

The influence of practical factors on the benefits of condition-based maintenance over time-based maintenance



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ABSTRACT

Recent developments in condition monitoring technology have led to an ongoing shift from time-based maintenance (TBM) to condition-based maintenance (CBM). Although CBM allows for more effectively planned maintenance actions, its relative performance strongly depends on the behavior of the deterioration process, the severity of failures, the required setup time, the accuracy of the condition measurements, and the amount of randomness in the deterioration level at which failure occurs. The contribution of this paper is twofold. First, we review studies that compare CBM with TBM, and studies that consider the above factors in combination with a CBM model. Second, whereas existing studies confine themselves to a few examples, we perform a numerical investigation to derive insights on the effects of the various characteristics on the relative benefit of CBM. The results can be used by companies to decide what factors are most important when considering to implement CBM, and to assess whether the benefit of CBM during the operational phase outweighs the additional costs during the life cycle of equipment. This study allows for follow-up research to quantify and generalize the insights obtained, and to analyze interaction effects.

1. Introduction

Due to ongoing automation of production processes and increasing reliance on expensive production equipment, the importance of effectively planned and performed maintenance activities is growing, and both the portion of employees working in maintenance and the maintenance costs are increasing [78]. As an illustration, over a quarter of the total workforce in the process industry, and up to 30% in the chemical industry, deal with maintenance operations [71]. In refineries, the maintenance and operations departments are usually the largest [17]. Furthermore, maintenance costs typically account for 15–70% of the total value of the end product [8,44], the amount of money spent on maintenance of engineering structures and infrastructures is increasing continuously [68], and medical equipment maintenance nowadays demands large sums from hospital budgets [14].

Many firms still apply ‘traditional’ time-based maintenance (TBM) strategies, which are easy to implement as only the time that a unit is in service has to be recorded. However, substantial remaining useful life is wasted if the machine is still in reasonable condition when preventive maintenance is performed, and a breakdown might occur if it happens to deteriorate faster than expected. Due to the increasing technical

possibilities to monitor, store, and analyze conditions, condition-based maintenance (CBM) strategies are gaining popularity [10,18,28,59,64]. Condition-based maintenance generally results in more effectively scheduled preventive maintenance, and, in the ideal case, preventive maintenance that is performed just before failure.

The relative benefit of CBM, however, strongly depends on the behavior of the deterioration process and the severity of failures. Furthermore, it is affected by various practical factors that are often present in practice, viz., required planning time, imperfect condition monitoring, and variation in the deterioration level at which failure occurs. CBM should only be applied if this relative benefit outweighs the efforts and costs during the entire life cycle that are required to apply CBM [22,50,60,69]. The requisites to switch from time-based to condition-based maintenance include condition monitoring equipment and software to store, analyze, and initiate maintenance actions [3,59]. Companies that are interested in implementing condition-based maintenance must also consider the risks related to the lack of experience [78]. Furthermore, they should realize that CBM requires a dynamic scheduling of maintenance activities, whereas they might not have the capability for such flexible planning.

The first contribution of this paper is to review studies that compare

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condition-based and time-based maintenance, as well as studies that consider the above practical factors in a CBM model. Although both CBM and TBM have received ample attention in the scientific literature, few studies compare them. Moreover, existing comparative studies confine themselves to a few examples. Insights on how the various characteristics influence the performance of condition-based and time-based maintenance are lacking. Therefore, our second contribution is to derive insights on the effects of the various characteristics on the relative benefit of CBM from a numerical investigation. We start with the effects of the behavior of the deterioration process and the severity of failures. Thereafter, we extend our model and analyze the effects of the practical factors on the relative performance of CBM. The obtained insights are useful in practice to decide what factors are most important when considering to switch from TBM to CBM, and for avoiding the risk of switching from TBM to CBM in situations where benefits do not outweigh costs.

The remainder of this paper is organized as follows. In [Section 2](#) we review existing studies that compare condition-based maintenance with time-based maintenance, and studies that consider planning time, accuracy of condition measurements, and predictability of the failure deterioration level. The approach that we use to compare the two maintenance strategies is discussed in [Section 3](#). This section also contains formal definitions of the condition-based maintenance and the time-based maintenance strategy that we adopt. In [Section 4](#) we consider the effect of the behavior of the deterioration process and of the severity of failures on the relative performance of CBM. In [Section 5](#) we point out how this performance is influenced by required planning time, imperfect condition information, and predictability of the deterioration level at which failure occurs. We end with conclusions and suggestions for future research in [Section 6](#).

2. Literature review

We start this section with a review of studies that compare condition-based maintenance with time-based maintenance. Thereafter, in [Section 2.2](#), we review studies that consider various practical factors that influence the relative performance of condition-based maintenance.

2.1. Comparative studies

In this section we review studies that compare time-based maintenance with condition-based maintenance.

The most simple models are those that consider a small number of deterioration states. McKone and Weiss [42] consider the integration of condition-based maintenance with traditional periodic preventive maintenance. The available condition information is limited to a signal of a potential failure that might be received before the actual failure. The probability that this signal is received depends on the prediction accuracy, and the time between the signal and the failure depends on the prediction precision. The performance of the condition-based maintenance strategy depends on the prediction accuracy and precision. In some situations, periodic preventive maintenance or a combination of condition-based and periodic preventive maintenance is preferred. Paté-Cornell et al. [55] use a Markov chain with four states to model the deterioration process of a production system. Time-based maintenance and three types of condition-based maintenance are considered. The latter are based on inspections of the product, signals of the machine, and signals provided by the use of the product. Zhang et al. [77] develop an adaptive discrete-state model based on Bayesian Belief Network theory. Both Paté-Cornell et al. [55] and Zhang et al. [77] consider a single illustrative example, and no attempt is made to generalize the results.

