To the best of our knowledge, this has not been studied in detail before. Another important issue is related to the parameter estimation of the models representing the dynamics of both the spot and the futures. As a result of dealing with unobservable factors, a popular estimation method that has been proposed in the literature is the maximum likelihood estimation (MLE) method under the assumption that observations are corrupted with additive Gaussian noise. In this framework, the state

space representation is used together with the Kalman filtering techniques, and the parameter estimates are obtained through maximization of a likelihood functional. To make this approach mathematically feasible, some *ad hoc* observation noise has to be added to the observation equation in order to derive the filter as has been made by numerous authors, see [2] and the references therein. The additional noise in the observation has been interpreted to take into account bid-ask spreads, price limits, non-simultaneity of the observations, or errors in the data. The argument is clearly forced, unconvincing and hard to verify. Even if one ignores this factor, there is an additional complication of futures. Since there is no feedback of the observation noise to the spot price, this leads to a model that is not anymore an arbitrage free model.

The purposes of this paper are twofold: first, starting from the two factor model of Schwartz-Smith, see [3], we formulate and implement a new arbitrage free model for the futures prices of energy. In this respect, we extend the idea proposed in [4, 5] to the energy market. Following this approach, we assume that the term structure of futures prices on electricity given by Schwartz-Smith model is perturbed by an error term. This error term is represented by a stochastic integral that generates infinite dimensional noise, as it should depend on all time of, or to maturity. In this model, we do not need to add artificial noises to the observation equation in order to estimate the model parameters. The factors are estimated as solutions of a filtering problem. Moreover, in this approach, the modeling of the correlation structure between the futures (observation) is a natural component of our formulation. Secondly, this paper estimates the parameters of the model without any modification to the nonlinear payoff of the futures (observation). As the new futures price formula is highly nonlinear with respect to the factors, we use the convolution particle filter algorithm proposed by [6] and [7]. This approach is based on kernel estimation techniques and has the advantage compared to other particle filtering algorithms in that, it is free of the analytical knowledge of both the state and observation variable distributions. Only the capability of simulating the state and observation noises is required. Moreover, it can handle the problem of small magnitude observation noise which is typical in financial data. Moreover, and on the empirical side, we test the feasibility of the filter using real data from the European energy market.

The paper is organized as follows. In Section 2, we review the Schwartz-Smith model [3]. In Section 3, we present the new model for the future price for one maturity date  $T$ , by using the idea proposed in [4]. In Section 4, we focus our attention on the electricity futures situation and we derive the explicit formula of the futures prices, and present the observation mechanism of the data. In Section 5, the convolution filter [7] based on the particle filtering approach is presented for the augmented state variable (state and parameters) as was used in [8, 9], The last two Sections, 6 and 7, contain the empirical work and conclusion, respectively.

**2. The Schwartz-Smith Spot Price Model.** Let  $S(t)$  represent the spot price of a commodity (electricity) at time  $t$ . Following [3], we decompose the logarithm of the spot price into two stochastic factors as

$$\ln(S(t)) = \chi(t) + \xi(t) + h(t) \quad (1)$$

where  $\chi(t)$  represents the short-term deviation in the price,  $\xi(t)$  is the equilibrium price level and  $h(t)$  is a deterministic seasonality function. Assume that the risk-neutral stochastic process for the two factors are of the form

$$\begin{cases} d\chi(t) = (-\kappa\chi(t) - \lambda_\chi)dt + \sigma_\chi dW_\chi^*(t) \\ d\xi(t) = \lambda_\xi dt + \sigma_\xi dW_\xi^*(t) \end{cases} \quad (2)$$

where  $W_\chi^*$  and  $W_\xi^*$  are correlated standard Brownian motions, where  $dW_\chi^*dW_\xi^* = \rho dt$ . We denote the current time by  $t$ , the maturity of the futures by  $T$ , the time to maturity by  $x$ , where  $x = T - t$ , and by  $T^*$  a fixed time horizon, where  $t \leq T < T^*$ . The futures price  $F(t, T)$  is given by

$$F(t, T) = \exp(B(t, T)\chi(t) + C(t, T)\xi(t) + A(t, T)) \quad (3)$$

where

$$B(t, T) = e^{-\kappa(T-t)}, \quad C(t, T) = 1 \quad (4)$$

$$A(t, T) = \frac{\lambda_\chi}{\kappa} (e^{-\kappa(T-t)} - 1) + \lambda_\xi(T-t) + \frac{1}{2}\sigma_A^2(t, T) + h(T) \quad (5)$$

and

$$\begin{aligned} \sigma_A^2(t, T) &= \frac{\sigma_\chi^2}{2\kappa} (1 - e^{-2\kappa(T-t)}) + \sigma_\xi^2(T-t) \\ &\quad + 2\frac{\rho\sigma_\chi\sigma_\xi}{\kappa} (1 - e^{-\kappa(T-t)}) \end{aligned}$$

