function of a function. This is much the same material as in Wilde and Beightler [1], and is good preparation for reading current journals. The Appendix summarizes Stocker's tests [2] of five nonlinear programming codes. The reports of numerical tests are much less extensive than those of Kowalik and Osborne [3]. It is unfortunate that, perhaps due to a publication deadline, Beveridge and Schechter did not reference Kowalik and Osborne.

Part II of the book by Beveridge and Schechter exceeds in length the entire text (469 pages) of the book by Wilde and Beightler [1]. This is partly due to the Beveridge and Schecter practice of elaborating the obvious. For example, we read on p. 103 the following:

"A function can be represented in several ways. In general, the dependent variable is expressed in terms of the independent variables, $y(x)$ being said to be a function of $x$, defined in some region, if we have any rule which allows us to determine $y$ for any set of $x$ in the region. Any analysis of the function is then carried out by examining this expression."

On p. 105, we read the following:

"Although higher-dimensional systems cannot be represented geometrically, it is convenient to discuss and visualize the objective function as a geometric surface—a practice which seems to be derived from the work of statisticians. This lends a considerable simplification to the concepts, even if the mathematical labor in optimization is not diminished, and allows one to discuss the scaling of mountains or the descent into valleys, even if these are in an $n$-dimensional terrain. In general, within this text, most procedures will be illustrated by a simplified geometric representation to indicate the significant features."

For each new idea, there is excessive preamble and excessive recapitulation.

The length of Part II is also due to a commendable concern with details when at last the authors describe mathematical techniques. For example, in Section 3.2.2, which develops the necessary conditions for an extremum subject to constraints, the authors do not hesitate to aid the reader by writing out each argument of the Jacobian in long equations involving summations of Jacobian determinants. There is even an Appendix on the properties of determinants and Cramer's rule.

Part III (133 pages), titled "Optimization in Practice," concerns the difficulty of optimizing systems of high dimension, systems with some states unknown, and systems in which an opponent controls certain variables. Here the authors treat optimization in the context of corporate decision making. For example, a company considers the profitability of producing phenol. Four chemical processes for producing phenol are considered. For each process the cost and availability of raw materials and the market for by-products are assessed. For each process the profit per ton of phenol is computed based on the following assumptions: 1) all by-products are sold; and 2) no by-products are sold. For each process the required capital investment is estimated and the rate of return on investment is estimated based on hypothesis 1) or 2).

The authors are generous in assisting the reader with numerical examples and problems. There are approximately 547 references, of which many are from journals of chemical engineering, industrial engineering, management science, and operations research. Approximately 86 percent of these references were published in 1965 or earlier.

I recommend the book as an introductory text in optimization for college seniors or first year graduate students of chemical engineering. Many chapters would be good supplemental reading in a course in business administration or industrial engineering.

### References

[1] D. J. Wilde and C. S. Beightler, *Foundations of Optimization.* Englewood Cliffs, N.J.: Prentice-Hall, 1967.
[2] D. C. Stocker "A comparative study of nonlinear programming codes," M.S. thesis, Univ. Texas, Austin, 1969.
[3] J. Kowalik and M. R. Osborne, *Methods for Unconstrained Optimization Problems.* New York: Elsevier, 1968.

**Feedback Systems**—J. B. Cruz, Ed. (New York: McGraw-Hill, 1972, 324 pp., $16.50). *Reviewed by Huibert Kwakernaak.*

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This book is Volume 14 in the McGraw-Hill Inter-University Electronics Series. Consistent with the format of other volumes in this series, the book is built up from a number of chapters written by different authors, each of whom is an acknowledged expert on the topic of his chapter. The editor, J. B. Cruz, wrote Chapters 1, 3, and 6. W. R. Perkins is responsible for Chapter 2; P. V. Kokotović authored Chapter 4; E. Kreindler wrote Chapter 5, while Chapters 7 and 8 originate from I. W. Sandberg and P. E. Sarachik, respectively.

It is gratifying to note a revival of interest in feedback theory among control theoreticians. During the optimal control decade (1960–1970) the overriding importance of feedback in control was all too often lost from view. It is significant that the last book on control published in the United States before the present book that contains the word "feedback" in its title seems to be Horowitz's *Synthesis of Feedback Systems* [1], which appeared in 1963. Of course, interest in feedback theory never completely died, and the present book describes the developments that are currently taking place.