Other studies consider deterioration processes with a continuous state space. Pandey et al. [53] use gamma deterioration processes and linear deterioration processes with a random (but fixed) rate. Xiang

et al. [75] also adopt linear processes, but consider a rate that depends on the environment in which the system operates. This environment is represented by a continuous-time Markov chain with three states. Crowder and Lawless [13] consider gamma and Wiener processes, and Zio and Compare [78] adopt the randomized Paris-Erdogan fatigue crack growth model. For the main part of their analysis, Pandey et al. [53] consider the threshold deterioration level that triggers preventive maintenance as fixed. Only a limited investigation also includes the threshold deterioration level as a decision variable. Condition-based maintenance turns out to be preferred over time-based maintenance only if the coefficient of variation of the lifetime exceeds a certain level. Crowder and Lawless [13] and Zio and Compare [78] compare the optimal condition-based maintenance policy with the optimal time-based maintenance policy, but they only do so for a single specification of the parameters. In both studies, the performance of condition-based maintenance turns out to be much better than that of time-based maintenance for the considered parameter settings, but general insights are lacking. Xiang et al. [75] include randomness in the deterioration level at which failure occurs, and show that there is potential cost saving through implementing a condition-based maintenance policy as opposed to time-based maintenance. No insights are presented on the effect of changes in the other model parameters on these cost savings.

More sophisticated models are considered by Huynh et al. [26] and by Bouvard et al. [9]. The former combine failures due to deterioration with failures due to shock events. Because failures are not self-announcing but should be identified by inspections, a cost is introduced for system inactivity. A condition-based strategy with periodic inspections is compared with a purely time-based block replacement strategy. A clear effect of the type of condition deterioration is lacking, as only two deterioration processes are considered (high and low variance). The influence of the values of the cost parameters is studied in more detail and the relative benefit of the condition-based strategy turns out to increase in the preventive replacement cost. Bouvard et al. [9] develop a maintenance model that dynamically optimizes the maintenance decisions for a multi-component system at each periodic inspection time. Maintenance actions are grouped to reduce maintenance costs. A system with three components is considered as an example and it is shown that the use of condition information leads to lower costs compared with the case that this information is not used. Cost savings are most significant for short times between inspections and moderate variances of the underlying gamma processes that are used to model deterioration of the components.

Summarizing, we conclude that only a few general insights on the benefits of condition-based maintenance compared with time-based maintenance are provided by the current literature.

2.2. Practical factors influencing the benefits of CBM

We continue with a review of studies that include various practical factors that influence the relative benefit of CBM. These factors are required planning time, imperfect condition monitoring, and uncertainty in the deterioration level at which failure occurs.

2.2.1. Planning time

In many practical situations, repairmen are not continuously available [27,34], and spare parts may not be on stock and have to be ordered [19,38,51]. If so, a certain planning time (in the literature also called lead time or delay time) is required between initiating and performing a maintenance action. We remark that joint optimization of maintenance and spare parts inventories has been considered by a number of authors. This is beyond the scope of our study; we refer to Van Horenbeek et al. [66] for a recent review.

A required planning time to perform preventive maintenance in combination with a continuously monitored unit that deteriorates according to a gamma process is considered by various authors. They

all assume that maintenance is scheduled if the level of deterioration exceeds an alarm level, and that failure occurs if the level of deterioration exceeds a fixed failure level. The alarm level can be specified and is the decision variable of the considered models. Bérenguer et al. [5] introduce a maintenance duration that depends on the current deterioration state. Approximations for the asymptotic unavailability are obtained and preventive maintenance policies that minimize these estimated asymptotic unavailabilities are determined. Two values for the delay time are compared and the longer planning time results in a greater unavailability. Grall et al. [25] focus on the asymptotic behavior of the reliability function and show how to compute the asymptotic failure rate of the system. Saassouh et al. [58] extend the model with a possible sudden change in the deterioration process. The latter two studies do not assess the effect of changing the required planning time.

Bouvard et al. [9] study a multi-component system with grouped maintenance actions to reduce maintenance costs. The proposed model includes a certain minimal time required to prepare and organize maintenance. However, the effect of the required planning time on the performance of the maintenance policies is not studied. Van Oosterom et al. [70] allow for both immediate and postponed preventive maintenance actions upon the identification of a defect at an inspection. This results both in a better utilization of the useful life and in a reduced maintenance cost. They find that if the cost difference between a planned maintenance action and an immediate maintenance action is sufficiently large, maintenance actions should always be planned in advance.

2.2.2. Imperfect condition information

According to Ghasemi et al. [23], condition information may contain noise due to errors of measurement and interpretation, and due to the limited accuracy of the measurement's instruments. Typical condition monitoring techniques like vibration and oil debris monitoring, which are widely applied in industry, generally result in such inaccuracies. These techniques can therefore be considered as imperfect [73]. Also when considering the crack growth of a mechanical component subject to fatigue degradation, observations of the crack depth at inspections are just estimations of the true values [78].

The most simple models that include imperfect condition monitoring contain two [6,52] or three [7] deterioration states, and inspections reveal the true system state with specific given probabilities. These basic models are extended with inspections that might induce failures [21], a distinction between minor and major inspections [74], and aperiodic inspections [37]. Ghasemi et al. [23] model the deterioration state using a discrete time Markov process, Makis and Jiang [41] and Moghaddass and Zuo [45] use a continuous time Markov process. They all relate the unobservable system state to the observed system state through an observation probability matrix. Le and Tan [36] also use a continuous time Markov process and combine imperfect continuous monitoring with perfect inspections. MacPherson and Glazebrook [40] consider continuously monitored two-phase systems. Transitions from the fault free to the worn state might not be observed, and indications of transitions might also be false.

