Theory of macroscopic quantum tunneling in Nb/Au/YBCO Josephson junctions

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Abstract

We have theoretically investigated macroscopic quantum tunneling (MQT) in $s$-wave/$d$-wave (Nb/Au/YBCO) Josephson junctions, and the influence of the nodal-quasiparticle and the zero energy bound states (ZES) on MQT. In contrast to $d$-wave/$d$-wave junctions, low-energy quasiparticle excitations resulting from nodal-quasiparticles and ZES are suppressed due to the finite isotropic gap of the $s$-wave superconductor. Therefore, the inherent dissipation of these junctions is weak. This result suggests the high potential of $s$-wave/$d$-wave hybrid junctions for applications in quantum information devices.

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1. Introduction

Since the experimental observations of macroscopic quantum tunneling (MQT) in YBCO grain boundary [1,2] and BSCCO intrinsic [3–5] Josephson junctions, the quantum dynamics of high-$T_c$ junctions has become a hot topic in the field of superconductor quantum electronics, e.g. quantum computation. Recently we have theoretically investigated the effect of nodal-quasiparticle and zero energy bound states (ZES) on MQT in $d$-wave junctions [6,7]. We have shown that suppression of MQT due to nodal-quasiparticles is very weak [6]. This result is consistent with recent experimental observations of MQT in the BSCCO intrinsic junctions [3–5]. However, it is found that ZES give a strong dissipation effect. Therefore it is important to avoid the formation of ZES in order to observe MQT.

In this paper, motivated by recent experimental studies of phase sensitive spectroscopy [8,9], we will develop a theory of MQT in Nb/Au/YBCO Josephson junctions ($s$-wave/$d$-wave hybrid junctions) as shown in Fig. 1. In contrast to $d$-wave/$d$-wave junctions [6,7], low-energy quasiparticle excitations resulting from the nodal-quasiparticles and ZES are suppressed due to the finite isotropic gap of the $s$-wave superconductor. Therefore, the weak dissipation is expected in such junctions.

2. Effective action

By using the functional integral method [10,11] the partition function of the system can be described by a functional integral over the macroscopic variable (the phase difference $\phi$)

$$Z = \int D\phi(\tau) \exp\left(-\frac{S_{\text{eff}}[\phi]}{\hbar}\right),$$

(1)
where \( \phi \) is the complex Grassmann fields. In the high barrier limit, i.e., \( z_0 \equiv m w_0 / h^2 k_F \gg 1 \) (\( m \) is the mass of the electron, \( w_0 \) is the height of the delta function-type insulating-barrier \( I \) (see Fig. 1), and \( k_F \) is the Fermi wave length), the effective action \( S_{\text{eff}} \) is given by

\[
S_{\text{eff}}[\phi] = \int_0^{\beta} d\tau \left[ \frac{M}{2} \left( \frac{\partial \phi(\tau)}{\partial \tau} \right)^2 + U(\phi) \right] + S_s[\phi],
\]

where \( \beta = 1 / k_B T \), \( M = C (h/2e)^2 \) is the mass (\( C \) is the capacitance of the junction) and the potential \( U(\phi) \) is described by

\[
U(\phi) = \frac{\hbar}{2e} \int_0^1 d\lambda \phi I_1(\lambda \phi) - \phi I_{\text{ext}},
\]

(3)

Here \( I_1 \) is the Josephson current and \( I_{\text{ext}} \) is the external bias current, respectively. The dissipation kernel \( \kappa(\tau) \) is related to the DC quasiparticle current \( I_{\text{qp}} \) by

\[
\kappa(\tau) = \frac{\hbar}{e} \int_0^{\infty} \frac{d\omega}{2\pi} e^{-\omega t} I_{\text{qp}}(\omega) \left( \frac{\hbar \omega}{e} \right)
\]

at the zero temperature.

Below, we derive the effective action for two types of \( s \)-wave/\( d \)-wave junctions \( s/d_0 \) and \( s/d_{\text{ev}} \) in order to investigate the effect of nodal-quasiparticles and ZES on MQT. In the case of the \( s/d_0 \) junction (Fig. 1a), no ZES are formed. However, in the case of the \( s/d_{\text{ev}} \) junction (Fig. 1b), the ZES are formed near the surface of the right superconductor \( d_{\text{ev}} \).

