

ON THE INFLUENCE OF HEAT CONDUCTION ON PARAMAGNETIC DISPERSION AND ABSORPTION CURVES

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Received 23 October 1969

Synopsis

Starting from the model of Casimir and Du Pré, which has been refined by Eisenstein, an expression for the differential paramagnetic susceptibility of a paramagnetic material, placed in a (gaseous or liquid) bath, is derived. This expression contains among others the coefficient of heat conduction and heat capacity of the surroundings and a heat resistance between sample and bath.

Numerical analysis and a short comparison with experiments show that in the temperature region of liquid helium the above-mentioned quantities have a remarkable influence on the dispersion and absorption curves.

1. *Introduction.* In a number of investigations concerning paramagnetic spin-lattice relaxation phenomena¹⁾ the magnetic material is placed in a field of the form

$$\mathbf{H} = \mathbf{H}_e + \mathbf{h}_0 \exp i\omega t. \quad (1.1)$$

\mathbf{H}_e is a constant field, $\mathbf{H}_e \parallel \mathbf{h}_0$. In the following considerations the sample is assumed to be isotropic and saturation phenomena are neglected. If $|\mathbf{h}_0|$ is sufficiently small, the magnetization can be described by

$$\mathbf{M} = \mathbf{M}_e + \mathbf{m}_0 \exp i\omega t. \quad (1.2)$$

\mathbf{M}_e is the constant part of the magnetization.

As a consequence of relaxation there is a phase difference between

$$\mathbf{m}_0 \exp i\omega t \quad \text{and} \quad \mathbf{h}_0 \exp i\omega t.$$

To describe this relaxation phenomenon Casimir and Du Pré²⁾ developed a thermodynamical theory. The essential features of their model are:

A. the magnetic sample is composed of two separate thermodynamic systems:

1. the collection of the magnetic moments (usually called the spin system);
2. the lattice.

The former is characterized by the state parameters M , H and T_s (spin-temperature), the latter by the lattice-temperature T_1 .

B. the energy transport from the lattice to the spin-system is given by

$$\frac{dQ_s}{dt} = -\alpha(T_s - T_1) \quad (\alpha > 0). \quad (1.3)$$

Supposing that:

1. the specific heat of the lattice is infinitely large,
 2. the lattice and the spin system are in internal equilibrium at every moment,
 3. the change of the state parameters can be described by a sufficiently small variation around the equilibrium values,
- (1.4)

Casimir and Du Pré arrived at the following formula for the reduced differential susceptibility:

$$\chi_{\text{def}} = \frac{m_0/h_0}{(\partial M/\partial H)_{T_s}} = 1 - \frac{F i \omega \tau_s}{1 + i \omega \tau_s}, \quad (1.5)$$

with $F = (C_H - C_M)/C_H$; C_H resp. C_M is the specific heat per volume of the spin system at constant H resp. M ; τ_s , the spin-lattice relaxation time, equals C_H/α .

Splitting 1.5 into a real and imaginary part ($\chi = \chi' - i\chi''$) one finds from 1.5 formulae for the dispersion χ' and the absorption χ'' :

$$\left. \begin{aligned} \chi' &= 1 - F + \frac{F}{1 + \omega^2 \tau_s^2} \\ \chi'' &= \frac{F \omega \tau_s}{1 + \omega^2 \tau_s^2} \end{aligned} \right\} \quad (1.6)$$

Usually χ' and χ'' are plotted as functions of $^{10}\log \omega$ (called resp. dispersion and absorption curve).

It has been found that in the temperature region of liquid helium in many cases the experimental results cannot be described by this formulation^{3,4)}. Several efforts have been made to give an explanation of this discrepancy. In some considerations the sample is assumed to consist of more than two thermodynamic systems between which heat exchange is possible. In literature⁵⁾ are mentioned *e.g.* the spin system, the lattice vibrations on speaking terms with the spin system, and the lattice. Another possibility is to subject the assumptions mentioned in section 1 (1.4) to a critical consideration or one may look for disturbing influences.