Imperfect condition information has also been combined with deterioration processes modeled by continuous stochastic processes. Kallen and Van Noortwijk [32] use a gamma deterioration process, Ye et al. [76] a Wiener process with positive drift, Peng and Tseng [56] a linear trend with random coefficient plus a Brownian motion as a second random effect, and Zio and Compare [78] a Randomized Paris-Erdogan fatigue crack growth model. In all cases, inspections have to be performed to obtain condition information, and the measurement error is modeled by independent normal distributions in all cases.

Most of the aforementioned studies lack a clear translation from the level of uncertainty in the observed condition information to the performance of condition-based maintenance. Xiang et al. [75] consider a model in which the rate of deterioration depends on the environment, and the measurement error is modeled by a normal

random variable. However, the assumption is made that this error is constant over time, which may often be unrealistic. Their analysis indicates that even small levels of measurement errors can render condition-based maintenance no better or even worse than time-based maintenance. The study of Zio and Compare [78] indicates that for large measurement errors the performance of condition-based maintenance gets worse if the number of inspections increases. However, they model all measurement errors by independent normal distributions and perform preventive maintenance when a single observed deterioration level exceeds a safety threshold. However, if the number of inspections is large, it becomes likely that one of the observed deterioration levels exceeds the safety threshold while the real deterioration level is still low. This can be avoided by using a decision rule for initiating preventive maintenance that does not only take the most recent condition measurement into account. An alternative is to model the noise by a Brownian motion, as for example done by Elwany et al. [20] and Li and Ryan [38]. According to Elwany and Gebraeel [19], this is suitable for applications where correlation exists between successive error fluctuations in sensor readings. Furthermore, such a continuous-time process may also be appropriate for modeling measurement errors when conditions are monitored continuously.

2.2.3. Uncertain failure level

According to Jiang [30], uncertainty in the failure level is especially relevant if the deterioration process is represented by several condition variables that are combined into a composite condition variable. Abdel-Hameed [2], Van Horenbeek and Pintelon [67], Wang et al. [72], and Xiang et al. [75] model the deterioration level at which failure occurs as a random variable. Some of them assume a specific distribution (the normal, exponential, Weibull, gamma, and lognormal distribution are used). Jiang [30] makes this random variable time-dependent.

Other models where failure does not occur at a fixed level of deterioration are those in which the failure rate depends on condition variables. Kong and Park [33] and Park [54] consider a failure rate that depends on a single condition variable. Examples of models that allow the failure rate to depend on multiple condition variables are the proportional hazards model (PHM) and the accelerating failure time (AFT) model. The PHM is used most often; see Kumar and Klefsjö [35] for a review. Authors that use this model include Ghasemi et al. [23], Jiang and Jardine [31], Louit et al. [39], Tian and Liao [65], and Zuashkiani et al. [80]. The AFT model is used by Newby [46].

3. Approach

We consider a single-unit system, i.e., a unit that can only be maintained in its entirety. The unit is monitored continuously and its deterioration is modeled using a gamma process. If the unit fails, a corrective maintenance action has to be performed. Preventive maintenance can be performed before failure of the unit. Both maintenance types make the unit as-good-as-new, maintaining and replacing the unit are thus interchangeable notions.

Corrective maintenance is assumed to be more expensive than preventive maintenance because failures occur unexpectedly and are likely to have severe consequences. This is a common assumption in studies on preventive maintenance planning (e.g. [15,16,61–63,79]). The cost of corrective maintenance is normalized to 1, and the cost of performing preventive maintenance (based on condition or time) is denoted by $c < 1$. We use the cost rate, i.e. the mean cost per unit of time, as the optimality criterion. Standard renewal theory can be used because of the as-good-as-new property of maintenance, see for example [57].

Section 3.1 discusses our selection of the gamma deterioration process and summarizes its most important properties. The condition-based and time-based maintenance strategy that we compare are formally described in Section 3.2. Because, different from most of the existing literature, the model that we consider takes all practical factors

discussed in Section 2.2 into account, it is rather complex. Therefore, simulation is used to assess the performance of the maintenance strategies. Section 3.3 motivates the choice for this research method and discusses the setup of the simulations.

3.1. Deterioration process

Various stochastic processes can be used to model stochastic deterioration. Examples are discrete-time or continuous-time Markov processes, Brownian motions with drift, compound Poisson processes, and gamma processes. In this paper we will use the latter. The gamma process was introduced in the area of reliability by Abdel-Hameed [1]. It is a rather flexible process that is applicable to model a wide variety of deterioration processes. Therefore, by considering various gamma processes, we obtain a broad impression of the relative performance of CBM. According to Van Noortwijk [68], the gamma process is most appropriate to model monotonic and gradual deterioration. He also lists some studies in which the gamma process fits well to deterioration data. Cinlar et al. [12] give a comprehensive justification of the gamma process from a physical and practical point of view.

We model the level of deterioration of the unit using a stationary (also called homogeneous) gamma process. We use the following definition for the density function f of the gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$:

$$f_{\alpha,\beta}(t) = \frac{1}{\Gamma(\alpha)\beta^\alpha} t^{\alpha-1} e^{-\frac{t}{\beta}}, \quad t > 0,$$

in which $\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} e^{-z} dz$ denotes the gamma function. The stationary gamma process has a shape function at with shape parameter $a > 0$ and a scale parameter $b > 0$. It is a continuous-time process $\{X(t) : t \geq 0\}$ with $X(\tau) - X(t) \sim f_{a(\tau-t),b}$ for $\tau > t \geq 0$. The expectation and the variance of the deterioration level $X(t)$ at time t are respectively given by $E(X(t)) = abt$ and $\text{Var}(X(t)) = ab^2t$. For convenience, we let $\sigma = \sqrt{ab}$ denote the standard deviation of the level of deterioration at time 1. Fig. 1 shows some illustrative sample paths of the stationary gamma process. The gamma process is a jump process, and the occurrence of larger jumps becomes more likely if σ increases. Gamma processes with larger variations are suitable to model deterioration processes with possible sudden large increases in the deterioration level on which a CBM strategy cannot respond quickly enough.