**3. A New Model for the Electricity Prices.** In term structure modeling, as in modeling the futures prices of electricity, the state is a function of two variables;  $t$  (the time) and  $x$  (the time to maturity). For stochastic modeling, it is then natural to introduce two-parameter Brownian motion  $w(t, x)$ . One way of defining this is to consider  $w(t, x)$  to be a stochastic process in  $t$  with values in the space of functions of  $x$ . If these functions are in a (separable) Hilbert space, we may think of  $w(t, x)$  as a Hilbert space valued stochastic process in  $t$ , see [10]. Hence, we assume that the correct model for the spot price is not exactly the same as in (1), but is close to it. Given this, the futures price will be somewhat perturbed from the formula given in (3). Suppose that the correct futures price at time  $t$  where  $t \leq T$  is given by

$$\begin{aligned} F^{corr}(t, T) &= \exp[\bar{B}(t, T)\chi(t) + \bar{C}(t, T)\xi(t) \\ &\quad + \bar{A}(t, T) + \int_0^t \sigma dw(s, T-s)] \end{aligned} \quad (6)$$

where

$$\int_0^t \sigma dw(s, T-s) = \sum_{k=1}^{\infty} \int_0^t \sigma \frac{1}{\lambda_k} e_k(T-s) d\beta_k(s) \quad (7)$$

and where  $e_k$  is a sequence of differentiable functions forming an orthonormal basis in  $L^2(0, T^*)$  and  $\{\beta_k(t)\}$  are mutually independent Brownian motion processes. Let  $q(y_1, y_2)$  represent the correlation of  $w(t, y_1)$  and  $w(t, y_2)$ . The extra stochastic integral term (7), represents the modeling error between the futures price given by (3) and the correct futures price. When  $T = t$ , the correct spot price process is given by

$$S^{corr}(t) \equiv F^{corr}(t, t) \quad (8)$$

To get the corresponding (correct) dynamics for the spot, we need the dynamics of the futures taking into account its dynamics under the risk-neutral measure to be a martingale. Applying Ito's formula to (6), we get  $\bar{B}(t, T) = B(t, T)$ ,  $\bar{C}(t, T) = C(t, T)$  given by (4) and

$$\bar{A}(t, T) = A(t, T) + \frac{1}{2}\sigma^2 \int_0^{T-t} q(z, z) dz \quad (9)$$

where  $A(t, T)$  is given by (5). Substituting  $\xi(t)$  in (6), we obtain

$$\begin{aligned} F^{corr}(t, T) &= \exp(B(t, T)\chi(t) + \tilde{A}(t, T) \\ &\quad + \int_0^t [\sigma dw(s, T-s) + \sigma_\xi dW_\xi^*(s)]) \end{aligned} \quad (10)$$

where

$$\tilde{A}(t, T) = \bar{A}(t, T) + \lambda_\xi t + \xi(0). \quad (11)$$

Using (8), the correct spot price process is given by

$$\begin{aligned} S^{corr}(t) &= F^{corr}(t, t) \\ &= \exp(\chi(t) + h(t) + \lambda_\xi t \\ &\quad + \int_0^t \{\sigma dw(s, t-s) + \sigma_\xi dW_\xi^*(s)\}). \end{aligned}$$

From now on, we omit writing the expression "corr" for  $S(t)$  and  $F(t, T)$  processes.