If one maintains the idea of two thermodynamic systems, the principal object of the experiments is to find the value of the spin-lattice relaxation

time. From the experimental dispersion and absorption curves (χ' resp. χ'' vs. $10 \log \omega$) quantities with the dimension of time are derived. Some of these quantities are:

$\tau_{\frac{1}{2}}$: the reciprocal frequency for which $\chi' = 1 - \frac{1}{2}F$,

τ_{disp} : the reciprocal frequency for which the dispersion curve has a point of inflection,

τ_{abs} : the reciprocal frequency for which the absorption curve has a maximum,

$$\tau_{\text{hf}} \stackrel{\text{def}}{=} \lim_{\omega \rightarrow \infty} \frac{1}{\omega} \frac{\chi''}{\chi' - (1 - F)} \quad (6).$$

Furthermore, explaining the experimental results with a distribution of relaxation times, a mean relaxation time $\bar{\tau}$ is defined⁷). If the salt obeys the formulae of Casimir and Du Pré (1.6), all these quantities have the same value and equal τ_s . In practice, however, they are different and their relation to τ_s is unknown.

An important step to find the spin-lattice relaxation time has been taken by Eisenstein⁸). He computed the influence of a finite coefficient of heat conduction and of a finite heat capacity of a spherical sample, assuming the heat conductivity of the surroundings to be infinitely large. By adapting theoretical curves to experimental ones (varying τ_s in the expressions for χ' and χ'') one finds a remarkable agreement between τ_{hf} and the resulting value of τ_s ⁴).

The aim of this paper is to calculate the influence of thermal properties of the surroundings on the differential magnetic susceptibility on the basis of the considerations of Casimir–Du Pré and Eisenstein. These properties are the coefficient of heat conduction, the heat capacity and a possible heat resistance between the surface of the sample and the surroundings. The calculations will be made for a spherical sample placed in a homogeneous infinitely large bath (*e.g.* liquid or gaseous helium).

2. *A general expression for the differential magnetic susceptibility.* We shall now derive a general expression for the differential magnetic susceptibility, taking into account the finite specific heat and coefficient of heat conduction of the lattice. Assumptions 2 and 3, mentioned in the preceding section under (1.4), hold with the following considerations. The basic formulae are:

$$dQ_s = C_H \left(\frac{\partial T_s}{\partial M} \right)_H dM + C_M \left(\frac{\partial T_s}{\partial H} \right)_M dH, \quad (2.1)$$

$$\frac{dQ_s}{dt} = -\alpha(T_s - T_1), \quad (2.2)$$

$$dM = \left(\frac{\partial M}{\partial T_s} \right)_H dT_s + \left(\frac{\partial M}{\partial H} \right)_{T_s} dH, \tag{2.3}$$

$$\frac{\partial}{\partial t} C_1 T_1 = \lambda_1 V^2 T_1 + P(x, y, z, t), \tag{2.4}$$

$$P(x, y, z, t) = \alpha(T_s - T_1). \tag{2.5}$$

(2.1) is related to the first law of thermodynamics for the spin system and, together with (2.2) and (2.3), it forms the basis for the theory of Casimir and Du Pré. (2.4) and (2.5) have been introduced by Eisenstein⁸); C_1 is the specific heat per volume of the lattice; λ_1 is the coefficient of heat conduction of the lattice, $P(x, y, z, t)$ is a heat source, which in general is a function of position and time. T_s and T_1 can be written in the form (*cf.* (1.1) and (1.2)):

$$T_s = T_e + \theta_s \exp i\omega t; \quad T_1 = T_e + \theta_1 \exp i\omega t. \tag{2.6}$$

From (2.3) follows, with (1.1), (1.2) and (2.6) an expression for the reduced differential magnetic susceptibility:

$$\chi = \frac{m_0/h_0}{(\partial M/\partial H)_{T_e}} = 1 + \left(\frac{\partial H}{\partial M} \right)_{T_e} \left(\frac{\partial M}{\partial T_s} \right)_{H_e} \frac{\theta_s}{h_0}. \tag{2.7}$$

Generally θ_s will be a function of position. So we define the (measurable) mean susceptibility $\bar{\chi}$:

$$\bar{\chi} \stackrel{\text{def}}{=} \frac{1}{V} \int \frac{m_0}{h_0} \frac{1}{\left(\frac{\partial M}{\partial H} \right)_{T_e}} d\tau, \tag{2.8}$$

V is the volume of the sample, $d\tau$ a volume element.