We assume that the unit fails if the level of deterioration exceeds a certain threshold level L . The distribution function F_L of the time until failure equals

$$F_L(t; a, b) = P(X(t) > L) = \int_L^\infty f_{at,b}(x) dx = \frac{\Gamma(at, Lb^{-1})}{\Gamma(at)}, \quad t > 0,$$

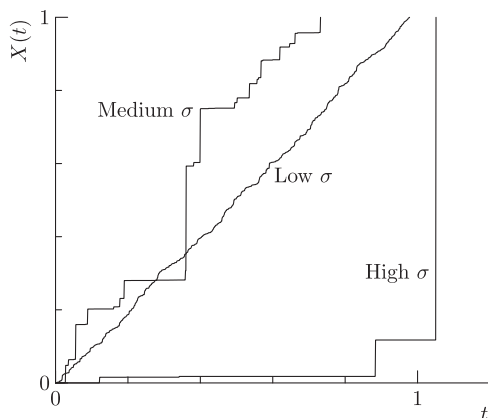


Fig. 1. Sample paths of a stationary gamma process with low ($\sigma = 0.05$), medium ($\sigma = 0.5$), and high ($\sigma = 5$) standard deviation, respectively.

in which

$$\Gamma(\alpha, x) = \int_x^\infty z^{\alpha-1} e^{-z} dz, \quad x \geq 0, \quad \alpha > 0,$$

denotes the upper incomplete gamma function.

For the ease of representation, we change the scales that we use to express deterioration levels and time. Firstly, we rescale the values of the deterioration levels such that failure occurs if deterioration level $L=1$ is exceeded. Secondly, in order to obtain a cost rate of 1 under a purely corrective maintenance strategy, we rescale the time such that the mean time to failure equals 1. We note that the latter does not imply that the mean level of deterioration at time 1 equals 1, and thus that $ab=1$. Realizing that the probability of failure between time t_1 and time t_2 equals $F_L(t_2; a, b) - F_L(t_1; a, b)$, and using a sufficiently small number Δ , the mean time to failure can be accurately approximated by

$$MTTF(a, b) = \sum_{i=0}^{\infty} i\Delta \left[F_L\left(\left(i+1\right)\Delta; a, b\right) - F_L\left(i\Delta; a, b\right) \right].$$

The mean time to failure equals 1 for combinations of values for a and b that satisfy $MTTF(a, b) = 1$. These combinations are determined numerically.

3.2. Maintenance strategies

Under the condition-based maintenance (CBM) strategy, we perform preventive maintenance if the level of deterioration exceeds a certain threshold level M , which is the decision variable of this strategy. This commonly used strategy is often called the control-limit strategy, see for example [29]. Obviously, M should not be chosen too low, as this results in performing preventive maintenance too often. On the other hand, if M is chosen too high, and if, for example, the deterioration process is a jump process or if a planning time is required before preventive maintenance can be performed, the failure level L might be exceeded before preventive maintenance is performed.

The time-based maintenance (TBM) strategy that we adopt is the age-based maintenance strategy, see [4]. Under this strategy, preventive maintenance is performed if the unit reaches a certain age T , which is the decision variable of this strategy. A too low value for T leads to preventive maintenance that is performed too frequently, while a too high value results in too many failures. The level of deterioration is obviously not observed if this strategy is applied, only failures are observed.

3.3. Simulation

The evaluation of maintenance strategies for units with a deterioration level that evolves according to a gamma process involves some integrals that are very complicated and burdensome to compute numerically [43]. The main reason for this is the ‘overshoot behavior’ [47] of the gamma process caused by the fact that the gamma process is a jump process. This behavior complicates the exact determination of the distribution of the level of deterioration at the moment that it exceeds a certain threshold value. Nicolai et al. [49] mention that in most studies the overshoot of the gamma process is not mentioned at all. To cope with these complexities of evaluating maintenance strategies with an underlying gamma process, approximations have been studied (e.g. [5,26]) and simulations have been used (e.g. [9,25,32,43]). Furthermore, the inclusion of various practical factors in Section 5 makes a numerical analysis even more complex. In the current study we use simulation. Although this obviously leads to approximations of the cost rate, by using a sufficiently large number of iterations these approximations become very accurate.

We have simulated gamma processes using gamma increment sampling, see [68] for details. This method starts with choosing a small time interval and iteratively simulates the level of deterioration at

the end of subsequent time intervals by taking samples of the additional deterioration during a time interval. In our simulations we use time steps with length 0.01. Furthermore, for specific values of the parameters and the relevant decision variable (M or T), we simulate 100,000 paths of the gamma process until failure occurs. This number of iterations is sufficiently large to obtain smooth curves for the cost rates as functions of the decision variables, implying that the selected optimal values of the decision variables are not caused by coincidence. We note that, instead of using simulation, the performance of the time-based maintenance strategy could also have been determined based on the lifetime distribution function F_L as stated in Section 3.1.

4. A comparison of CBM and TBM

We start our comparison of the performances of condition-based maintenance and time-based maintenance with an initial setting of the parameters, the so-called base case. Thereafter, we will assess the effect of changing the (values of the parameters that drive the) behavior of the deterioration process and the cost structure.