Firstly, we will calculate the potential energy \( U \) in the effective action (2). As mentioned above, \( U \) can be described by the Josephson current through the junction in the high barrier limit. In order to obtain the Josephson current we start from the Bogoliubov-de Gennes (B-dG) equation [12]

\[
\int dr' \left( \delta(r-r') \hbar(r') - \frac{\Delta(r-r') \delta(r-r')}{\Delta(r-r') \hbar(r') - \delta(r-r') \hbar(r')} \right) \left( \begin{array}{c} u(r) \\ v(r) \end{array} \right) = E \left( \begin{array}{c} u(r) \\ v(r) \end{array} \right),
\]

(5)

where \( \phi \) is the phase of order parameter, \( u \) (\( v \)) is the amplitude of the wave function for the electron (hole)-like quasiparticle, \( \hbar(r) = -\hbar^2 \nabla^2 / 2m - \mu + w_0 \delta(x) \), and \( \Delta(r-r') = \Omega^{-1} \sum \Delta_k \exp[ik \cdot (r-r')] \) is the order parameter (\( \Omega \) is the volume of the superconductor). In the superconductor regions \( s, d_0 \) and \( d_{\text{ev}} \), the B-dG equation (5) can be transformed into the eigenequation

\[
\left( \begin{array}{cc} \xi_k & \Delta_k e^{i\phi} \\ \Delta_k e^{-i\phi} & -\xi_k \end{array} \right) \left( \begin{array}{c} u_k \\ v_k \end{array} \right) = E \left( \begin{array}{c} u_k \\ v_k \end{array} \right),
\]

(6)

where \( \xi_k = \hbar^2 k^2 / 2m + \hbar^2 p^2 / 2m - \mu (p = 2\pi n/D \text{and } D \text{ is the width of the junction}) \). The amplitude of the order parameter \( \Delta_k \) is given by \( \Delta_k \cos2\theta \equiv \Delta_{k,\text{ev}}(\theta) \) for \( d_0 \), and \( \Delta_k \sin2\theta \equiv \Delta_{k,\text{ev}}(\theta) \) for \( d_{\text{ev}} \), where \( \cos \theta = k/k_F \). The Andreev reflection coefficient for the electron (hole)-like quasiparticle \( r_{\text{ev}}(r_{\text{eh}}) \) is calculated by solving the eigenequation (6) together with the appropriate boundary conditions. By substituting \( r_{\text{ev}}(r_{\text{eh}}) \) into the formula of the Josephson current for unconventional superconductors [12]

\[
I_1 = \frac{e}{\hbar} \sum_p \frac{1}{\beta} \sum_{\text{ev}} \left( \frac{A_{k,\text{ev}}}{Q_+} r_{\text{ev}} - A_{k,\text{ev}}(r_{\text{eh}}) \right),
\]

(7)

we can obtain \( \phi \) dependence of the Josephson current. Here \( A_{k,\text{ev}} = A_{k,\text{ev}(p)} \), \( Q_+ = \sqrt{(\hbar \omega_n)^2 - |A_{k,\text{ev}}|^2} \), \( \omega_n = (2n + 1) \pi / \beta \) is the fermionic Matsubara frequency. In the case of low temperatures \( (\beta^{-1} \ll \Delta_0) \) and the high barrier limit \( (z_0 \gg 1) \), we get

\[
I_1(\phi) \approx \left\{ \begin{array}{ll}
I_1 \sin \phi & \text{for } s/d_0, \\
-I_2 \sin 2\phi & \text{for } s/d_{\text{ev}},
\end{array} \right.
\]

(8)

where \( I_1 \equiv (3\delta/(2\pi)) \delta(I_{\text{ev}}) \), \( I_2 \equiv 3\delta \Delta_0 B_0 I_{\text{ev}}/10N_c R_N \), \( \delta \equiv \delta_{0;/} \Delta_0 \), \( f(\delta) \equiv \int_{-1}^{1} dx (1 + x)^{-1/2} \sqrt{1 - xK(1 - x^2 \delta^2)} \) (\( K \) is the elliptic integral), \( I_0 \) is the Josephson critical current for \( s \)-wave junctions, \( R_N \) is the normal state resistance of the

\[
\text{Fig. 1. Schematic drawing of the in-plane } s \text{-wave}/d \text{-wave hybrid Josephson junction: (a) } s/d_0, \text{ (b) } s/d_{\text{ev}}, \text{ and (c) potential } U(\phi) \text{ vs. the phase difference } \phi \text{ between two superconductors.}
\]
junction, and \( N_c \) is the number of channel at the Fermi energy.

