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A Brief Summary of L. van Wijngaarden's Work Up Till His Retirement

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Abstract. This paper attempts to provide an overview of Professor Leen van Wijngaarden's scientific work by briefly summarizing a number of his papers. The review is organized by topic and covers his work on pressure waves in bubbly liquids, bubble dynamics, two-phase flow, standing waves in resonant systems, and flow cavitation noise. A list of publications up till his retirement in March 1997 is provided in the Appendix.

Key words: multiphase flow, cavitation, nonlinear waves.

1. Introduction

The title of the present paper is patterned after that used by Leendert (Leen) van Wijngaarden in writing a note on the work of his mentor, L.J.F. Broer [39].⋆ There he observed: "We know that retiring as Professor will not bring Broer's scientific activities to a stop. A review therefore can only give a momentary picture, but the occasion seems a good one to make such a survey". This is the spirit in which I write this paper, warning the reader that it has already become out of date in the few months elapsed since Leen's official retirement in March 1997. Another sentence from Leen's paper on Broer bears repeating here, with an obvious adaptation: "It would be impossible, given the number of them, to discuss here all of van Wijngaarden's papers . . . We mention only a few of them, which in particular brought [him] wide reputation".

Once Leen told me that his plan was to remain with a certain area of research only for about five years, after which he would change topic. A glance at his list of publications in the Appendix shows indeed a diversity of topics, but it is also obvious that, once bewitched by multiphase bubbly flow, he found it very difficult to leave this subject. This is the work for which he is best known and with which I am most familiar. Even though it will take up the lion's share of my remarks, I cannot hope to do more than give a sense of its scope, adding a little bit of historical perspective.

⋆ Numbers in brackets refer to van Wijngaarden's publications listed in the Appendix.

2. Pressure Waves in Bubbly Liquids

Cavitating flows are a major concern in any naval research laboratory, and the Wageningen Netherlands Ship Model Basin – which Leen joined in 1962 shortly after receiving his doctorate from the Technical University of Delft – was no exception. In hindsight, it seems obvious that cavitation and bubbles were to prove particularly congenial to him. His doctoral dissertation had the title *Applications of the One-Fluid and Two-Fluid Models in Magnetohydrodynamics*. Thus, he was already familiar with the problem of modeling particle systems as complex continua, which is precisely the problem one encounters in trying to describe a bubbly liquid either as a single continuum with a complex structure (one-fluid model), or as two interpenetrating continua (two-fluid model). In addition, he had a very solid background in potential theory, which is of course a particularly suitable modeling tool for free-surface flow, such as that around bubbles.

The first problem that caught Leen's attention at Wageningen was the observed bending of the trailing edge of ship propeller blades. Van Manen (1963) had put forward the hypothesis that the effect was associated in some way with cavitation, but it was not clear from what was known at the time about single-bubble collapse that the necessary impulse could be generated in this way. Indeed, as Leen observes in his paper [13], "A collapsing bubble can . . . have effect only over a range comparable in magnitude with the radius of the bubble. The bending of trailing edges must be due to high pressures, effective over a region which is many orders of magnitude larger than the radius of an individual bubble". This observation prompted him to examine the possibility that cumulative effects develop in the course of the collapse of a large number of bubbles. Not only this hypothesis has proven correct, but it has also spawned a rich literature on bubble cloud behavior that continues strong to this day.

In this paper [13] (which carries a publication date of 1966, but was actually presented at a conference in 1964), Leen considers an effectively one-dimensional situation with a layer of bubbly liquid adjacent to a plane rigid boundary. Pure liquid extends beyond the bubbly layer and all the bubbles are assumed to have the same initial radius. He formulates his model in terms of mixture velocity and pressure fields, a bubble velocity field, and a bubble number density, for which he writes two continuity and two momentum equations. The key aspect of the model is the connection between the local instantaneous radius of the bubbles and the mixture pressure. His sentence "For an individual bubble the average [mixture] pressure p is the 'pressure at infinity' of single-bubble theory" contains the insight that makes the whole machinery work.

Unknown to him at the time, a similar idea had been introduced by Foldy in a 1945 paper devoted to the theory of multiple scattering. The important difference, however, was that the path followed by Foldy would only work for linear problems, while Leen's inspiration led to a model of much broader validity that actually contains Foldy's in the small-amplitude limit. And indeed, shortly thereafter, he started publishing papers on the nonlinear aspects of wave propagation in bubbly liquids [14, 16].* The latter paper [16] “On the equations of motion of liquid and gas bubbles” is especially well known as it has formed the basis for many subsequent analyses of bubbly liquids and, in particular, wave phenomena in such media. It is remarkable that Leen’s assumption has been put on a satisfactory mathematical basis only two decades later (Caflisch et al., 1985), in a paper that proves several points that, from a reading of Leen’s original work, appear to have been quite evident to him.

