Erratum


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ABSTRACT

We correct representations for the endpoints of the true interval of orthogonality of a sequence of orthogonal polynomials that were stated by us in the Journal of Computational and Applied Mathematics 233 (2009) 847–851.

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In [1, Theorem 1] representations are given for the smallest zero $x_{n1}$ and the largest zero $x_{nn}$ of the polynomial $P_n$, $n > 0$, for when these polynomials satisfy a three-term recurrence relation of the type

$$
P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x), \quad n > 1,
$$

$$
P_0(x) = 1, \quad P_1(x) = x - c_1, \quad (1)
$$

where $c_n$ is real and $\lambda_n > 0$, and therefore constitute a sequence of orthogonal polynomials. Since the smallest point $\xi_1$ and largest point $\eta_1$ of the true interval of orthogonality for these polynomials are the limits as $n \to \infty$ of $x_{n1}$ and $x_{nn}$, respectively, the representations for $x_{n1}$ and $x_{nn}$ lead to representations for $\xi_1$ and $\eta_1$. However, an unjustified step in the limiting procedure has led to two incorrect statements in [1, Corollary 2]. Specifically, the second representation for $\xi_1$ is not correct and should be replaced by

$$
\xi_1 = \lim_{n \to \infty} \min_{a > 0} \left\{ \max_{1 \leq i \leq n} \left\{ c_i - a i + \lambda_i \right\} \right\}, \quad (2)
$$

where $\delta_{in}$ denotes Kronecker’s delta and $a = (a_1, a_2, \ldots)$. Also, the second representation for $\eta_1$ is not correct and should be replaced by

$$
\eta_1 = \lim_{n \to \infty} \max_{a > 0} \left\{ \min_{1 \leq i \leq n} \left\{ c_i + a i + \lambda_i \right\} \right\}. \quad (3)
$$

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These corrections have consequences for the applications in [1, Section 4]. Thus the second representation for the decay parameter $\delta$ of a nonergodic birth–death process with killing in [1, Theorem 3] should be replaced by

$$
\delta = \lim_{n \to \infty} \min_{a > 0} \left\{ \max_{0 \leq i \leq n} \left\{ \alpha_i + \beta_i + \gamma_i - a_{i+1} - \frac{\alpha_{i-1}\beta_i}{a_i} + \delta_{\min}a_{n+1} \right\} \right\},
$$

and the second representation for the decay parameter $\delta$ of an ergodic birth–death process in [1, Theorem 4] should be replaced by

$$
\delta = \lim_{n \to \infty} \min_{a > 0} \left\{ \max_{0 \leq i \leq n} \left\{ \alpha_i + \beta_i + 1 - a_{i+1} - \frac{\alpha_i\beta_i}{a_i} + \delta_{\min}a_{n+1} \right\} \right\}.
$$

Here $\alpha_i$, $\beta_i$, and $\gamma_i$ are, respectively, the birth, death, and killing rate of the process in state $i$.

The hitch in the argument leading to the erroneous representation for $\xi_1$ in [1, Corollary 2] was caused by neglecting the requirement $a_{n+1} = 0$ when taking limits as $n \to \infty$ in [1, Eq. (11)], that is, in the inequalities

$$
\min_{1 \leq i \leq n} \left\{ c_i - a_{i+1} - \frac{\lambda}{a_i} \right\} \leq x_n \leq \max_{1 \leq i \leq n} \left\{ c_i - a_{i+1} - \frac{\lambda}{a_i} \right\}.
$$

This oversight invalidates the resulting upper bound for $\xi_1$ but not the lower bound, and therefore affects the second representation for $\xi_1$ but not the first. Similar remarks pertain to the representations for $\eta_1$.

One can easily see that the second representation for $\delta$ in [1, Theorem 3], and hence the second representation for $\xi_1$ in [1, Corollary 2], cannot be correct by considering a transient, pure birth–death process with $\gamma_0 = 0$, and noting that, on choosing $a_i = \alpha_{i-1}$, this representation leads to the conclusion $\delta \leq 0$, and hence $\delta = 0$, which is well known to be false in general.

References