In our base case, we select parameter values rather arbitrarily. Other values reveal similar patterns, and those are of interest to us rather than specific outcomes. We set the shape parameter a of the gamma process equal to 5. In order to obtain a mean time until failure of 1, the value of the scale parameter b follows to be approximately 0.2246 (see Section 3.1). Furthermore, we assume that the relative cost of performing preventive maintenance equals $c=0.2$. Fig. 2 shows the cost rate for both strategies as functions of the respective decision variables. The upper dotted lines show the cost rate if only corrective maintenance is performed. This cost equals 1 because both the time until failure and the failure cost are normalized to 1. The lower dotted lines show the cost rate in the ideal case in which we would be able to perform preventive maintenance just before each failure. This cost obviously equals $c=0.2$.

A comparison of Fig. 2 (a) and (b) indicates that the cost rate under the optimal condition-based maintenance strategy is lower than the cost rate under the optimal time-based maintenance strategy, and thus that the use of condition information allows for cost savings. However, there is still a gap between the cost in the ideal case and the cost under the optimal condition-based maintenance strategy, implying that we are not able to perform condition-based maintenance just before failure. This is caused by the fact that the deterioration process makes jumps; if we choose the maintenance threshold M very high, it is very likely that failure occurs at the same moment at which the threshold is exceeded. Thus, the potential cost saving of applying condition-based maintenance decreases if sudden large increases in the deterioration level occur. A similar effect occurs if the deterioration process would be continuous, but if condition information is only observed by inspections. In that case, the observed levels of deterioration are also non-

continuous. In this paper we consider continuously monitored equipment; studies on inspection-based CBM include Chen et al. [10], Golmakani and Fattahipour [24], and Jiang [29].

We continue to explore the effect of the behavior of the deterioration process on the performance of CBM and TBM. As outlined in Section 3.1, the variation in the deterioration process depends on the values of a and b . We use the standard deviation $\sigma = \sqrt{ab}$ of the level of deterioration at time 1 as a measure of this variation. Fig. 3 (a) shows the cost rate under the optimal TBM and under the optimal CBM strategy, for various values of σ . Fig. 3 (b) shows the difference between these costs.

In the extreme case with $\sigma = 0$, the level of deterioration accrues linearly without any variation and the time until failure is deterministic. Under both maintenance strategies this enables us to perform preventive maintenance just before failure, resulting in a cost rate of $c=0.2$. Although the costs are low in this case, there is no cost benefit of condition-based maintenance compared with time-based maintenance. In the other extreme case with σ very high, failure is caused by a single shock occurring at a random moment in time, resulting in an exponentially distributed lifetime. Both time-based maintenance and condition-based maintenance cannot prevent such a failure, implying that only corrective maintenance will be performed and that the cost rate is 1. Again, the use of condition information does not allow for any cost savings. In between these extremes, CBM does offer a benefit and we next explore when this benefit is largest.

Fig. 3 (a) indicates that a small level of variation in the deterioration process has a relatively large impact on the performance of TBM. This results from the fact that the increased variance in the time until failure makes it impossible to determine a TBM strategy that always performs maintenance just before failure. The impact on CBM is smaller; for minor variance in the deterioration process maintenance can still be scheduled relatively effectively if condition information is available. If the variance increases further, failure caused by sudden large increases in the deterioration process on which a CBM strategy cannot respond quickly enough become more likely. This results in a declining performance of this strategy, both absolute and relative compared to TBM. The benefit of CBM over TBM is thus largest for small but positive levels of variation in the deterioration process, as depicted in Fig. 3 (b).

We now return to the initial gamma deterioration process and consider the effect of the cost structure on the performance of CBM and TBM. To that end, we vary the relative cost c of performing preventive maintenance. Fig. 4 (a) shows the cost rate under the optimal TBM and under the optimal CBM strategy, for various values of c . Because the cost rate in the ideal case with preventive maintenance just before failure depends on the cost c of preventive maintenance, the corresponding dotted line is now a diagonal line. Fig. 4 (b) shows the difference between the cost under the optimal TBM and the optimal

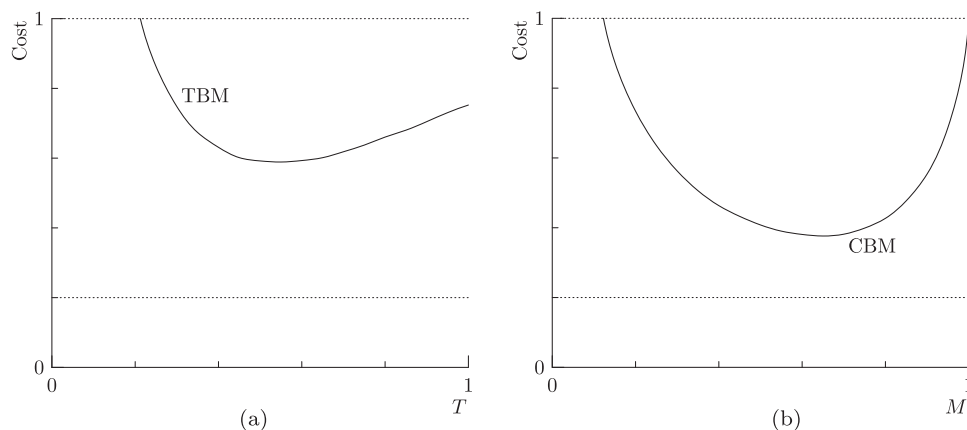


Fig. 2. The cost rate for $a=5$, $b \approx 0.2246$, and $c=0.2$ as a function of the maintenance age T for the TBM strategy (a) and of the maintenance threshold M for the CBM strategy (b).

