KINETIC MODELLING OF A SELF-SUSTAINED TEA CO_2 LASER

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Recently new experimental values for the electron-ion recombination and the avalanche coefficients have been obtained. The results have been used to predict the time behaviour of the laser current and voltage. Excellent agreement between theory and experiment could be obtained.

1. Introduction

In order to predict the time behaviour of the gain and output power of a TEA CO_2 laser detailed knowledge is required of the time behaviour of the gas discharge. At a specified time the values of the current and the voltage determine the electron density and the electron energy distribution function; these two quantities determine the excitation rates to the various excitation levels relevant for the lasing process.

Several more or less sophisticated calculations have been made to determine the behaviour of the gain and output power as a function of time from excitation rates calculated from a known current pulse. However, to our knowledge no results have been published where the current and voltage pulse of a self-sustained TEA CO_2 laser have been predicted from first principles, i.e. starting from the electrical scheme of the system. The only investigation that has been made deals with the formation of the discharge [1]. The reason why so little attention has been paid to the problem may be a lack of knowledge of the values of the electron-ion recombination coefficient and the electron-amplification coefficient, especially in the important voltage region around the stabilization voltage.

In a recent publication [2] we published data for the electron amplification coefficient $\delta = (\alpha - a)v_d$, where α and a are the ionization and attachment coefficients respectively and v_d the drift velocity, and the electron-ion recombination coefficient γ in a $CO_2 : N_2 : He = 1 : 1 : 3$ mixture for E/N values ranging up to 5.3×10^{-16} V cm^2. We will use these values for calculating the current and voltage pulses in a 1 : 1 : 3 mixture for different charge voltages of the storage capacitor of our laser system and compare it with measurements.

2. Experimental

The experiments have been performed with the system described in [3]. The gap distance between the electrodes is 5 cm. The electrodes are profiled according to Chang's paper with $k = 0.02$ and $v = \arccos(-k)$. The discharge length is 40 cm. Our system has a built-in UV source, which produces a large amount of radiation because of the rapid current rise and the large area of the source. In this way a very high level of preionization is reached, resulting in a very homogeneous discharge without streamers and an excellent shot-to-shot reproducibility. A more detailed description can be found in [3] and [4]. The profiles are activated by a 5-stage marx generator designed for low self-inductance. The connection between laser head and marx generator acts as a peaking capacitor and plays an important role.

The current measurements have been carried out by measuring the voltage over a resistance of 0.1 Ω, placed directly in series with the laser head. For the voltage

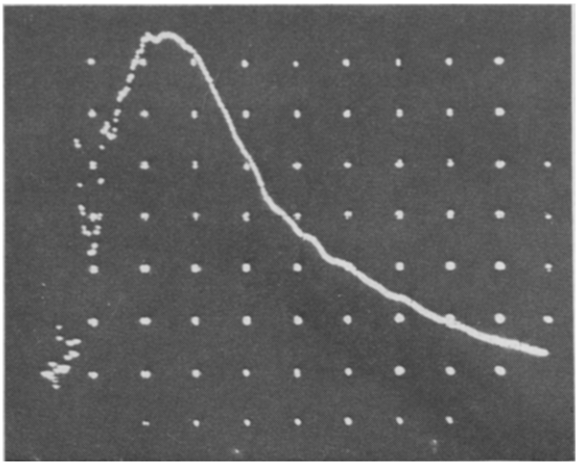

Fig. 1. The current as a function of time. At the horizontal axes the time is plotted in 100 ns/div. The total marx-generator voltage was 97.5 kV and the capacity 16 nF. The laser mixture was $CO_2 : N_2 : He = 1 : 1 : 3$.

measurements a copper sulphate resistance divider has been used. The divider was connected directly to both laser profiles. The attenuation measured with a 200 ns pulse line charged to 20 kV turned out to be 213 times. The rise time of both probes lie in the nanosecond region. Further details can be found in [2]. A typical example of the current and voltage measurements can be found in figs. 1 and 2 respectively. For both pictures the marx generator was charged to 19.5 kV. The gas mixture consisted of 1 part CO_2, 1 part N_2 and 3 parts He. At the horizontal axis of both pictures the time is plotted in 100 ns/div. At the vertical axis of fig. 1 the current is plotted in units of 1017 A/div. and at the vertical axis of fig. 2 the voltage in units of 21.66 kV/div. The oscillatory behaviour in both pictures is due to the peaking capacitors.