By substituting the Josephson current into Eq. (3), we can obtain the analytical expression of the potential \( U \), i.e.,

\[
U(\phi) \approx \begin{cases} 
-\frac{\hbar}{2e} (\cos \phi + \eta \phi) & \text{for } s/d_0, \\
-\frac{\hbar}{2e} (-\cos 2\phi + 2\eta \phi) & \text{for } s/d_{x/4},
\end{cases}
\]

where \( \eta \equiv I_{\text{ex}}/I_{(1/2)} \). As in the case of the s-wave junctions, \( U \) can be expressed as the tilted washboard potential (see Fig. 1c).

3. Dissipation due to nodal-quasiparticles and ZES

Next we will calculate the dissipation kernel \( \alpha(\tau) \) in the effective action (2). In the high barrier limit, the quasiparticle current is given by [12]

\[
I_{\text{qp}}(V) = \frac{2e}{h} \sum_p |t_p|^2 \int_{-\infty}^{\infty} dE N_L(E, 0) N_R(E + eV, 0) \times [f(E) - f(E + eV)],
\]

(10)

where \( t_p \approx \cos \theta/2_0 \) is the transmission amplitude of the barrier \( I \), \( N_{L/R}(E, \theta) \) is the quasiparticle surface density of states \( (L = s \text{ and } R = d_0 \text{ or } d_{x/4}) \), and \( f(E) \) is the Fermi-Dirac distribution function. The explicit expression of the surface density of states is obtained by Matsumoto and Shiba [13]. In the case of \( d_0 \), there are no ZES. Therefore the angle \( \theta \) dependence of \( N_{d_0}(E, 0) \) is the same as the bulk and is given by

\[
N_{d_0}(E, 0) = R e - \frac{|E|}{\sqrt{E^2 - \Delta_{d_0}(\theta)^2}}.
\]

(11)

On the other hand, \( N_{d_{x/4}}(E, 0) \) is given by

\[
N_{d_{x/4}}(E, \theta) = R e \sqrt{E^2 - \Delta_{d_{x/4}}(\theta)^2} + \pi |\Delta_{d_{x/4}}(\theta)| \delta(E).
\]

(12)

The delta function peak at \( E = 0 \) corresponds to the ZES. Because of the bound state at \( E = 0 \), the quasiparticle current for the \( s/d_{x/4} \) junctions is drastically different from that for the \( s/d_0 \) junctions in which no ZES are formed. By substituting Eq. (10) into (4), we can obtain the analytical expression of the dissipation kernel \( \alpha(\tau) \). In the limit of low temperatures \( (\beta^{-1} \ll \Lambda_s) \), this can be approximated as

\[
\alpha(\tau) \approx \begin{cases} 
\frac{3h}{8\pi^2} \frac{\hbar}{R_{0} K_{1}(\frac{\Delta_{d_0}}{\hbar})} & \text{for } s/d_0, \\
\frac{3h}{8\pi^2} \frac{\hbar}{R_{0} K_{1}(\frac{\Delta_{d_{x/4}}}{\hbar})} & \text{for } s/d_{x/4},
\end{cases}
\]

(13)

where \( K_{1} \) is the modified Bessel function. For \( |\tau| \gg \hbar/\Lambda_s \), the dissipation kernel decays exponentially as a function of the imaginary time \( \tau \), i.e., \( \alpha(\tau) \sim \exp(-\Lambda_s |\tau|/\hbar) \). If the phase varies only slowly with the time scale given by \( \hbar/\Lambda_s \), we can expand \( \phi(\tau) - \phi(\tau') \) in Eq. (2) about \( \tau = \tau' \). This gives \( S_{\phi(\tau)} \approx (8C/2) \int_0^{\hbar} d\tau (\hbar/2e)\tilde{\epsilon}(\tau)\tilde{\epsilon}(\tau')^2 \). Hence the dissipation action \( S_{\alpha} \) acts as a kinetic term so that the effect of the quasiparticles results in an increase of the capacitance, \( C \rightarrow C + \delta C \equiv C_{\text{ren}} \). This indicates that the quasiparticle dissipation in s-wave/d-wave junctions is qualitatively weaker than that in in-plane d-wave/d-wave junctions in which the super-Ohmic and Ohmic dissipation appears [7].