One striking aspect of Leen’s 1964/1966 paper [13] is the brevity of the reference list: the paper by van Manen, two references to the single-bubble dynamics literature used to substantiate simplifying assumptions, and Lamb’s book in connection with what is known today as the Rayleigh–Plesset equation. Since I know for a fact that Leen is very conscientious when it comes to citing others’ work, the implication is that he invented the whole mathematical model by himself. In particular, he was unaware of papers published in Russian at about the same time by Kogarko (1964) and a few years earlier by Iordanskii (1960), both of whom had developed similar models.

His own interest, Broer’s teaching, and the rapidly developing field of nonlinear waves, kept Leen interested in pressure waves in bubbly liquids even as he was working on other problems. The subject of shock waves was particularly intriguing. The data of Campbell and Pitcher (1958), and those that Leen’s student L. Noordzij was gathering in the laboratory, clearly showed the existence of shocks possessing a characteristic undular structure on the high-pressure side. This was incompatible with the results of Leen’s 1968 paper [16] in which he had reduced his system of equations to the Korteweg–De Vries form as this latter equation is known not to possess shock-like solutions. Lighthill’s work on viscous effects in nonlinear sound waves (1956), as well as Broer’s work on relaxation gasdynamics (1970), pointed to the need of including dissipative effects in the theory. This Leen did in his first paper on the subject [19], identifying, with Batchelor (1969), the agent responsible for the observed shock structure in the viscosity of the liquid. Leen’s 1974 paper with Noordzij [25] presents beautiful data and an elaborate attempt to explain them on the basis of the velocity slip between the phases. This insistence on viscosity is understandable as at the time the essential role that gas-liquid heat transfer plays in the dynamics of gas bubbles was just beginning to be realized (Nigmatulin and Khabeev, 1974). We now know that thermal, rather than viscous, relaxation accounts for most of the shock wave structure in bubbly liquids (Watanabe and Prosperetti, 1994; Kameda and Matsumoto, 1996), but it was Noordzij and van Wijngaarden’s paper that showed how to go about proving this fact.

In conclusion, I would like at least to mention Leen’s influential 1972 *Annual Review of Fluid Mechanics* paper [24], that served as an introduction to bubbly

* The comments made by Brooke Benjamin in a discussion of [14] formed the basis for the analysis of nonlinear waves in [16]. Leen has more than once recognized the influence that Brooke Benjamin’s work exerted on his own.

liquids for many – including myself – and his very recent tribute to a life-long fascination, "Evolving solitons in bubbly flows" [60]. This latter paper reports data gathered with his Master's student Roelofsen about ten years earlier. Leen intended it as a tribute to Korteweg and de Vries 100 years after their seminal paper (Korteweg and de Vries, 1895).

3. Bubble Motion

Leen's interest in the modeling of bubbly flows prompted him to study several aspects of the motion of single bubbles and bubble pairs to be used as building blocks for averaged equations models. The best known paper in this group is probably "Hydrodynamic interaction between gas bubbles in liquids" [29] which he completed during his stay at Caltech as Fairchild Scholar in 1974/1975. In this study he considers the velocity acquired by a cloud of bubbles when the liquid in which they are immersed is impulsively accelerated. For a single massless sphere in a uniform flow subject to an acceleration $\mathrm{d}u/\mathrm{d}t$ the answer follows directly from a consideration of the Kelvin impulse:

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\frac{1}{2}\rho\tau(v-u)=\rho\tau\,\frac{\mathrm{d}u}{\mathrm{d}t}\,, \tag{1}$$

where ρ is the liquid density, and τ and v the volume and velocity of the sphere. The left-hand side of (1) is the rate of change of the impulse of the liquid, and the right-hand side the external force. For a dilute suspension, the problem can be reduced to that of a pair of bubbles, and then Batchelor's renormalization technique (1972) can be used to determine the velocity of the cloud. I mentioned before Leen's mastery of potential flow theory. A particularly elegant example can be found in section 2 of this paper and, since it is brief and can be fully appreciated even outside its context, it is worthwhile to cite it in its entirety:

The motion of a spherical bubble in an arbitrary potential flow

We consider a perfect fluid, at rest for times $t<0$, in which a spherical gas bubble of radius a is immersed. At $t=0$ a velocity field which has, in the absence of the bubble, a potential ϕ_0, is instantaneously generated in the fluid. As a result of the motion of the liquid the bubble will assume a velocity $\mathbf{v}$, and the resultant potential will be $\phi_0+\phi_1$. Since the bubble can be regarded as massless, the resultant force exerted on the bubble by the liquid is zero at all times and therefore, with pressure p and surface element $\mathbf{dA}$,

$$\int_{t=0-}^{t=0+}\mathrm{d}t\int p\,\mathbf{dA}=0, \tag{2}$$

where $t=0\mp$ indicate times just before and just after $t=0$, respectively. By using Bernoulli's theorem it follows from (2) that

$$\int(\phi_0+\phi_1)\;\mathbf{dA}=0. \tag{3}$$

The meaning of (3) is that the impulsive forces on the sphere generated by the original motion of the liquid and by the relative motion between liquid and bubble are equal but opposite in sign. Apart from (3), the resulting potential has also to satisfy the boundary condition

$$\nabla(\phi_0 + \phi_1) \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n} \tag{4}$$

on the sphere, $\mathbf{n}$ being a unit vector normal to the surface of the sphere. If $\mathbf{v}$ were known, (4) would, for given ϕ_0, uniquely determine ϕ_1. However, $\mathbf{v}$ is unknown and must be such as to satisfy (3). In solving this problem, two properties of the potential flow around a sphere are of great assistance. The first is that, for any harmonic function with singularities outside the sphere, the surface and volume averages over the sphere are equal to the value of that function at the centre of the sphere (see, for example, Kellog, 1953, p. 223). Thus, if the value of $\nabla\phi_0$ at the centre of the sphere is denoted by $(\nabla\phi_0)_c$, we have

$$\int \phi_0 \, \mathbf{dA} = \int \nabla\phi_0 \, \mathrm{d}\tau = (\nabla\phi_0)_c \, \tau, \tag{5}$$

τ being, as before, the volume of the sphere. The second useful property is given by Weiss's sphere theorem (Milne-Thomson, 1968, p. 520). This states that, if in an unbounded potential flow with potential ϕ a sphere is placed at $\mathbf{r} = 0$ and if the singularities of ϕ are all outside the sphere, the resulting potential is

$$\phi + \frac{1}{a} \int_0^{a^2/r} R \frac{\partial}{\partial R} \{\phi(R, \omega, \theta)\} \, \mathrm{d}R, \tag{6}$$

where (R, ω, θ) are spherical co-ordinates with origin at $\mathbf{r} = 0$. We cannot apply Weiss's theorem to ϕ_0 because in the motion described by (6) the sphere is kept, by external forces, at rest, whereas our spherical bubble moves as a result of the impulsive forces associated with the generation of ϕ_0. We can apply Weiss's theorem, however, to that part of ϕ_0 which produces no motion at the centre of the sphere. Denoting $(\nabla\phi_0)_c$ by $\mathbf{u}$, we write ϕ_0 as

$$\phi_0 = \mathbf{u} \cdot \mathbf{r} + \tilde{\phi}_0. \tag{7}$$

Because $\int (\mathbf{u} \cdot \mathbf{r}) \, \mathbf{dA} = \mathbf{u}\tau$, we have, on account of (5),

$$\int \tilde{\phi}_0 \, \mathbf{dA} = 0. \tag{8}$$

The response of the bubble to $\tilde{\phi}_0$ can be obtained from Weiss's theorem (6), whereas the response of the uniform flow with velocity $\mathbf{u}$ is well known: a doublet of strength $(1/2)a^3(\mathbf{v} - \mathbf{u})$ at the centre of the bubble. The potential $\phi_0 + \phi_1$ therefore becomes

$$\phi_0 + \phi_1 = \mathbf{u} \cdot \mathbf{r} + \tilde{\phi}_0 - \frac{(\mathbf{v} - \mathbf{u})a^3 \cdot \mathbf{r}}{2r^3} + \frac{1}{a} \int_0^{a^2/r} R \frac{\partial}{\partial R} \tilde{\phi}_0(R, \omega, \theta) \, \mathrm{d}R. \tag{9}$$