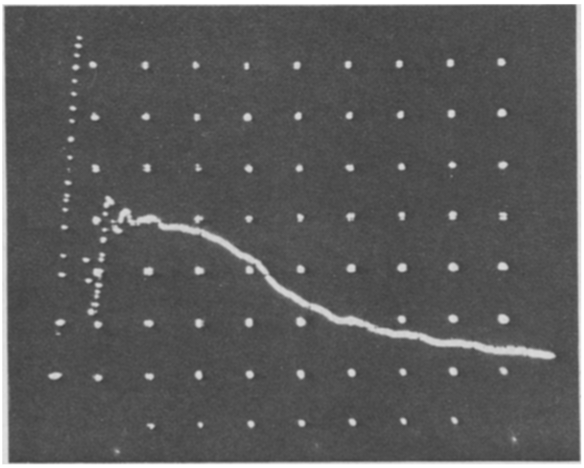

Fig. 2. The voltage as a function of time. The conditions are the same as for fig. 1.

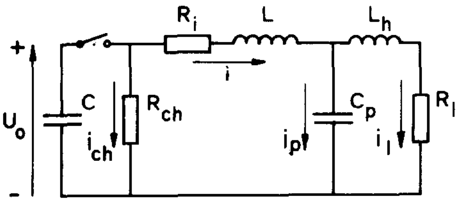

Fig. 3. Electrical scheme of the system.

The electrical circuit of the system is schematically represented in fig. 3. The total marx-generator capacity C was 16 nF for all measurements and the peaking capacity C_p, originating from the connection between the marx generator and laser head, appeared to be 180 pF. The total self-inductance of the discharge circuit has been determined by replacing the laser head by a low resistance. The resulting oscillatory behaviour gave us an accurate value for this self-inductance L (= 1.07 μH). The self-inductance L_h of the laser head is not easy to determine, but does not play an important role. Its estimated value is 50 nH. R_{ch} is the charge resistance of the capacity C. Its value is 2.3 kΩ. A difficult problem is the determination of the internal impedance of the marx generator R_i, which originates from the six spark gaps. It is well-known that the resistance of a spark gap is strongly time-dependent and is accompanied by a time dependence of the self-inductance [5]. The latter, however, can be neglected compared to the large total self-inductance of the system. In order to find the time-dependence of the resistance of the marx generator we replaced the laser head by a 13 Ω copper sulphate resistor, giving almost critical damping. We measured the current through and the voltage over this resistance for several charge voltages. A computer analysis then gives the resistance characteristic. We found the fall time of the resistance to be slightly dependent on the charge voltage and the minimum value of the resistance to be a function of the current through the circuit. The resistance increases again when the current in the circuit decreases. Although for every charge voltage a

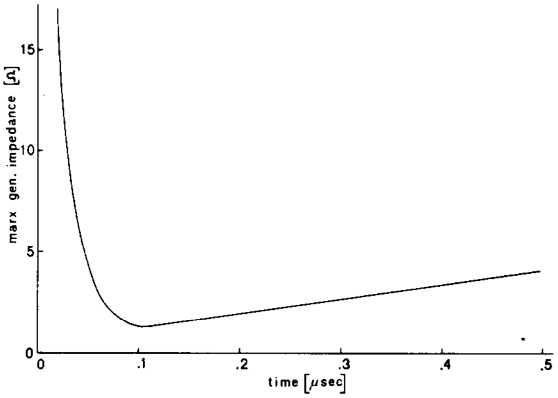

Fig. 4. The internal resistance of the marx generator plotted as a function of time.

different characteristic should be used for the sake of simplicity and also because of uncertainties in the measured values, only one characteristic is used throughout the calculations, which is presented in fig. 4. It basically is the measured characteristic for a marx-generator charge voltage of 100 kV (20 kV per stage) adapted for a 3.3 kA laser current.

3. Computer modelling

For the computer simulation of the laser current and voltage we used the scheme of fig. 3. Because the preionization discharge is also fed from the marx-generator energy, one should take this discharge into account. However, very little is known about such a sliding discharge and because the total energy dissipated in it is small (typically not more than a few percent of the marx-generator energy), neglecting it only leads to a small error. Instead we assumed a uniform electron density is present at time $t = 0$ between the laser electrodes, due to the preionization discharge. This electron density is the only parameter left in our calculations.

The time-dependent electron density n_e can be found by integrating the following equation:

$$\mathrm{d}n_e/\mathrm{d}t = (\alpha - a)v_d n_e - \gamma n_e^2. \tag{1}$$

For the recombination coefficient γ and the avalanche coefficient $\delta = (\alpha - a)v_d$ we used the values published in [2]. Because no values for δ could be measured for E/N values exceeding 5.3×10^{-16} V cm^2, we extrapolated our curve linearly from $\delta = 1.15 \times 10^7$ s^{-1} at $E/N = 5.3 \times 10^{-16}$ V cm^2 to $\delta = 1.8 \times 10^8$ cm^{-1} at $E/N = 10^{-15}$ V cm^2. Also the recombination coefficient values γ have been extrapolated for high and low E/N values. It appeared, however, that the recombination at those E/N values has little effect on the current and voltage characteristics so that the extrapolated values are unimportant.