From (2.1), (2.2) and (2.3) we can derive a relation between T_1 and T_s :

$$\begin{aligned} \frac{\partial}{\partial t} C_H T_s &= -\alpha(T_s - T_1) + \\ &- \left[C_H \left(\frac{\partial T_s}{\partial M} \right)_H \left(\frac{\partial M}{\partial H} \right)_{T_s} + C_M \left(\frac{\partial T_s}{\partial H} \right)_M \right] \frac{\partial}{\partial t} H \end{aligned} \tag{2.9}$$

and so (with (2.6)):

$$\theta_s - (1 + i\omega\tau_s)^{-1} \left(\frac{\partial T_s}{\partial H} \right)_{M_e} h_0 F i\omega\tau_s - (1 + i\omega\tau_s)^{-1} \theta_1 = 0. \tag{2.10}$$

$\bar{\chi}$ can now be written in the form:

$$\bar{\chi} = 1 - \frac{F i\omega\tau_s}{1 + i\omega\tau_s} + \left(\frac{\partial M}{\partial T_s} \right)_{H_e} \frac{1}{\left(\frac{\partial M}{\partial H} \right)_{T_e}} \frac{1}{h_0} \frac{1}{1 + i\omega\tau_s} \frac{1}{V} \int \theta_1 d\tau. \tag{2.11}$$

For θ_1 shall be derived a differential equation.

From (2.4) and (2.5) we get:

$$\frac{\partial}{\partial t} C_1 T_1 = \lambda_1 V^2 T_1 + \alpha(T_s - T_1). \quad (2.12)$$

With the aid of (2.6) and (2.10) this equation can be transformed into:

$$V^2 \theta_1 - L(\omega) \theta_1 - M(\omega) = 0. \quad (2.13)$$

In (2.13) the following abbreviations are used:

$$\tau_1 = \frac{C_1}{\alpha}, \quad (2.14)$$

$$L(\omega) = \frac{C_1}{\lambda_1 \tau_1} \left[1 + i\omega\tau_1 - \frac{1}{1 + i\omega\tau_s} \right], \quad (2.15)$$

$$M(\omega) = - \frac{C_1}{\lambda_1} \frac{\tau_s}{\tau_1} \frac{1}{1 + i\omega\tau_s} \left(\frac{\partial T_s}{\partial H} \right)_{M_e} h_0 F i \omega. \quad (2.16)$$

To determine $\bar{\chi}$ we must solve (2.13) with the correct boundary conditions. For a spherical sample in a homogeneous infinitely large bath, $\bar{\chi}$ can be expressed in terms of the value of θ_1 on the boundary of the sample. In this case θ_1 will be spherically symmetric. We define a new function of the spherical coordinate r :

$$\zeta_1 \stackrel{\text{def}}{=} \frac{r\theta_1}{M(\omega)}. \quad (2.17)$$

Now (2.11) and (2.13) can be rewritten in the form:

$$\frac{\partial^2 \zeta_1}{\partial r^2} - L(\omega) \zeta_1 - r = 0, \quad (2.18)$$

$$\bar{\chi} = 1 - \frac{F i \omega \tau_s}{1 + i\omega\tau_s} + \frac{C_1}{\lambda_1} \frac{\tau_s}{\tau_1} \frac{F i \omega}{(1 + i\omega\tau_s)^2} \frac{1}{V} \int_V \frac{\zeta_1}{r} d\tau. \quad (2.19)$$

The solution of (2.18) has to obey the boundary conditions:

$$\zeta_1(0) = 0, \quad \zeta_1(r_0) = \zeta_B. \quad (2.20)$$

Here r_0 is the radius of the sample; ζ_B will be determined in section 3. So an inhomogeneous differential eq. (2.18) with inhomogeneous boundary conditions (2.20) has to be integrated. For the solution we make use of the set of functions

$$v_n(r) = \sqrt{\frac{2}{r_0}} \sin n \frac{\pi}{r_0} r \quad n = 1, 2, 3 \dots, \quad (2.21)$$

which is an orthonormal set, complete for the solutions of (2.18) and (2.20).

These functions $v_n(r)$ are eigenfunctions of the operator $\partial^2/\partial r^2 - L(\omega)$ with eigenvalue μ_n :

$$\mu_n \stackrel{\text{def}}{=} -L(\omega) = -\frac{n^2\pi^2}{r_0^2}. \quad (2.22)$$

This leads to the following expression for $\zeta_1(r)$:

$$\zeta_1 = \zeta_B \cdot \frac{r}{r_0} + \sum_{n=1}^{\infty} \frac{1}{\mu_n} v_n(r) \int_0^{r_0} r \left[L(\omega) \frac{\zeta_B}{r_0} + 1 \right] v_n(r) dr. \quad (2.23)$$

For $\bar{\chi}$ then holds:

$$\begin{aligned} \bar{\chi} = 1 - \frac{Fi\omega\tau_s}{1 + i\omega\tau_s} + \frac{C_1}{\lambda_1} \frac{\tau_s}{\tau_1} \frac{Fi\omega}{(1 + i\omega\tau_s)^2} \times \\ \times \left[\frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \frac{1}{\mu_n} + \frac{\zeta_B}{r_0} + \frac{\zeta_B}{r_0} L(\omega) \cdot \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \frac{1}{\mu_n} \right]. \end{aligned} \quad (2.24)$$

So the problem has been reduced to the determination of ζ_B . This will be done in the next section.