The potential, while satisfying (4), must satisfy (3) as well. At $r = a$, the last term on the right-hand side of (9) may be written as

$$\tilde{\phi}_0(a, \omega, \theta) - \frac{1}{a} \int_0^a \tilde{\phi}_0\,(R, \theta, \omega)\;\mathrm{d}R.$$

On account of (8) and because $\tilde{\phi}_0$ is regular inside the sphere the integral of this expression over the surface of the sphere vanishes. We must choose **v** such that, upon substitution of (9) into (3), the integral comprising the first and third terms on the right-hand side of (9) is zero. As is easily verified, this is the case for

$$\mathbf{v} = 3\mathbf{u} = 3\,(\nabla\phi_0)_c\,. \tag{10}$$

We have in this way obtained the interesting result that a massless sphere moves, in an impulsively generated flow, with three times the velocity that this flow has at the location at $t = 0$ of the centre of the sphere. For a uniform flow, this follows immediately from (1). Note that the result expressed by (10) is independent of the radius of the sphere.

I have always found this argument very beautiful in its generality and economy of means. I think it deserves to find its way into textbooks.

Another contribution to this area is the proof of the following result for the force **F** on a sphere moving with velocity **v** in a flow field **u** [42]:

$$\begin{aligned}\mathbf{F} &= -\int_A p\,\mathbf{n}\,\mathrm{d}A \\ &= \rho\tau\frac{D\mathbf{v}}{Dt} - \frac{3}{2}\,\rho\tau\,\frac{D}{Dt}\,(\mathbf{v} - \mathbf{U}) - 4\pi\rho\sum_s M_{ij\ldots k}(\partial_i\partial_j\ldots\partial_k)\,\mathbf{u}_R(\mathbf{x}_s).\end{aligned} \tag{11}$$

Here p is the pressure, **n** the outwardly directed unit normal, **U** the velocity field **u** evaluated at the center of the sphere, and D/Dt the convective derivative with the velocity **v**. The summation is over all the singularities located at points $\mathbf{x}_s$ inside the sphere, the M's are the corresponding multipole moments, and $\mathbf{u}_R$ is the regular part of the velocity field. This equation was originally given by Landweber and Miloh (1980), but their argument left some room for doubt due to a questionable interchange of integration and differentiation. Leen's proof, simpler and more direct, showed that it is nevertheless correct. As an application, he considered the flow of a dilute bubbly liquid through a diffuser and pointed out the possibility of coalescence due to the deceleration of the bubbles. Generalizations of (11) have recently been derived by Galper and Miloh (1994, 1995).

As shown in the well-known paper by Batchelor and Green (1972) on the determination of the bulk stress in a suspension to second order in the volume fraction, ensemble-average calculations with this order of accuracy require a knowledge of the two-particle probability distribution function. For this reason Leen has devoted considerable attention to the mutual interaction of two bubbles rising in a liquid.

His early 1982 paper with Biesheuvel [41] was followed by the more complete study of his student Kok (1993a, 1993b, 1993c; see also [55]). With the neglect of viscosity, if the angle that the line joining the bubble centers makes with the direction of rise is smaller than a critical value the radial force is repulsive, while it is attractive in the opposite case. The interaction force in the angular direction, however, is always such as to tend to align the bubbles normally to their direction of rise. As a consequence, even if the bubbles start out with a small angle so that they repel each other, eventually the angle will grow and the force will become attractive. This argument has important implications for the stability of bubbly flows that will be mentioned at the end of the next section. Viscous effects change the picture somewhat as the relative motion may be damped out before the critical angle is reached but, in any event, these results suggest that an arrangement with the bubbles aligned horizontally should be much more likely than a random one. A generalization to unequal bubble radii has been given by Kumaran and Koch (1993a, 1993b) who found attraction for radii ratios between 0.93 and 1.07. Outside this narrow range the bubbles do not touch during the interaction provided their size is not too different.

To conclude, I would like to mention the work on the shape of rising bubbles and the effect of surfactants recently carried out by one of the last students of Leen's, P.C. Duineveld (1995, 1996; Bel Fdhila and Duineveld, 1996). With his characteristic generosity, Leen does not appear as a co-author although his influence is easy to surmise.

4. Two-Phase Flow

The work on pressure waves briefly described in Section 2 was in a way a preparation for the modeling of more general bubbly flows to which Leen has devoted a great deal of energy and ingenuity.