Once the electron density is known, the laser impedance R_L can be determined from

$$R_L = V/n_e e v_d S, \tag{2}$$

where V is the laser voltage, S the cross-section of the discharge and e the electron charge. For the drift velocity v_d of the electrons we used the values from [6].

In calculating the laser current and voltage we included the influence of the charge resistor and the voltage and current probe resistances. From the scheme of fig. 3 the following equations can be deduced:

$$U_0 - \frac{1}{C}\int_0^t (i + i_{ch})\,\mathrm{d}t = i_{ch}R_{ch}, \tag{3}$$

$$U_0 - \frac{1}{C}\int_0^t (i + i_{ch})\,\mathrm{d}t = L\frac{\mathrm{d}i}{\mathrm{d}t} + iR_i + \frac{1}{C_p}\int_0^t i_p\,\mathrm{d}t, \tag{4}$$

$$\frac{1}{C_p}\int_0^t i_p\,\mathrm{d}t = L_H\frac{\mathrm{d}i_\ell}{\mathrm{d}t} + i_\ell R_L. \tag{5}$$

U_0 is the initial charge voltage of the marx generator, so that the left-hand side of eq. (1) gives the voltage on C at time t. i_{ch} is the leak current in the marx generator flowing through R_{ch}, i is the total current delivered by the marx generator, i_p is the current through the peaking capacitor and i_ℓ is the laser current. The latter three currents have to satisfy the relation

$$i = i_p + i_\ell. \tag{6}$$

Eqs. (3)–(6) have been solved numerically under the simplifying assumption that $\mathrm{d}i/\mathrm{d}t$ (= k) is constant in the time interval between t and $t + \Delta t$. For that case differentiation of (5) and substitution of the relations

$$di_\ell/dt = k - di_p/dt$$

and

$$d^2 i_\ell/dt^2 = -d^2 i_p/dt^2$$

leads to

$$\frac{d^2 i_p}{dt^2} + \frac{R_L}{L_h}\frac{di_p}{dt} + \frac{1}{C_p L_h} i_p - k\frac{R_L}{L_h} = 0. \qquad (7)$$

This equation can easily be solved for the time interval Δt, if the initial conditions at time t are known. Of course the laser current and voltage are then easy to determine.

Figs. 5, 6, and 7 show the result of our calculations and experiments for different marx-generator charge voltages. Figs. 5a and 5b show the voltage and current pulses respectively for 85.3 kV charge voltage, figs. 6 for 97.5 kV and figs. 7 for 117 kV. The solid lines are the experimental curves and the dashed lines the theoretical ones. As can be seen, the resemblence between the theoretical and experimental curves is quite good. Our model, however, cannot explain the fast rise time of the first current peak. This fast rise is due to the fact that the current in the early stage of the pulse flows alongside the glass plate and the avalanche of a surface discharge is much faster than in a CO_2–N_2–He mixture. This is also the reason why the experimental current peaks are higher than the theoretical ones. For all three charge voltages there is a slight discrepancy: the experimental pulses are a little shorter than the theoretical ones. This can probably be improved by taking the energy dissipated in the preionization discharge into account. However, we think the discrepancy is within the experimental