3. *The influence of the thermal properties of the surroundings and of heat resistance at the surface of the sample.* In this section we derive an expression for ζ_B , which depends on the coefficient of heat conduction of the surroundings and on a heat resistance on the surface of the sample.

There are two conditions for the energy flow through the surface:

$$\lambda_1(\nabla T_1)_{r_0} = \lambda_g(\nabla T_g)_{r_0}, \quad (3.1)$$

$$\lambda_1(\nabla T_1)_{r_0} = \frac{1}{R_k} (T_g - T_1)_{r_0}; \quad (3.2)$$

λ_g is the coefficient of heat conduction of the bath; T_g is the bath temperature; R_k is a heat resistance, which causes a temperature jump on the surface (at temperatures below the λ -point of liquid helium this will be the Kapitza resistance).

For T_g holds:

$$\lambda_g \nabla^2 T_g = \frac{\partial}{\partial t} C_g T_g; \quad (3.3)$$

C_g is the specific heat per volume of the bath.

Writing

$$T_g = T_c + \theta_g \exp i\omega t \quad (3.4)$$

and

$$\zeta_g(r) \stackrel{\text{def}}{=} \frac{r\theta_g}{M(\omega)}, \quad (3.5)$$

we find

$$\frac{\partial^2}{\partial r^2} \zeta_g - i\omega \cdot \frac{C_g}{\lambda_g} \zeta_g = 0. \tag{3.6}$$

The general solution of (3.6) can be written:

$$\zeta_g = A e^{\beta r} + B e^{-\beta r} \quad (A, B \text{ constants}), \tag{3.7}$$

with

$$\beta = -(1 + i) \sqrt{\frac{\omega C_g}{2\lambda_g}}. \tag{3.8}$$

It is obvious that B has to be zero and so we have:

$$\zeta_g = A \cdot e^{\beta r}. \tag{3.9}$$

(3.1) and (3.2) can be expressed in terms of ζ_1 and ζ_g :

$$\left(\frac{\partial}{\partial r} \zeta_1 \right)_{r_0} - \left(\frac{\zeta_1}{r_0} \right)_{r_0} = \frac{\lambda_g}{\lambda_1} \left(\frac{\partial}{\partial r} \zeta_g \right)_{r_0} - \frac{\lambda_g}{\lambda_1} \left(\frac{\zeta_g}{r_0} \right)_{r_0}, \tag{3.10}$$

$$\left(\frac{\partial}{\partial r} \zeta_1 \right)_{r_0} - \left(\frac{\zeta_1}{r_0} \right)_{r_0} = \frac{1}{R_k \lambda_1} (\zeta_g - \zeta_1)_{r_0}. \tag{3.11}$$

By substituting (3.9) and (2.23) into (3.10) and (3.11) we have come to two linear equations with two unknown variables A and ζ_B . For ζ_B it is finally found:

$$\frac{\zeta_B}{r_0} = \left[1 - \frac{\lambda_g}{r_0} R_k (r_0 \beta - 1) \right] 2 \sum_{n=1}^{\infty} \frac{1}{\mu_n} \times \\ \times \left[(r_0 \beta - 1) \left(2 \sum_{n=1}^{\infty} \frac{L(\omega)}{\mu_n} \frac{\lambda_g}{r_0} R_k - \frac{\lambda_g}{\lambda_1} \right) - 2 \sum_{n=1}^{\infty} \frac{1}{\mu_n} L(\omega) \right]^{-1}. \tag{3.12}$$

In the limiting case $R_k = 0$, $\lambda_g \rightarrow \infty$ the expression for $\bar{\chi}$ (combination of (3.12) and (2.24)) must turn into the formula of Eisenstein⁸). However, the form of this expression, is quite different from that of Eisenstein. It can be shown that they are identical.