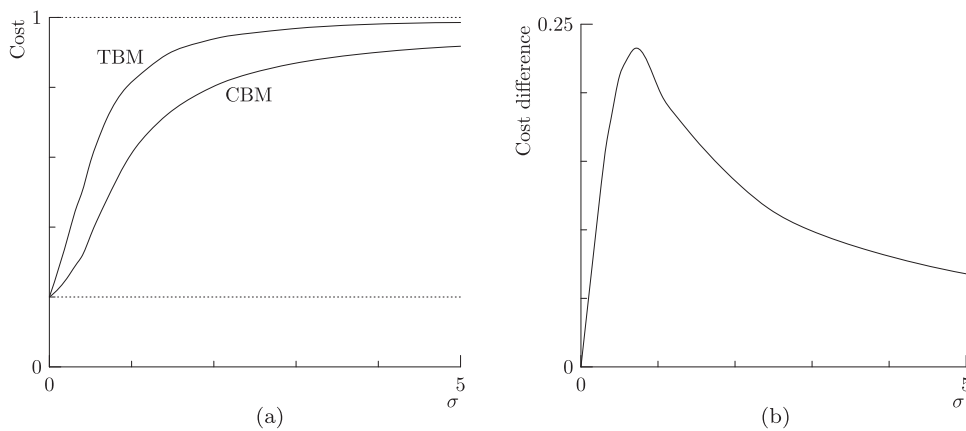


Fig. 3. The cost rate under the optimal CBM and TBM strategy (a) and the difference between these cost rates (b) for a varying standard deviation σ of the gamma deterioration process.

CBM strategy.

Again, two extreme cases can be distinguished. If the cost of preventive maintenance is extremely small, preventive maintenance can be performed very frequently. Both under CBM and under TBM no failures occur, resulting in a cost rate close to 0. If, on the other hand, the cost of preventive maintenance is almost equal to the cost of a failure, both maintenance strategies do not allow for any cost savings. Thus, in both extreme cases, applying condition-based maintenance does not result in any cost savings compared with time-based maintenance.

Fig. 4 (a) shows that the cost of TBM increases more steeply in the preventive maintenance cost for moderate values of that cost, due to the increasing costs of maintaining the system when it is not close to failure and of preventive maintenance that is scheduled too late. Fig. 4 (b) shows that this results in a substantial relative cost benefit of CBM already for low preventive maintenance costs c . This cost benefit is retained for all realistic preventive maintenance costs and only vanishes if the preventive maintenance cost is very high.

5. Practical factors affecting the benefits of CBM

So far, we assumed that maintenance can be performed immediately when a certain level of deterioration is reached, that the exact level of deterioration can be observed without any errors, and that failure always occurs at the exact same level of deterioration. In this section, we relax these assumptions and assess the influence on the relative performance of CBM.

5.1. Planning time

Let us assume that performing preventive maintenance requires a

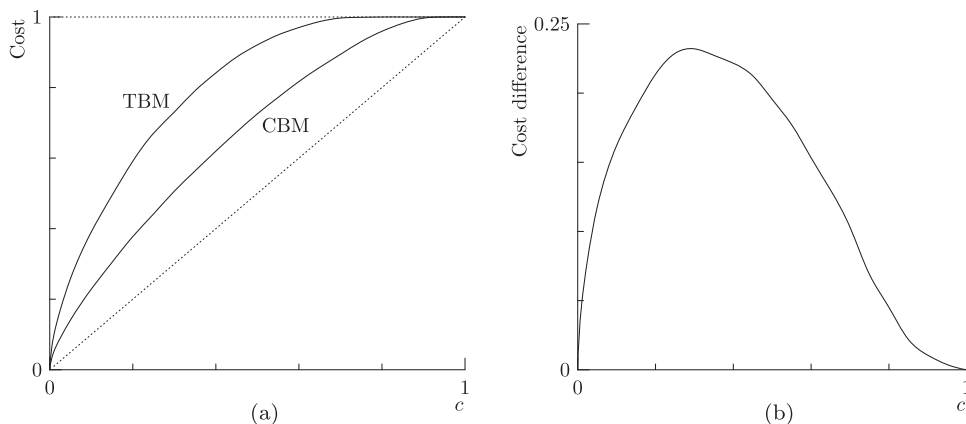


Fig. 4. The cost rate under the optimal CBM and TBM strategy (a) and the difference between these cost rates (b) for a varying relative preventive maintenance cost c .

fixed planning time, denoted by s . The addition of this planning time slightly changes the CBM strategy. Preventive maintenance is scheduled at the first time that the level of deterioration exceeds level M . Denoting this time by $t(M) = \min\{t: X(t) > M\}$, preventive maintenance is performed at time $t(M) + s$. Because the unit still deteriorates during the planning time, a breakdown might occur between planning and performing preventive maintenance. In this case, we assume that the cost c of the preventive maintenance action that is already scheduled does not have to be paid, but the higher corrective maintenance cost of 1 is incurred instead. Note that as long as the planning time s is smaller than the optimal maintenance age, the planning time does not affect the TBM strategy as preventive maintenance will just be planned s time units before the optimal maintenance age is reached.

Fig. 5 (a) shows the cost rate as a function of the decision variable M for the base case considered in Section 4, and for the same case with a planning time of $s=0.1$ time units. Under the presence of a planning time, condition-based maintenance should already be initiated at a lower deterioration level. Furthermore, the cost rate under the optimal CBM strategy increases if a planning time is required. This is easily explained by the fact that the condition information cannot be utilized anymore once preventive maintenance has been planned. Fig. 5 (b) gives a more complete view of the effect of a planning time on the cost rate under the optimal CBM strategy. This cost turns out to increase more or less linearly in the planning time, resulting in a cost benefit of CBM over TBM that decreases linearly in the planning time. If the planning time equals the maintenance age under the optimal TBM strategy, the condition information does not allow for any cost savings anymore. Preventive maintenance is scheduled at time 0 under both strategies in that case.