**4. A Practical Model for the Electricity Prices.** The market prices of electricity futures are different from the standard futures traded in other markets. The electricity futures prices are based on the arithmetic averages of the spot prices over a delivery period  $[T_0, T]$ , given by

$$\frac{1}{T - T_0} \int_{T_0}^T S(\eta) d\eta. \quad (12)$$

where we use continuous-time arithmetic average to conform to our model formulation. Now, for  $t < T$ , we can calculate the futures price by

$$F(t, T_0, T) = \mathbf{E}\left\{\frac{1}{T - T_0} \int_{T_0}^T S(\eta) d\eta \middle| \mathcal{F}_t\right\}, \quad (13)$$

where  $\mathcal{F}_t = \sigma\{S(\eta); 0 \leq \eta \leq t\}$ . Assuming that  $S(t) \in L^2(T_0, T)$ , and using the linearity of the expectation operator, (13) can be written as

$$F(t, T_0, T) = \frac{1}{T - T_0} \int_{T_0}^T \mathbf{E}\{S(\eta) d\eta \middle| \mathcal{F}_t\}, \quad (14)$$

Using the definition of futures price, it can be simplified as

$$F(t, T_0, T) = \frac{1}{T - T_0} \int_{T_0}^T F(t, \eta) d\eta, \quad (15)$$

This price using (10) satisfies

$$\begin{aligned} F(t, T_0, T) &= \frac{1}{T - T_0} \int_{T_0}^T \exp\left[B(t, \eta)\chi(t) + \tilde{A}(t, \eta) \right. \\ &\quad \left. + \int_0^t \{\sigma dw(s, \eta - s) + \sigma_\xi dW_\xi^*(s)\}\right] d\eta, \end{aligned} \quad (16)$$

where  $B$  and  $\tilde{A}$  satisfy the same equations (4) and (9).

4.1. **A forward model.** In (16), setting  $x = \eta - t$  and

$$f(t, x) = B(t, x + t) \chi(t) + \tilde{A}(t, x + t) + \int_0^t \{ \sigma dw(s, x + t - s) + \sigma_\xi dW_\xi^*(s) \}$$

we get

$$df(t, x) = \frac{\partial f(t, x)}{\partial x} dx - \frac{1}{2} \tilde{q}(x, x) dt + d\tilde{w}(t, x), \quad (17)$$

where

$$\tilde{w}(t, x) = \sigma w(t, x) + e^{-\kappa x} \sigma_\chi W_\chi^*(t) + \sigma_\xi W_\xi^*(t), \quad (18)$$

and

$$\begin{aligned} \tilde{q}(x_1, x_2) = \sigma^2 q(x_1, x_2) + \frac{\rho \sigma_\chi \sigma_\xi}{2} (e^{-\kappa x_1} + e^{-\kappa x_2}) \\ + \sigma_\chi^2 e^{-\kappa(x_1 + x_2)} + \sigma_\xi^2. \end{aligned} \quad (19)$$

Hence the futures price becomes

$$F(t, T_0, T) = \frac{1}{T - T_0} \int_{T_0}^T \exp[f(t, x)] dx.$$

**Remark 4.1.** *Usually the identification is performed under the real world measure. This implies that the market price of risk terms are included in (17). Here we neglect these terms because our identification procedure in Section 5 is easily applied to the model under the real world measure. See [5] for the detailed procedure.*

4.2. **Observation mechanism.** In practice, the observation data for  $m$  different futures are available on a daily basis. These data are transformed in terms of a fixed time to delivery for each futures contract. That is, for each time  $t$ , if we denote by  $\tau_i = T_0^i - t$  the time to delivery of future  $i$ , where  $i = 1, 2, \dots, m$ . Then  $\tau_i$  represents is kept fixed through time  $t$ . Moreover, the delivery period  $\theta^i = T^i - T_0^i$  of all the available futures is also transformed, and set to be equal to constant period  $\theta$  of 1-month. Hence the usual observation data becomes

$$\begin{aligned} y_i(t) &= \log F(t, \tau_i + t, \tau_i + t + \theta) \\ &= \log \frac{1}{\theta} \int_{\tau_i}^{\tau_i + \theta} \exp\{f(t, x)\} dx, \\ &\text{for } \tau_1 < \tau_2 < \dots < \tau_m. \end{aligned} \quad (20)$$

and we set the observation vector as

$$\vec{Y}(t) = [y_1(t), y_2(t), \dots, y_m(t)]'.$$

5. **Algorithm of the Convolution Particle Filter.** In this section, we are interesting in estimating the parameters denoted by  $\Theta$ , of the new nonlinear state space model given by equation (17) for the state, and equation (20) for the observation. One way to handle this nonlinear estimation problem, is often based on an approximation of the optimal nonlinear filter by using the extended Kalman filter (EKF) and its various alternatives, coupled with maximum likelihood estimation techniques. However, the EKF methods sometimes encounter problems in practice. Another approach, is to employ the Bayesian framework, in which  $\Theta$  is considered as a random variable with a prescribed a priori density