4. *Discussion.* In this section the result of the preceding sections will be applied to some experimental cases. The calculations are related to a spherical sample of CrK-alum, placed in (gaseous or liquid) helium and, if the value of the magnetic field H_e is not mentioned, this amounts:

$$H_e = 179 \times 10^3 \text{ A/m}. \tag{4.1}$$

The following parameters characterize the sample:

$$\left. \begin{aligned} r_0 &= 8 \times 10^{-3} \text{ m}, & C_1 &= 15.07 \times T^3 \text{ J/K}\cdot\text{m}^3, \\ \mu_0 C &= 109 \times 10^{-9} \text{ J}\cdot\text{K/A}^2\cdot\text{m}, & b/\mu_0 C &= 4.5 \times 10^9 \text{ A}^2/\text{m}^2. \end{aligned} \right\} \tag{4.2}$$

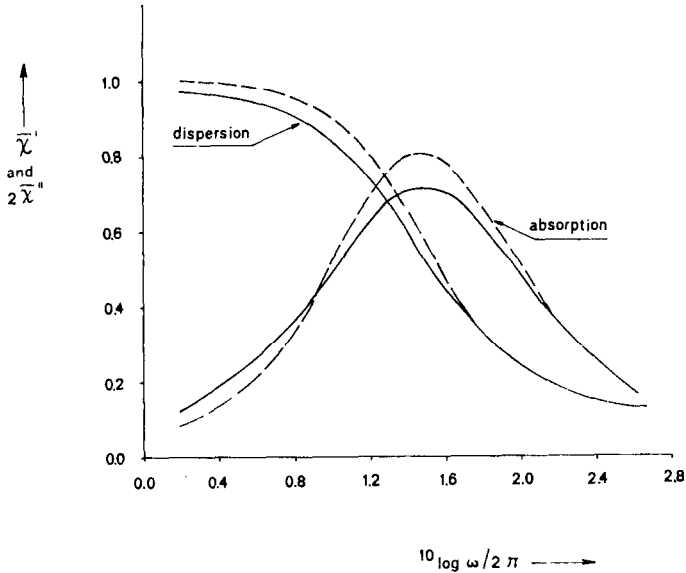


Fig. 1. Calculated dispersion and absorption curves according to formulae (2.24) and (3.12); $\tau_s = 4 \times 10^{-3}$ s, $\lambda_1 = 1.5$ J/s·K·m.; fully drawn: $C_g = 3.95 \times 10^4$ J/K·m³, $\lambda_g = 2.2 \times 10^{-2}$ J/s·K·m, (values from Wilks¹⁰); dashed: $C_g = 4 \times 10^5$ J/K·m³, $\lambda_g = 10^{20}$ J/s·K·m (in agreement with the theory of Eisenstein).

C is Curie's constant, b is related to C_M by the formula $C_M = b/T^2$. C_1 is given by Kapadnis⁹), $b/\mu_0 C$ follows from experimental dispersion curves, performed on a sample with parameters (4.2)⁴).

A) The sample is placed in a liquid He bath, with temperature above the λ -point of liquid He.

Calculations are carried out, the temperature of the sample being $T = 3.021$ K, ignoring the heat resistance ($R_k = 0$). From a comparison with the result of Eisenstein (this means in our formulation $\lambda_g \rightarrow \infty$) follows a remarkable influence of a real value of λ_g on $\bar{\chi}$. In fig. 1 the real and imaginary part of the theoretical value of $\bar{\chi} = \bar{\chi}' - \bar{\chi}''$ are drawn as functions of $10 \log \omega/2\pi$ for both cases.

By varying τ_s and λ_1 we tried to adapt a theoretical curve to an experimental one; the result is shown in fig. 2.

Remark that τ_{abs} and τ_{disp} (*cf.* sect. 1) differ considerably from τ_s :

$$\tau_{abs} = 4.4 \times 10^{-3} \text{ s}, \quad \tau_{disp} = 5.5 \times 10^{-3} \text{ s } ^4).$$

On account of these facts we conclude that the finite value of the coefficient of heat conduction in the He bath may have a measurable influence on $\bar{\chi}$.