He early on realized the importance of added mass effects in such flows (see, e.g., [24]). When the issue of hyperbolicity was raised in the multiphase flow community in a well-known round table discussion at the Fifth International Heat Transfer Conference held in Tokyo in 1974 (Gidaspow et al., 1974), his interest increased further as he thought that such effects – neglected in the simpler models – could be the answer to the problem. In our – regrettably only – joint paper [28], it was shown that a hyperbolic model could be formulated provided the added mass interaction had a certain form. At the time it was not clear how to derive the proper expression for the added mass of bubbles in a general flow and it was later shown (among others, by Leen himself in his paper [42] where he proved Equation (11)) that our proposed form was not correct. The equations remained therefore ill-posed.

Leen's paper [29] on the hydrodynamic interactions of bubbles, already mentioned in the previous section, was at least in part an attempt to explore the consequences of bubble-bubble effects on the added mass interaction. In this paper he derived his well-known result for the mean added mass coefficient in a uniform, dilute bubbly liquid:

$$m(\beta) = \frac{1}{2}\rho\beta\ (1 + 2.78\beta)\,, \tag{12}$$

where β is the bubble volume fraction. A later calculation by Biesheuvel (1984; see Biesheuvel and Spoelstra, 1989) gave a coefficient 3.32 in place of 2.78 and, for a while, it was thought that (12) was incorrect. As a matter of fact, on p. 313 of Leen's (1984) paper with Biesheuvel [45], we find: "We take this opportunity to report that the value 2.78…is incorrect owing to a computational error, and should be 3.32"! Only later was it realized that the numerical value of this coefficient depends on the assumed velocity distribution (Biesheuvel and Spoelstra, 1989; Sangani et al., 1991; Zhang and Prosperetti, 1994a). In [29] Leen had started with spheres at rest, and did not restrict their velocity distribution after the acceleration, while Biesheuvel and Spoelstra assumed that all the spheres had the same velocity. A continuous interpolation between the two extreme cases can be made by assigning to the particles densities between 0 and infinity (Zhang and Prosperetti, 1994a). It is then found that in the former case, corresponding to Leen's calculation, (12) is recovered ⋆ while, for infinite density (i.e., equal – vanishing – velocities), the correct numerical factor is 3.32.

In any event, it was clear that the answer to the puzzle provided by the lack of hyperbolicity did not rest with added mass alone. After a few years mainly devoted to resonance in open tubes (see next section), Leen's group was joined by a new doctoral student, Arie Biesheuvel. Thus started a most fruitful collaboration on bubbly flows that continues to this day. In their 1984 paper [45] already cited, they were able to derive an expression for the bulk and Reynolds stresses in a dilute bubbly liquid by ensemble averaging and potential flow considerations. These results have recently been confirmed by a different approach (Zhang and Prosperetti, 1994b). The final system of equations, however, still would not have been hyperbolic had they retained the convective part of the acceleration in the equation for the relative motion between the phases. On the basis of an argument of local uniformity they dropped this term and were able to obtain a fully hyperbolic model.

Leen's satisfaction with this result, however, was not long-lived due to the somewhat artificial argument about the convective derivatives. He, Biesheuvel, and a new student, Kapteyn, started working on void fraction disturbances in bubbly liquids. His 1990 paper with Kapteyn [50] (published in the *J. Fluid Mech.* volume dedicated to his long-time friend George Batchelor) contains the following argument highlighting the essential nature of the dependence of the stability of concentration waves on the relation between added mass and drag.

⋆ A more precise value of the coefficient is 2.76.

Consider the one-dimensional upward motion of a bubbly liquid. The equation of motion of the bubbles may be expected to have the following structure:

$$\frac{\mathrm{d}}{\mathrm{d}t}\,[m(u - U_0)] + f\,(u - U_0) = \rho g \tau, \tag{13}$$

where m is the added mass, $u - U_0$ the mean bubble velocity relative to the (constant) volume velocity of the mixture U_0, $f(u - U_0)$ the drag force, and g the acceleration of gravity. Suppose that a simple compressive concentration wave connecting two states with uniform concentrations β_a ahead, and β_b behind the wave, with $\beta_b < \beta_a$, propagates in the mixture. Upstream and downstream of the wave the flow is steady and buoyancy can only be balanced by friction, so that

$$f\,(u - U_0) = \rho g \tau. \tag{14}$$

If f were equal to m, then (14) would also be valid throughout the wave: as f increases due to the increasing concentration, the relative velocity would decrease by an equal amount and the added mass interaction would remain identically zero. But suppose that m increases more rapidly than f with increasing β. In this case the fluid impulse would tend to increase along the wave and, as a consequence, the force on the bubbles would be downward. Conversely, if m were to increase more slowly than f, the force would be upward. The effect is thus stabilizing in the first case, and destabilizing in the second one, which points to the importance of the sign of

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\left(\frac{m}{f}\right) \tag{15}$$

for the stability of the concentration wave.