function. Then an extended state variable  $(X_k, \Theta)$  joining all the unknown quantities is considered, the posterior density of  $(X_k, \Theta)$  given the observation till time  $k - 1$ , denoted by  $\mathbf{Y}_{k-1} = \{Y_1, Y_2, \dots, Y_{k-1}\}$ , is approximated using particle filters. In general, the classical particle filters using sample importance resampling (SIR) or auxiliary sampling importance resampling (ASIR), can handle the augmented state vector if a dynamic noise term is artificially added to the parameters. But, the drawback of this, is the reduction of the estimation performance of the filter. On the other hand, another algorithm which avoids adding extra artificial noises to the parameters is the convolution particle filter. This approach is based on kernel estimation techniques and it is free of the analytical knowledge of both the state and observation variable distributions. Only the capability of simulating the state and observation noises is required. Moreover, it can handle the problem of small magnitude observation noise which is typical in financial data. For more details about the algorithm, we refer to [7, 6]. Before writing the necessary steps that cast our estimation problem into the convolution filter algorithm, we assume that the forms of the seasonality function  $h(x)$  and the covariance kernel  $q(x, x)$  are known, (the details of obtaining these, will be explained in the following section). With this in mind, the following steps of the filter are as follows:

- First we need to generate the initial values of the state  $f(0, x)$ , using equation (17), with  $t = 0$ , we get

$$\begin{aligned} f(0, x) = & \frac{1}{2}\sigma^2 \int_0^x q(z, z)dz + \xi(0) + \frac{\lambda_\kappa}{\kappa}(e^{-\kappa x} - 1) + \lambda_\xi x \\ & + h(x) + \frac{\sigma_\chi}{\kappa}(1 - e^{-2\kappa x}) + \frac{1}{2}\sigma_\xi^2 x + e^{-\kappa x} \chi(0) \\ & + \rho \frac{\sigma_\chi \sigma_\xi}{\kappa}(1 - e^{-\kappa x}). \end{aligned} \quad (21)$$

- Set all the unknown parameters as

$$\Theta = \sigma, \xi(0), \kappa, \dots$$

- Generation of  $N$  i.i.d particles:  $\Theta^{(1)}, \Theta^{(2)}, \dots, \Theta^{(N)}$  in some bounded regions.
- From above generation, we get from (21)

$$f^{(i)}(0, x) \sim f(0, x; \Theta^{(i)}) \text{ for } i = 1, 2, \dots, N$$

- Evolving step:  $f^{(i)}(t_{j+1}, x) \sim$  from (17)
- From (20), we automatically get  $N$  observation data:

$$\begin{aligned} \tilde{Y}^i(t_{j+1}) = & \left[ \log \frac{1}{\theta} \int_{\tau_1}^{\tau_1+\theta} \exp\{f^{(i)}(t_{j+1}, x)\} dx, \dots, \right. \\ & \left. \log \frac{1}{\theta} \int_{\tau_m}^{\tau_m+\theta} \exp\{f^{(i)}(t_{j+1}, x)\} dx \right]' \end{aligned}$$

for  $i = 1, 2, \dots, N$ .

- Calculate the time differences:

$$\Delta \tilde{Y}_j^i = \tilde{Y}^i(t_{j+1}) - \tilde{Y}^i(t_j)$$

and similarly for the observation data  $\vec{Y}(t)$

$$\Delta \vec{Y}_j = \vec{Y}(t_{j+1}) - \vec{Y}(t_j)$$

- Approximation of  $p(Y(t)|f(t, x))$ :

$$\hat{p}^{(N)}(\vec{Y}(t_{j+1})|f^{(i)}(t_{j+1}, x)) = \frac{1}{Nh^q} \sum_{i=1}^N K_h(\Delta\vec{Y}_j - \Delta\tilde{Y}_j^i)$$

where  $K_h(\cdot)$  is a Parzen-Rosenblatt kernel, see [7].