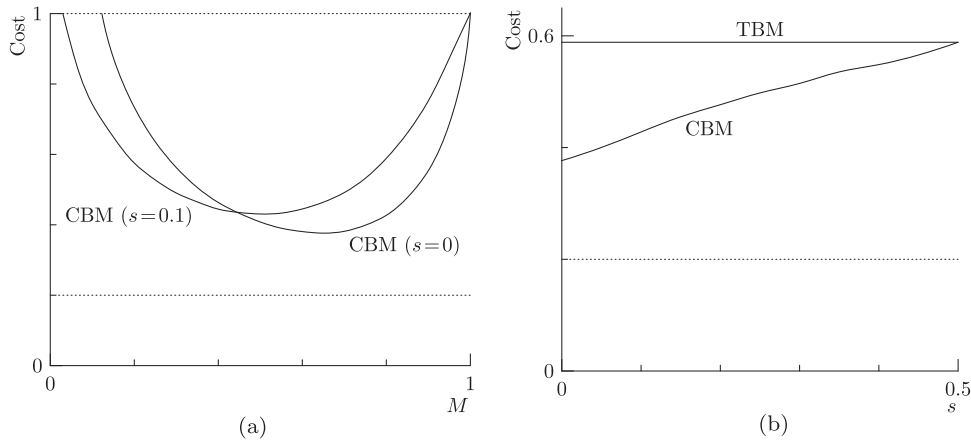


Fig. 5. The cost rate for the CBM strategy for the base case with $s=0$ and with $s=0.1$ as a function of the preventive maintenance threshold M (a) and the cost rate under the optimal TBM strategy and CBM strategy as a function of the planning time s (b).

5.2. Imperfect condition information

We use a Brownian motion to model the difference between the real level of deterioration $X(t)$ and the observed level of deterioration. The Brownian motion is a continuous-time process $\{W(t): t \geq 0\}$ with the following properties:

1. $W(0) = 0$ with probability 1;
2. $W(\tau) - W(t) \sim N(0, \tau - t)$ for $\tau > t \geq 0$, where $N(\mu, \sigma^2)$ denotes the normal distribution with mean μ and variance σ^2 ; and
3. $W(t)$ has independent increments.

The observed deterioration level is $X(t) + \sigma_p W(t)$, where $\sigma_p \geq 0$ can be seen as a measure of the level of uncertainty in the obtained condition information. Obviously, if $\sigma_p = 0$ then condition monitoring is perfect.

Under imperfect condition information, condition-based maintenance is performed if the observed level of deterioration exceeds a certain threshold level M . The threshold level that minimizes the cost rate constitutes the optimal CBM strategy. Because measurement errors make the obtained condition information less valuable, they result in a declining performance of condition-based maintenance. Measurement errors only influence the observed information; the underlying process and the lifetime distribution do not change. Therefore, they do not affect time-based maintenance.

Fig. 6 (a) shows the cost rate under the optimal CBM and the optimal TBM strategy, Fig. 6 (b) shows the difference between these costs. The base case introduced in Section 4 is again considered, and the level of uncertainty σ_p in the obtained condition information is varied. It turns out that a small level of uncertainty in the obtained

condition information only has a minor effect on the cost rate under the optimal CBM strategy. If the uncertainty increases further, the cost rate tends to increase linearly in the level of uncertainty. When the level of uncertainty exceeds a certain threshold, the use of condition information does not result in any cost benefits anymore compared to time-based maintenance and may even worsen performance.

We note that the control-limit strategy initiates maintenance actions based only on the actual observed deterioration level, which serves as an estimate for the true deterioration level when condition measurements are imperfect. In such cases with measurement errors, however, the estimate of the true deterioration level can be improved by using statistical inference methods that also take historical condition measurements into account. Such procedures improve condition-based maintenance strategies, see e.g. [11,41], and provide opportunities for future research.

5.3. Uncertain failure level

We model the uncertainty in the deterioration level at which failure occurs using a normal distribution with mean 1 and standard deviation σ_f . The value of σ_f can be seen as a measure of the level of uncertainty in the critical deterioration level. Because a negative deterioration level does not make sense we left truncate the normal distribution at value 0, and to maintain an average deterioration level of failure of 1, we also right truncate the normal distribution at value 2. These truncations have only a minor effect for the values of σ_f that we consider.

Contrary to imperfect condition information, uncertainty in the failure threshold does affect the lifetime distribution and thereby the performance of time-based maintenance. Uncertainty in the failure

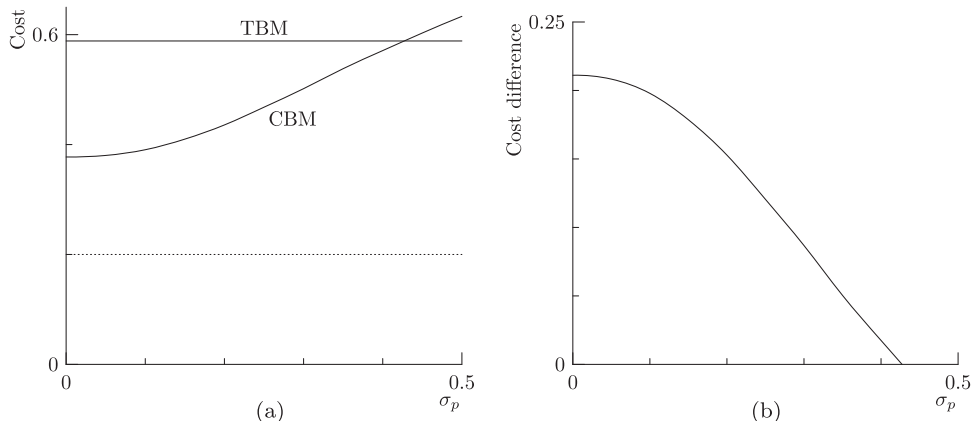


Fig. 6. The cost rate under the optimal CBM and TBM strategy (a) and the difference between these cost rates (b) for a varying level of uncertainty σ_p in the obtained deterioration information.