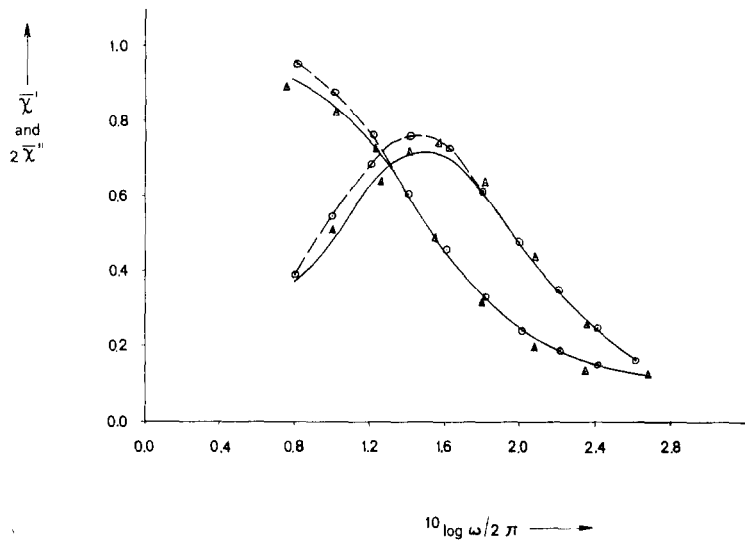


Fig. 2. Dispersion and absorption curves.

Δ : experimental curves, taken from a paper of v. d. Broek *c.s.* 4);
 fully drawn: curves according to formulae (2.24) and (3.12) which have the best fit with experimental results;
 $C_g = 3.95 \times 10^4 \text{ J/K}\cdot\text{m}^3$, $\lambda_g = 2.2 \times 10^{-2} \text{ J/s}\cdot\text{K}\cdot\text{m}$,
 $\tau_s = 4 \times 10^{-3} \text{ s}$, $\lambda_l = 1.5 \text{ J/s}\cdot\text{K}\cdot\text{m}$;
 \circ : the best fit according to the theory of Eisenstein (also from 4)); $\tau_s = 4 \times 10^{-3} \text{ s}$,
 $\lambda_l = 0.9 \text{ J/s}\cdot\text{K}\cdot\text{m}$.

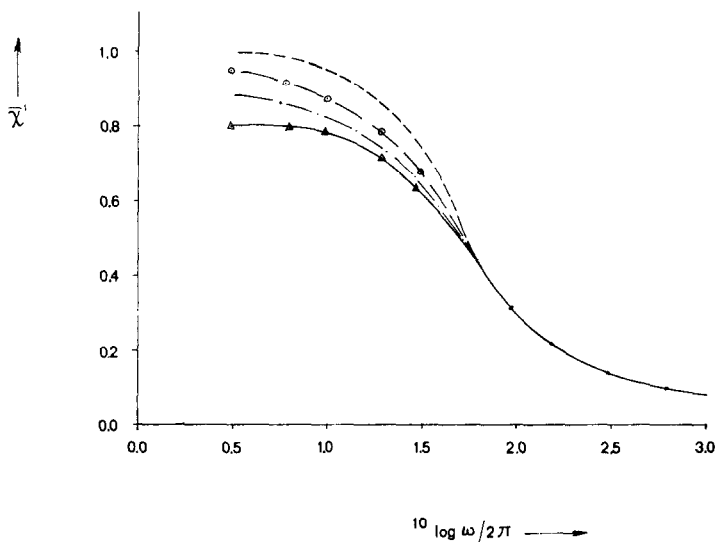


Fig. 3. Experimental dispersion curves of CrK-alum at various pressures of the surrounding helium gas.

$H = 192 \times 10^3 \text{ A/m}$, $T = 4.22 \text{ K}$, \circ : $p = 34.8 \text{ cm Hg}$,
 Δ : isolated salt, $---$: $p = 76 \text{ cm Hg}$,
 $- \cdot - \cdot$: $p = 2.5 \text{ cm Hg}$,

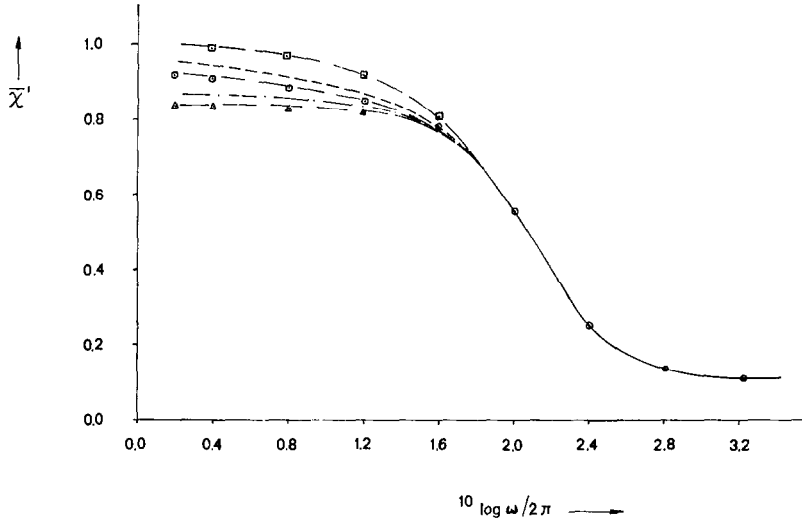


Fig. 4. Calculated dispersion curves of CrK-alum at various pressures of the surrounding helium gas.