- approximation of the weight of  $f^{(i)}(t, x)$ :

$$\omega_{t_{j+1}}^i = \hat{p}^{(N)}(\vec{Y}(t_{j+1})|f^{(i)}(t_{j+1}, x))$$

- normalization of the weight :

$$\hat{\omega}_{t_{j+1}}^i = \frac{\omega_{t_{j+1}}^i}{\sum_{i=1}^N \omega_{t_{j+1}}^i}$$

- approximation steps:

$$\begin{aligned} p(f(t, x)|Y(t)) &= \sum_{i=1}^N \hat{\omega}_{t_{j+1}}^i \delta_{f^{(i)}(t_{j+1}, x)} \\ p(\Theta|Y(t)) &= \sum_{i=1}^N \hat{\omega}_{t_{j+1}}^i \delta_{\Theta^{(i)}}, \end{aligned}$$

where  $\delta$  is a  $\delta$ -measure.

- resampling for  $\Theta^{(i)}$ . ( This implies the resampling for the state  $f^{(i)}$ .)

Here the kernel  $K : \mathbb{R}^d \mapsto \mathbb{R}$  is a bounded positive symmetric function such that  $\int K(x) dx = 1$ . For example, the Gaussian kernel is  $K(x) = \left(\frac{1}{\sqrt{2\pi}}\right)^d e^{-\frac{\|x\|^2}{2}}$ . The Parzen-Rosenblatt kernel is a kernel such that  $\|x\|^d K(x) \rightarrow 0$  as  $\|x\| \rightarrow \infty$ . Typically the following notation is normally used

$$K_{h_n}(x) \equiv \frac{1}{h_n^d} K\left(\frac{x}{h_n}\right) \quad (22)$$

where  $h_n > 0$  is the bandwidth parameter. and  $n$  is the number of particles. The value of  $n, h_n$  and the kernel must be chosen by the user.

**6. Simulation Studies.** First, we simulate the observation data such that it will be similar to the real data. We used a real data set which includes a historical time-series of UK-Gas-NBP spot and futures prices quoted daily from 2-Jan-2007 to 28-Dec-2008.

Following [11], from the spot data we identify the parameters as follows:

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$$[\hat{a}, \hat{b}] = \operatorname{argmin}_{a,b} \int_0^1 |(S(t) - (a + bt))|^2 dt.$$

We get  $\hat{a} = 3.0362, \hat{b} = 0.2698$ .

- By using FFT, we picked up the first 2 frequencies  $\omega_1, \omega_2$  from the biggest magnitude:

$$\begin{aligned} S(t) - (\hat{a} + \hat{b}t) \\ \sim \sum_{k=1}^2 [m s_k \sin(2\pi f_k \tau) + m c_k \cos(2\pi f_k \tau)] \end{aligned}$$

Table 1 shows the obtained estimates of these parameters. The real data and the fitted curve are shown in Figure 1.

TABLE 1. Estimated parameters

$k$	1	2
$f_k$	1.0040	2.0080
$ms_k$	7.9838	3.1169
$mc_k$	1.4593	0.4337

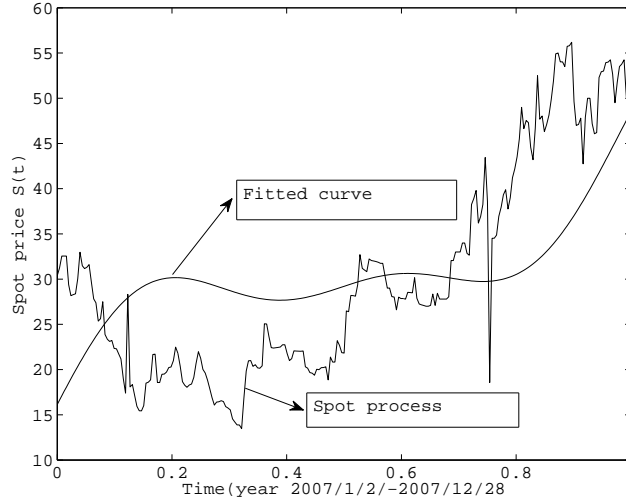


FIGURE 1. Real and estimated spot curves

The periodic part  $h_p(t)$  of  $h(t)$  is set as

$$h_p(t) = \sum_{k=1}^2 [ms_k \sin(2\pi f_k \tau) + mc_k \cos(2\pi f_k \tau)]. \quad (23)$$