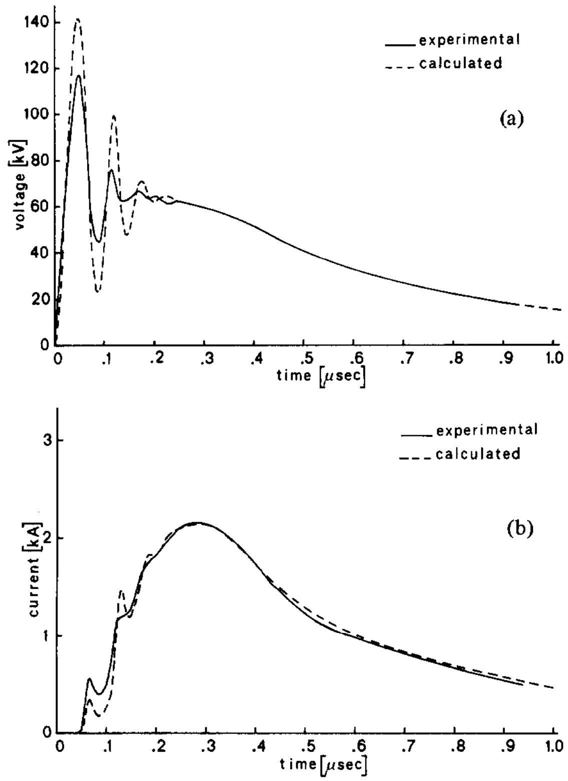

Fig. 5. (a) shows the voltage and (b) the current as a function of time. The time scale is 100 ns/div. The total marx-generator capacity is 126 nF. The solid curves are experimental and the dashed ones are theoretical. For the theoretical curves an initial electron dnseity of 10^9 cm^{-3} is used.

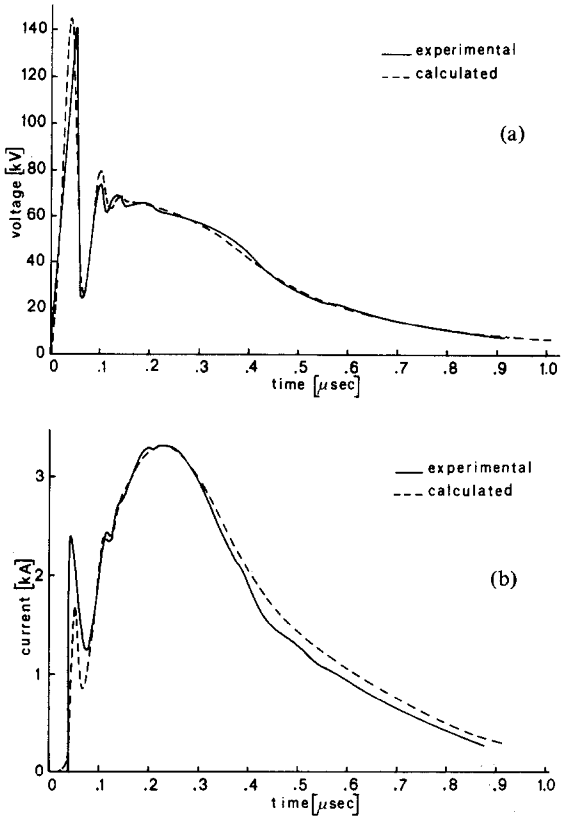

Fig. 6. (a) shows the voltage and (b) the current as a function of time for a marx-generator voltage of 97.5 kV. For the theoretical curves an initial electron density of 5×10^{10} cm^{-3} is used.

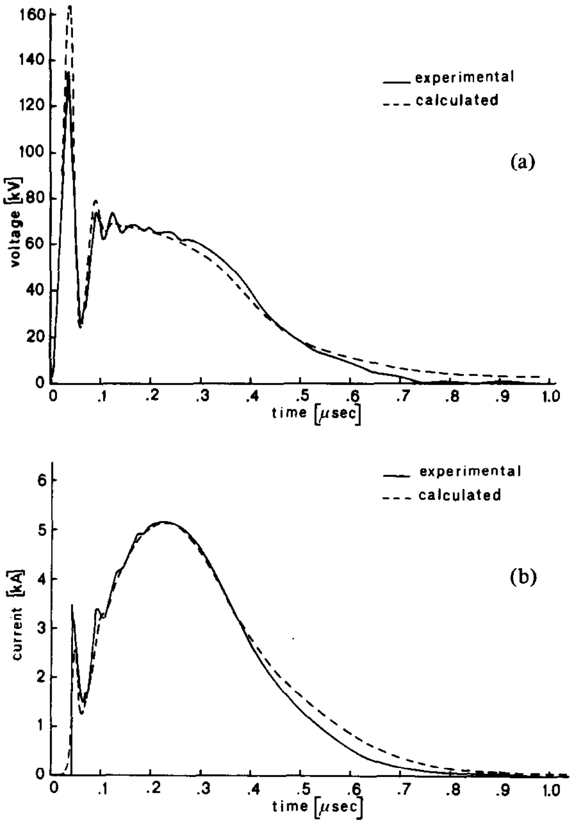

Fig. 7. (a) shows the voltage and (b) the current as a function of time for a marx-generator voltage of 117 kV. For the theoretical curves an initial electron density of 5×10^{10} cm^{-3} is used.

inaccuracy, with which all different parameters and especially the gap impedance have been determined.