$\tau_s = 1.5 \times 10^{-3}$ s, $\lambda_1 = 2$ J/s·K·m, $H = 192 \times 10^3$ A/m, $T = 4.22$ K,
 Δ : isolated salt, $---$: $p = 76$ cm Hg,
 $- \cdot -$: $p = 2.5$ cm Hg, \square : the salt is placed in liquid helium,
 \circ : $p = 34.8$ cm Hg,

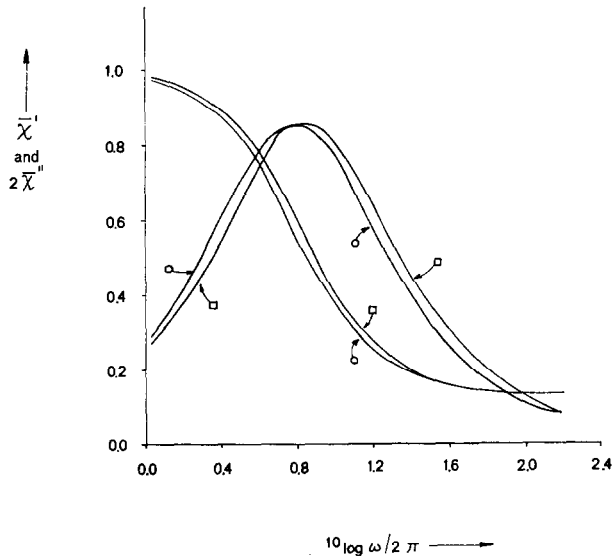


Fig. 5. Calculated absorption and dispersion curves of CrK-alum at different values of R_k .

\square $R_k = 0$, \circ $R_k = 10^{-3}$ s·K·m²/J, $\left. \begin{array}{l} \tau_s = 9.5 \times 10^{-3}$ s, \\ \lambda_1 = 0.9 J/s·K·m. \end{array} \right\}

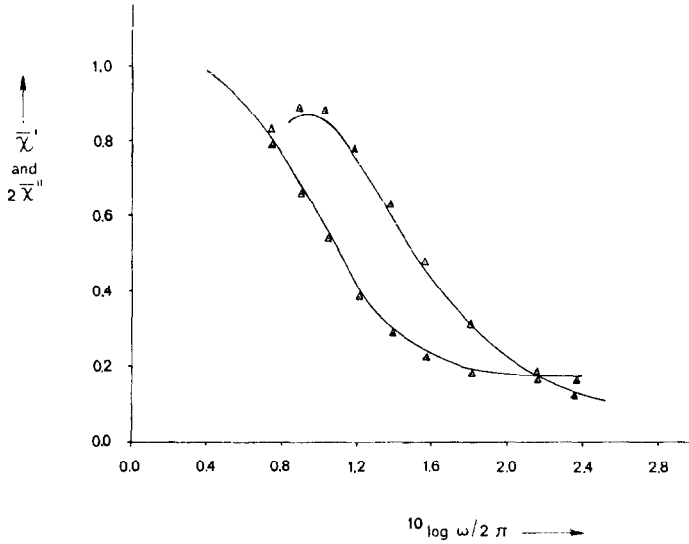


Fig. 6. Dispersion and absorption curves below the λ point.

Δ : unpublished experimental curves of CrK-alum, measured at the Kamerlingh Onnes Laboratory, Leiden in relation to the investigations mentioned in ref. 4.

$H = 179 \times 10^3$ A/m, $T = 2.068$ K.

fully drawn: calculated curves, $R_k = 10^{-3}$ s·K·m²/J,

$$\tau_s = 9.5 \times 10^{-3}$$
 s, $\lambda_1 = 0.7$ J/s·K·m.

Note: variation of τ_s and λ_1 within a range of respectively 10^{-4} s and 0.3 J/s·K·m gives no better fit with the experimental curves.

B) The sample is placed in gaseous He, at various pressures.