We set the system parameters as follows:

$$\begin{aligned} \kappa &= 1.321, \quad \lambda_\chi = 0.623, \quad \sigma_\chi = 0.3, \\ \lambda_\xi &= 0.04, \quad \sigma_\xi = 0.05, \quad \rho = 0.6 \end{aligned}$$

The seasonality function is set as

$$h(t) = 3.0362 + 0.2698t + h_p(t).$$

For the initial conditions of  $\chi, \xi$ , we use

$$\chi(0) = 0.02, \quad \xi(0) = 0.5.$$

We assume that the covariance kernel of  $\sigma w(t, x)$  is given by

$$\sigma^2 q(x, y) = \sum_{k=1}^{100} \sigma^2 e^{-c(x+y)} \left( \frac{\sin(k\pi x)}{5} \right) \left( \frac{\sin(k\pi y)}{5} \right),$$

with  $\sigma = 0.02, c = 0.2$ .

The simulated observation data of the futures is shown in Figure 2. For  $T - T_0$  fixed to 1-month, the factor process  $f(t, x)$  is also demonstrated in Figure 3.



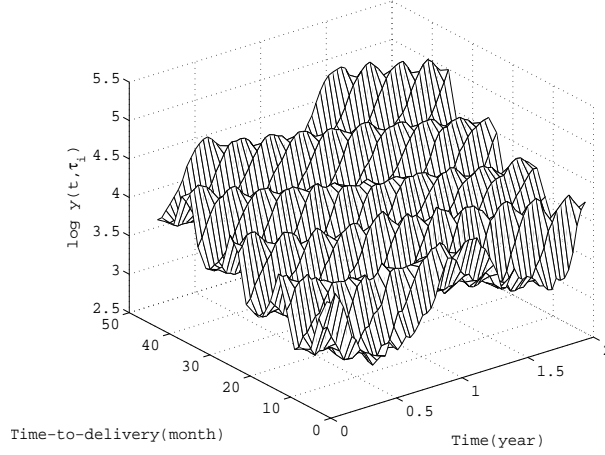
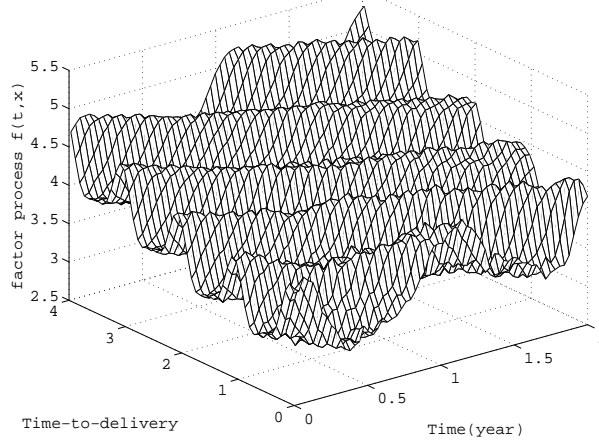


FIGURE 2. Observation data

FIGURE 3.  $f(t, x)$ -process

**6.1. Simulation for particle convolution filter.** We assume that the unknown parameters  $\{\kappa, \lambda_\chi, \sigma_\chi, \lambda_\xi, \sigma_\xi, \rho, \sigma, c\}$  are random constants where each follows a bounded uniform distribution. The upper and lower bounds are:

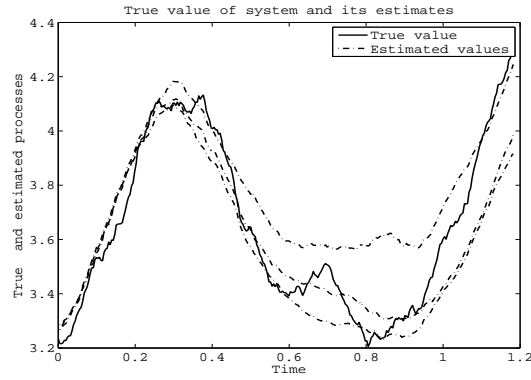
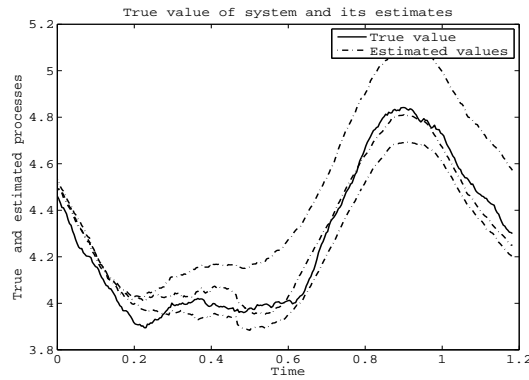
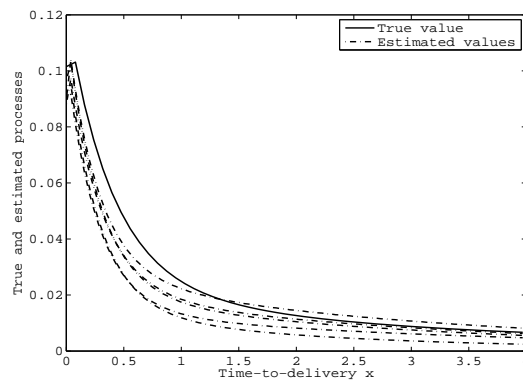
$$\begin{aligned} 1.00 &\leq \kappa \leq 2.00 & , & & 0.1 &\leq \lambda_\chi \leq 1.00 \\ 0.28 &\leq \sigma_\chi \leq 0.32 & , & & 0.001 &\leq \lambda_\xi \leq 0.06 \\ 0.02 &\leq \sigma_\xi \leq 0.06 & , & & 0.20 &\leq \rho \leq 0.90 \\ 0.01 &\leq \sigma \leq 0.03 & , & & 0.1 &\leq c \leq 0.3 \end{aligned}$$

The initial conditions of  $\chi(0)$  and  $\xi(0)$  are assumed to be  $N(0.02, 0.005)$  and  $N(0.9, 0.005)$ , respectively.

Now we generate 500 particles for  $(\kappa, \lambda_\chi, \sigma_\chi, \lambda_\xi, \sigma_\xi, \rho, \sigma)$  vector. Hence, we get the 500 initial conditions  $f(0, x)$  from (21) and also 500  $\tilde{q}(x, x)$  functions from (19). The Parzen-Rosenblatt kernel is set as

$$K_h(\cdot) = \frac{1}{(2\pi h)^m} \exp\left\{-\frac{\|\cdot\|^2}{2h^2}\right\},$$

with  $h^2 = 0.09$  and  $m$  is the dimension of  $\vec{Y}$  ( $m = 44$ ). We performed simulation studies for 5 times. The estimates for  $f(t, x)$  at  $x = 0.79$  and  $x = 3.19$  are shown in Figure 4 and Figure 5, respectively. The estimate of  $\tilde{q}(x, x)$  is also shown in Figure 6.

FIGURE 4. Estimated  $f(t, 0.79)$ FIGURE 5. Estimated  $f(t, 3.16)$ FIGURE 6. Estimated  $\tilde{q}(x, x)$ 

Finally we demonstrated the estimation results for unknown parameters  $\kappa, \sigma_\chi, \lambda_\chi, \lambda_\xi, \sigma_\xi, \rho, \sigma$  and  $c$ . From Figure 6, we found that even if some of the parameters contained in the unknown function  $\tilde{q}(x, x)$  are not well fitted to its true values, the function  $\tilde{q}(x, x)$  as a whole is well identified. Hence, we may ignore the precise estimation of each parameter in  $\tilde{q}(x, x)$ .

**7. Conclusion.** In this paper, we propose a new arbitrage free model for the futures prices of energy. The new model can be used without adding any artificial noises to the observation equation in order to estimate the model parameters. The factors are estimated as solutions of a filtering problem. As our futures pricing formula for energy futures is nonlinear, and to handle the problem of small magnitude observation noise which is