The prerequisites for being able to profit from the book are not explicitly stated, but they seem to consist of an acquaintance with the state-space approach to continuous-time system theory and the principal results of optimal control theory, together with the appropriate mathematical background. The most important results that are needed are adequately summarized in all instances.

Chapter 1, which is very brief, sets the stage. The question "Why use feedback?" is answered by stating that it may serve: 1) to stabilize an unstable system; 2) to reduce sensitivity to parameter variations and noise, as well as to decrease nonlinear distortion; or 3) to maintain optimality.

Chapter 2 is devoted to a presentation of sensitivity analysis. The clear distinction that is made between various types of sensitivity analysis is very welcome indeed. First, *comparison sensitivity* analysis is introduced (Bode's familiar approach [2], such as generalized to the multivariable case and reinterpreted by Cruz and Perkins [3]). More suited to the time domain approach dominant in the optimal control era is the concept of *trajectory sensitivity*, which is next discussed. In this approach the changes in the system trajectories due to (small) parameter changes are analyzed. The great benefit of the "sensitivity model" is very clearly pointed out. The fact that *performance sensitivity* is not mentioned in this chapter, but is introduced only in Chapter 5, points to an inherent weakness of books such as this, caused by the near impossibility of coordinating the efforts of six different authors. This weakness is even more poignantly brought out by the fact that Fréchet derivatives are mentioned and used in this chapter 220 pages before the definition of such a derivative in Chapter 8. However, the reader is alerted on page 57 to a possible necessity for him to consult Chapter 8.

The subject of Chapter 3 is the effect of feedback on signal distortion in nonlinear systems. Because reduction of signal distortion may be a prime reason for introducing feedback, the inclusion of this chapter is very appropriate. The beneficial effect of feedback on signal distortion is first extensively demonstrated by means of a nondynamic example. That for extension of the results to dynamic feedback systems functional analysis is required does not come as a surprise; once again the fact that functional analysis is not treated until Chapter 8 strikes the reader as an imperfection. The derived results can also be applied to multivariable feedback systems, as is elaborated upon in a subsequent section. The chapter closes with the conclusion that the capacity of a feedback system to reduce signal distortion is directly in proportion to its capacity to reduce sensitivity to disturbances and parameter variations.

Chapter 4 is headed "Feedback Design of Large Linear Systems." This title promises a presentation of methods for designing linear feedback systems with specified properties. In fact, the chapter deals with certain approximation methods for solving high-dimensional Riccati equations. The author introduces his contribution as follows: "... the theory of optimal linear systems with quadratic performance indices, or, briefly, linear-quadratic theory, has become the main tool for feedback design of linear dynamic systems .... By using the formalism of the linear quadratic theory, every design problem is eventually reduced to the solution of a matrix Riccati equation ..." (p. 99). The author then proceeds to introduce two approximation methods for solving linear-quadratic problems. Although the reviewer can go a long way with the author in his conviction that linear-quadratic theory is a great advance in linear control theory, it would be a mistake to think that all difficulties have been overcome. The author's otherwise very competent contribution in the present chapter is to show that often a "small" parameter can be identified in a control problem. The solution of a linear-quadratic optimization problem then can be obtained by a suitable series expansion in this parameter. Impressive savings of computing effort can be obtained. It would have been very valuable if this chapter had offered advice on how to design a feedback system such as to achieve a *specified* decrease of sensitivity to parameters variations and noise, or a specified distortion reduction.

Chapter 5 deals with comparative sensitivity of optimal control systems. The author first proves the by now well-known and somewhat disconcerting fact that an open-loop implementation and a closed-loop implementation of a given optimal control system possess the same performance sensitivity (provided the terminal state is free). The author then rejects performance sensitivity as a tool for measuring the sensitivity of optimal control systems, and proposes to study their comparative sensitivity. It is proved that in terms of comparative sensitivity closed-loop optimal control systems are indeed less sensitive than their open-loop equivalents, provided that their sensitivities are properly compared. This result applies to nonlinear systems with continuous controls. A case of special interest is the problem of a linear system with a quadratic criterion, where various specific results are obtained, and a number of convincing examples are exhibited. It should be commented here that this chapter contains the germs for a *constructive* theory of feedback, since the sensitivity properties of optimal feedback systems are examined, and optimal control theory is a constructive theory.