Since only one real parameter, i.e. the initial electron density n_{e0} is left in our calculations, it is interesting to see the influence of this parameter on the current pulse. Fig. 8 shows the current calculations for three different values of n_{e0} for a marx-generator voltage of 97.5 kV. As can be seen, the parameter n_{e0} has only a slight influence on the pulse form. It mainly determines the delay between the current and voltage pulses. From the damping of the oscillation and the general shape of the current pulse the estimated value of n_{e0} is about 5×10^{10} cm^{-3}, a rather high value. For the other two charge voltages the n_{e0} value has been estimated in the same way. For 85.3 kV and 117 kV a value of 10^9 and 5×10^{10} cm^{-3} respectively gave a good fit. Since we now are able to predict the current and voltage pulse to a good accuracy, we like to apply our calculations to an important problem, i.e. the question how much energy can be coupled into a laser discharge under different experimental circumstances. We chose ideal circumstances: the internal resistance of the marx generator R_i and the charge resistance R_{ch} were neglected. In fig. 9 the calculated efficiency of the circuit, defined as the part of the total marx-generator energy that is coupled into the laser discharge, was plotted as a function of the marx-generator voltage with the total circuit inductance as a parameter. For the initial electron density the value of 5×10^{10} cm^{-3} and for the marx-generator storage capacity the value of 16 nF were

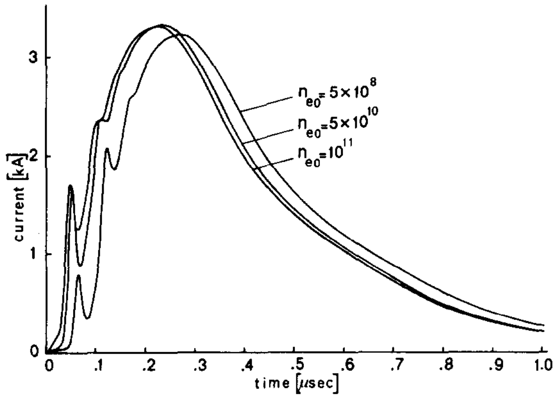

Fig. 8. The calculated current as a function of time for a marx-generator voltage of 97.5 kV. The initial electron density n_{e0} has been used as a parameter.

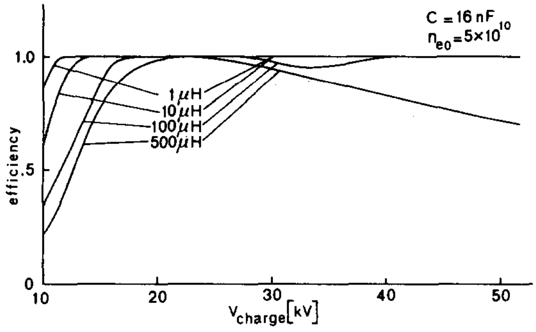

Fig. 9. The efficiency of the circuit as a function of the total marx-generator charge voltage. The total self-inductance of the circuit has been used as a parameter. For the initial electron density the value 5×10^{10} cm^{-3} has been used.

chosen. As can be seen, the efficiency is always higher for a lower self-inductance. To have an efficiency close to unity for a charge voltage of 80 kV the self-inductance must be lower than 10 μH. For large charge voltages the efficiency is always unity for moderate and low self-inductance values. Only for a large self-inductance a lower efficiency is found. This is due to the fact that for large self-inductances the electron-density level is lower and the time during which the laser has a voltage where attachment is important is long, so that the discharge finally goes out.

4. Conclusions

Our measurements and calculations have shown that it is possible to predict the temporal behaviour of the current and voltage of a CO_2–N_2–He laser mixture starting from the electrical scheme.

We may conclude that the published values of the recombination and attachment coefficient in [2] are sufficiently accurate to predict the time behaviour of a CO_2–N_2–He laser. The initial electron density of our system, determined from curve fitting, has a relatively high level, which is certainly one of the reasons why our laser construction leads to an excellent laser performance [3]. The time-dependent internal impedance of the marx generator has an important influence on the current and voltage behaviour of the system. A better performance can be obtained if the system is driven by a marx generator with less stages. From our calculations it must be concluded that the level of the initial electron density has only a slight influence on current and voltage characteristics, although it certainly influences the homogeneity and stability of the discharge.

References

[1] L.E. Kline and L.J. Denes, J. Appl. Phys. 46 (1975) 1567.
[2] G.J. Ernst and A.G. Boer, Optics Comm. 34 (1980) 235.
[3] G.J. Ernst and A.G. Boer, Optics Comm. 34 (1980) 221.
[4] G.J. Ernst and A.G. Boer, Optics Comm. 27 (1978) 105.
[5] J.C. Martin, Switching Note no. 9, AWRE, Dec. 1965.
[6] J.W. Limbeek and J. Lucas, Solid State and Electron Devices 2 (1978) 161.