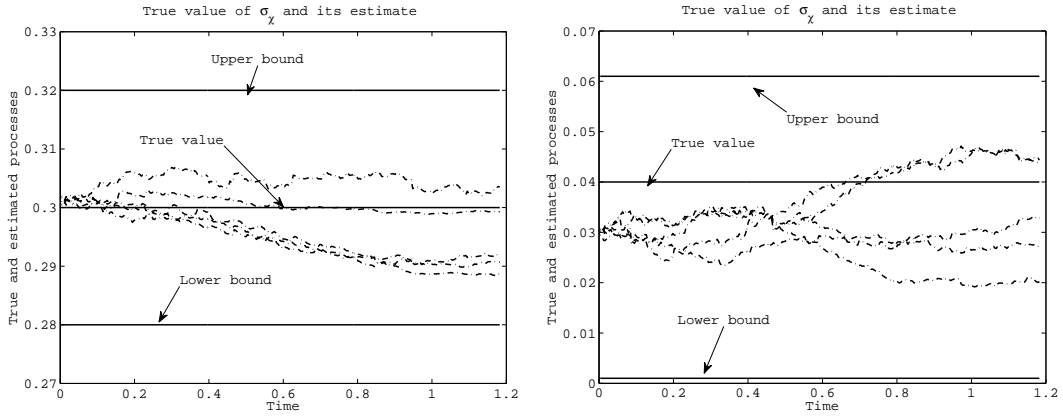


FIGURE 7. Estimated  $\sigma_\chi$  and  $\lambda_\xi$

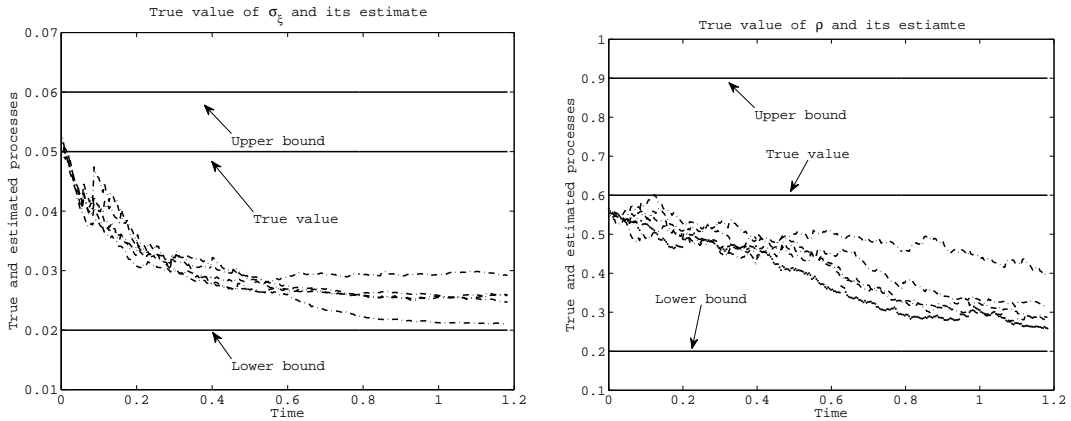


FIGURE 8. Estimated  $\sigma_\xi$  and  $\rho$

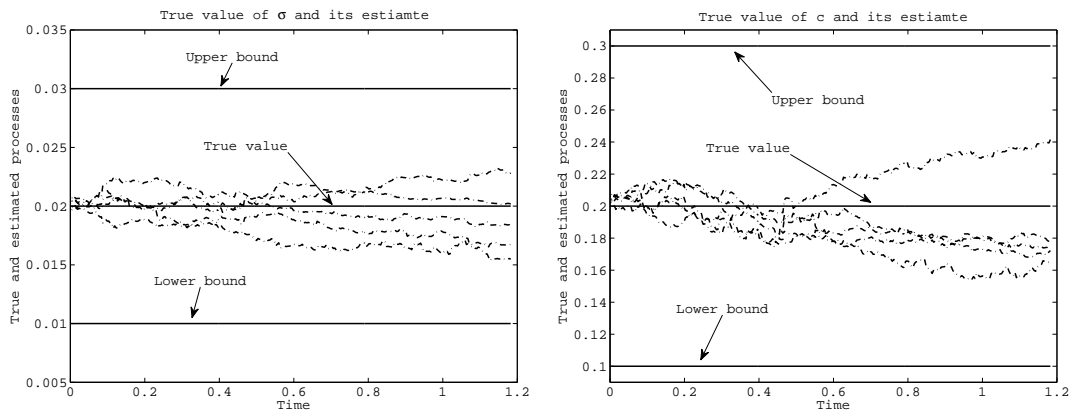


FIGURE 9. Estimated  $\sigma$  and  $c$

typical in financial data. We employ the particle filtering methodology, and in particular the convolution particle filter. We cast our state space model within this algorithm. The estimation procedures are all performed under the risk neutral measure, this means that parameters obtained here can be used in pricing other derivatives. Moreover, using UK-Gas-NBP spot and futures data, we run a simulation study to test the feasibility of the proposed filter.

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