The book continues on a somewhat different note with Chapter 6, which is given over to the topic of near optimal feedback control. It is well known that optimal control laws in explicit feedback form generally are very difficult to obtain. The problem of finding such feedback control laws is approached in this chapter by constructing a Taylor series expansion around a nominal trajectory. This expansion is matched up to a certain order by the near optimal controller that is proposed. The chapter also gives a prospect of a form of adaptive control by tackling the problem of unknown, time-varying parameters through the introduction of estimators. The examples that are shown are convincing, but rather simple. It is difficult to envisage what problems one would encounter when applying these methods to more realistic problems.

As a footnote confirms, Chapter 7, dealing with the theory of linear multiloop feedback systems, is essentially a reprint of a journal paper [4], which appeared in 1963. Is it a coincidence that neither the journal paper nor the present chapter are referenced anywhere in the book? It is a fact that all other parts of the book, with the possible exception of Chapter 3, are firmly oriented towards control, while the present chapter bears the stamp of circuit theory. The reviewer finds it regrettable that the author has not taken account of the work in recent years in linear multivariable system theory. The chapter treats generalizations of Blackman's classical equation and Bode's return difference considerations to multivariable linear systems. By way of example, the theory is applied to one or two simple electric circuits.

Chapter 8, concerned with applications of functional analysis to nonlinear control systems with unknown plants, begins with the long-awaited introduction to functional analysis. This introduction is concise, neat, and adequate for the purposes of the book. Not altogether standard is a survey of five methods for the iterative solution of nonlinear functional equations: contraction mapping; generalized Newton methods; steepest descent; Davidon's method; and the conjugate gradient method. The author continues by showing that these methods can be applied to various types of optimal control problems. The chapter, and with this the book, concludes with a description of how the method can be applied to control systems with unknown plants. It is suggested that the differential of the plant operator should be obtained by successive experimentation. The reviewer doubts the feasibility of this scheme as it is very unlikely that in any practical situation the protracted, repetitive experimenting period that is required would be acceptable, even if optimality is the reward.

In conclusion, it is only fair to state that the book lives up to its intention of "providing timely and comprehensive coverage that stresses general principles, and integrates newer developments into the overall picture," such as is claimed in the Foreword. The book brings together material that has been available in the literature only in scattered form. It does a good job in unifying terminology and standardizing some of the notions. Its weaknesses mainly derive from its multiple authorship—a slight (but not serious) lack of coordination between the various chapters, and a tendency with one or two of the authors to insist on their own pet topics.

In the opinion of the reviewer, the book has no great future as a textbook, because of its relatively limited scope. It is mandatory for supplementary reading at the graduate level, however. Researchers will find the book very valuable for reference purposes.

In a final overview of the book, one fact stands out very strikingly—the lack of a constructive theory of feedback in the case of multivariable systems. Such a theory would offer methods for designing multivariable feedback systems which exhibit prescribed capabilities of reducing sensitivity or distortion. Linear optimal control theory gives a partial solution, and is rightfully stressed in the present book. More development is needed, however, (and indeed is underway) before all problems are solved.

### References

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[3] J. B. Cruz, Jr., and W. R. Perkins, "A new approach to the sensitivity problem in multivariable feedback system design," *IEEE Trans. Automat. Contr.*, vol. AC-9, pp. 216–223, July 1964.
[4] I. W. Sandberg, "On the theory of linear multiloop feedback systems," *Bell Syst. Tech. J.*, vol. 42, no. 2, pp. 355–382, Mar. 1963.

**Stochastic Processes and Filtering Theory**—Andrew H. Jazwinski (New York: Academic, 1970, 376 pp., $18.50). *Reviewed by K. D. Senne.*

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As the title implies, A. Jazwinski has set himself the task of unifying the significant contributions in filtering theory in a single work, complete and self-contained, yet, as he claims (p. 3), "in a form accessible to engineers." I believe that, while the latter claim