4. MQT and crossover temperature

The MQT escape rate from the metastable potential at the zero temperature is given by \( \Gamma = \lim_{T \rightarrow -\infty} \frac{1}{2} \Im \ln Z \) [14]. By using the semiclassical (bounce) method [15], the MQT rate is approximated as

\[
\Gamma(\eta) = \frac{\omega_0(\eta)}{2\pi} \sqrt{120\pi B(\eta)} e^{-B(\eta)},
\]

(14)

where \( \omega_0(\eta) = \sqrt{\hbar C/2eM(1 - \eta^2)^{1/4}} \) is the Josephson plasma frequency and \( B(\eta) = S_{\text{eff}}(\phi_B)/\hbar \) is the bounce exponent, that is the value of the action \( S_{\text{eff}} \) evaluated along the bounce trajectory \( \phi_B(\tau) \). The analytic expression for the exponent \( B \) is given by

\[
B(\eta) = \frac{2}{5e} \sqrt{\frac{\hbar}{2e} I_{\text{C}} C_{\text{ren}} (1 - \eta^2)^{3/2}}.
\]

(15)

In MQT experiments, the switching current distribution \( P(\eta) \) is measured. \( P(\eta) \) is related to the MQT rate \( \Gamma(\eta) \) as

\[
P(\eta) = \frac{1}{v} \Gamma(\eta) \exp\left( - \frac{1}{v} \int_0^\eta \Gamma(\eta') d\eta' \right),
\]

(16)

where \( v \equiv |d\eta/d\tau| \) is the sweep rate of the external bias current. The average value of the switching current is expressed by \( \langle \eta \rangle \equiv \int_0^\infty d\eta P(\eta'/\eta) \). At high temperatures, the thermally activated decay dominates the escape process. Below the crossover temperature \( T^* \), the escape process is dominated by MQT and the escape rate is given by Eq. (14). The crossover temperature \( T^* \) is defined by [16]

\[
T^* = \frac{\hbar \omega_0(\eta = \langle \eta \rangle)}{2\pi k_B}.
\]

(17)

As was shown by Caldeira and Leggett, in the presence of a dissipation, \( T^* \) is suppressed [15].

In order to see explicitly the effect of the quasiparticle dissipation on MQT, we numerically estimate \( T^* \) by using the parameters for a high-quality Nb/Au/YBCO junction [17] (\( \delta = 10, C = 2.4 \text{ pF}, I_c = 102 \mu \text{A}, \nu I_c = 0.03 \text{ A/s} \)). In the case of \( s/d_0 \) junctions, we obtain \( T^* = 148 \text{ mK} \) for the dissipationless case \( (C_{\text{ren}} = C) \) and \( T^* = 148 \text{ mK} \) for the dissipation case \( (C_{\text{ren}} = C + \delta C) \). On the other hand, in the case of \( s/d_{x/4} \) junctions, we get \( T^* = 264 \text{ mK} \) for the dissipationless case and \( T^* = 119 \text{ mK} \) for the dissipation case. Therefore the reduction of \( T^* \) due to the node and ZES quasiparticle dissipation is small.

5. Summary

In conclusion, MQT in the in-plane s-wave/d-wave Josephson junctions has been theoretically investigated. By using the path integral and the bounce method, the effect of the low-energy quasiparticles on MQT is found to be weak. This is due to a quasiparticle tunneling block-
ade effect in the s-wave superconductor. This result suggests the high potentiality of the Nb/Au/YBCO junctions for applications in quantum logic circuits, i.e., qubit.

Finally we would like to comment the advantage of \( s/d_0 \) junctions over \( d_0/d_0 \) junctions. Regarding MQT, nodal-quasiparticles in \( d_0/d_0 \) junctions give negligibly small effect [6]. However, as was shown by Fominov et al. [18], the decoherence time of the \( d_0/d_0 \) qubit is not enough for practical quantum computation. This indicates that the nodal-quasiparticles give large influence on the qubit operation. On the other hand, due to the quasiparticle tunneling blockade effect in the \( s/d_0 \) junctions, the decoherence time of the \( s/d_0 \) qubit is expected to be much longer than that of the \( d_0/d_0 \) qubit.

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