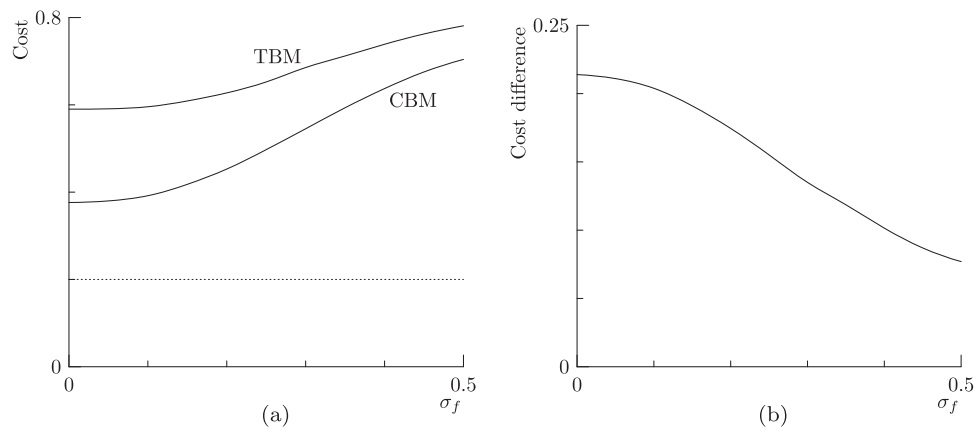


Fig. 7. The cost rate under the optimal CBM and TBM strategy (a) and the difference between these cost rates (b) for a varying level of uncertainty σ_f in the failure level.

level increases the variance of the time until failure, thereby worsening the performance of the time-based maintenance strategy. It also makes the observed condition information less valuable, leading to a declining performance of the condition-based maintenance strategy as well. Fig. 7 (a) shows the cost rate under the optimal time-based and the optimal condition-based maintenance strategy for a varying level of uncertainty in the failure threshold. Fig. 7 (b) shows the difference between these costs.

It turns out that uncertainty in the failure threshold has a greater impact on condition-based maintenance than on time-based maintenance, implying that the relative benefit of condition-based maintenance decreases if this uncertainty increases. Similar to uncertainty in the obtained condition information, a small level of uncertainty in the failure threshold only has a minor effect on the cost difference between CBM and TBM.

6. Conclusions and future extensions

We have considered the benefit of condition-based maintenance compared with time-based maintenance. We started with a literature review of studies that compare CBM with TBM, and of studies that consider required planning time, imperfect condition monitoring, and variation in the deterioration level at which failure occurs in a CBM model. These practical factors affect the relative benefit of CBM. It turned out that existing studies confine themselves to a few examples and that general insights on how the various characteristics influence the benefits of CBM are lacking. Subsequently, we considered a model for a single, continuously monitored unit that deteriorates gradually over time. We performed simulations to first analyze the effect of the behavior of the deterioration process and of the cost structure on the relative performance of CBM. Thereafter, we investigated the influence of the practical factors mentioned above.

The behavior of the deterioration process turns out to be more important for the relative cost benefit of condition-based maintenance than the cost of performing preventive maintenance. The cost difference between CBM and TBM is substantial for a small level of variation in the deterioration process, but diminishes quite rapidly if this variation increases. The actual preventive maintenance cost is less important for the magnitude of the relative cost saving of CBM; the cost benefit of CBM compared with TBM is substantial for a wide range of preventive maintenance costs. Only for extremely small or extremely large preventive maintenance costs, the potential cost benefit is limited.

From the practical factors that influence the relative benefit of CBM, required planning time and imperfect condition information only influence the performance of CBM. Uncertainty in the failure level has a negative effect on both CBM and TBM. However, because the effect on CBM is stronger, uncertainty in the failure level also worsens the relative performance of CBM. The cost benefit of CBM turns out to

decrease linearly in the planning time, and is negated completely when the planning time equals the maintenance age of the optimal TBM strategy. The effect of a planning time is thus substantial if it is large compared to the optimal maintenance age.

Both imperfect condition information and uncertainty in the failure level only have a minor effect on the relative cost benefit of CBM if the respective uncertainties are small. The marginal effects become stronger and significantly impact the cost benefit of CBM if the uncertainties increase further. A notable difference between these two effects is that the cost benefit gradually decreases but continues to be positive if the uncertainty in the failure level increases; whereas a large level of uncertainty in the obtained deterioration information might make CBM perform worse than TBM.

Our results show that all factors can significantly affect the benefit of condition-based maintenance over time-based maintenance. The obtained insights are useful for companies to assess the relative importance of the different factors in specific practical situations, and to judge whether the relative benefit of CBM outweighs the additional costs for e.g. monitoring equipment and collecting, storing and analyzing data.

Further research could be devoted to a more detailed quantification of the insights obtained, and the extent to which these are generalizable. We have modeled the deterioration using a stationary gamma process, which is a rather flexible and appropriate to model various deterioration processes in practice. Future studies could consider non-stationary gamma processes [48], or other stochastic processes as Wiener processes or inverse Gaussian processes [10]. The potential required planning time is assumed to be fixed, and studying the effect of uncertainty in the planning time is also of interest. Furthermore, we have made some modeling assumptions regarding the measurement errors and the randomness in the failure level. The obtained effects could be validated by making alternative modeling choices. Another reasonable stochastic process to model measurement errors is a mean-reverting Brownian process, which has the property that the probability distribution of measurement errors is stationary over time. The random distributions that model the uncertainty in the failure level and the random planning time, as well as the stochastic process that models the imperfect condition monitoring, could also be based on real life data. Finally, we have repeatedly considered the effect of a single factor on the benefit of CBM. The manner in which these factors interact with each other also constitutes interesting future research opportunities.

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