Experiments have been performed at the low temperature group of the Delft University of Technology on CrK-alum (not yet published) and on CoCs-tutton salt¹¹); these show a remarkable influence of the pressure in the surrounding gas on the dispersion curves (fig. 3). We computed a series of dispersion curves at various λ_g ¹²) and C_g , corresponding with different pressures (fig. 4). C_g has been calculated with the aid of experimental values of De Laet¹³) and with the equation of state of helium¹⁴).

The behaviour of these curves support the conclusion under A).

A remarkable experimental fact is that there is no difference between $\bar{\chi}$ measured at a sample in liquid He or at a sample in saturated He vapour (at the same temperature). Values of C_g and λ_g are different in both cases, and so are the calculated dispersion curves (fig. 4). This may be reason to suppose that there is some adsorption of He at the surface of the sample.

C) The sample is placed in a liquid He bath, at a temperature below the λ -point. We made calculations for $T = 2.068$ K. From Beenakker¹⁵) and Pollack¹⁶) we estimated the Kapitza resistance to be $R_k = 10^{-3}$ s·K·m²/J.

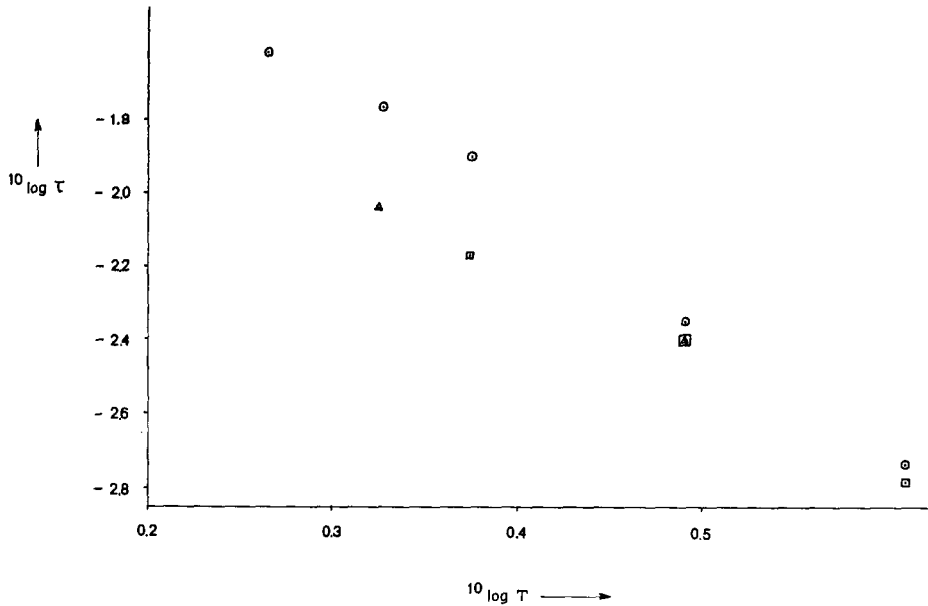


Fig. 7. Temperature dependency of τ .

- : τ_{abs} and τ_{disp}^4).
- : τ_{hf}^4).
- △: calculated values mentioned in the captions of figs. 2 and 6.

That the Kapitza resistance may have a measurable influence on the magnetic behaviour of the sample is to be seen in fig. 5.

Furthermore we tried to adapt theoretical curves to experimental ones by varying τ_s and λ_1 . The result is shown in fig. 6.

Note: variation of τ_s and λ_1 with respectively 10^{-4} s and 0.3 J/s·K·m gives no better fit between the theoretical and experimental curves. On account of the following facts we conclude that the Kapitza effect may have a measurable influence on $\bar{\chi}$: if we take $R_k = 0$, the best adaptation of the calculated curves to the experimental ones is reached for the values $\tau_s = 18 \times 10^{-3}$ s and $\lambda_1 = 10$ J/s·K·m. However, if for R_k the estimated value is taken ($R_k = 10^{-3}$ s·K·m²/J) the best fit is found at values $\tau_s = 9.5 \times 10^{-3}$ s and $\lambda_1 = 0.7$ J/s·K·m. This τ_s agrees rather well (fig. 7) with the temperature dependency of τ_{hf}^4) and a computed τ_s at $T = 3.021$ K (fig. 2). *Mutatis mutandis* the same holds for λ_1 .

This investigation has been set up on account of the results of relaxation experiments at the Delft University of Technology. The authors are indebted to Professor Dr. B. S. Blaisse, Dr. G. J. C. Bots, Mr. J. H. H. Hülsmann and Mr. A. A. Los, who took part in the experiments, for the pleasant co-operation.

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