The rest of the paper is devoted to a study of the internal structure of concentration waves. With the assumption that the mean rise velocity of bubbles in a uniform bubbly liquid has the form

$$U - U_0 = V\,(1 - \lambda\beta), \tag{16}$$

with V and λ obtained from experiment, the wave thickness is calculated with results in general agreement with experiment.

A relation such as (16) must have looked like a challenge to Leen. It has the same general form as the famous result of Batchelor (1972) on the sedimentation of a suspension and seems to beg for a theoretical derivation. This Leen attempted in a paper published in 1993 but originally submitted in 1988 [55]. Building on the results of Kok mentioned before, he assumed that the probability distribution of the mutual orientation of the bubbles was highly peaked in the direction normal to the direction of rise. With this simplification, he was able to calculate approximately the evolution of the pair probability of rising bubbles and to find a relation of the form (16) with $\lambda = 1.56$, which is in reasonable agreement with the measured value $\lambda = 1.78$. Perhaps more importantly, in this paper he showed analytically that his own results with Biesheuvel [41], and the more recent ones of Kok, implied a strong tendency of rising bubbles to organize into horizontal structures. This conclusion was confirmed in the same year by two papers reporting the results of direct numerical simulations (Sangani and Didwania, 1993; Smereka, 1993). This finding has spawned a considerable amount of work attempting to explain why these horizontal bands are not observed experimentally. The original version of Leen's paper suggested that turbulence would play a role in disrupting these structures. This suggestion is very likely one component of the puzzle (see, e.g., Spelt and Biesheuvel, 1997) but, for some reason, it did not pass muster with the reviewers of *J. Fluid Mech.* and was removed from the final version of the paper.

5. Standing Waves

Leen's first paper to be published in the *J. Fluid Mech.* [9] was devoted to the sloshing of a liquid in a harmonically tilting rectangular container. For small amplitudes, the natural period of the surface disturbances is just the time it takes for a wave to go back and forth from one end of the container to the other. In the shallow-water approximation, therefore, the natural angular frequency is given by

$$\omega_0 = \frac{\pi}{B} (g h_0)^{1/2}, \tag{17}$$

where B is the width of the container and h_0 the undisturbed water depth. Linear theory predicts that the amplitude of the oscillations is inversely proportional to $\cos(\pi\omega/2\omega_0)$, which diverges as the frequency ω of the tilting motion approaches ω_0. The paper studies the phenomena occurring in these conditions. Starting from their experimental observation that, near resonance, a hydraulic jump appears on the free surface, Leen and his co-author Verhagen use Lin's method of strained characteristic coordinates previously used by Chu and Ying (1963) in a study of thermoacoustically forced oscillations in a resonant gas tube. This problem was not revisited by Leen but, indirectly, led him to a very beautiful piece of work with his student Disselhorst a few years later.

The starting point was the mathematical similarity between the sloshing waves and standing waves in a gas-filled tube, that also develop into shocks when driven near one of the linear normal-mode frequencies. The situation had been studied when the tube was closed at both ends, but Leen realized that a host of fascinating fluid mechanical phenomena take place when one of the ends is open. In his classic *Theory of Sound*, Rayleigh had estimated the acoustic energy radiated by the open end and, by equating it to the work performed on the gas, had deduced that the amplitude U of the gas velocity oscillations should be of the order

$$\frac{U}{\omega\delta} \sim \frac{L}{R}, \tag{18}$$

where δ is the piston displacement, L the tube length, and R its radius. For typical values of the parameters, this formula gives amplitudes that are orders of magnitude too large. In his first paper on the subject [17], Leen pointed out that viscous effects cause separation of the boundary layer during the compressive stroke of the piston, so that a jet of gas exits the tube. He argued that, during the subsequent inflow phase, the pressure gradient is favorable and the flow into the tube mouth is similar to that induced by a sink. As a consequence of this asymmetry between the two phases of the wave, the kinetic energy carried away by the jet is not entirely recovered and this net loss of kinetic energy turns out to be much more important than the energy lost by radiation. With this picture in mind, a simple argument leads, in place of (18), to

$$\frac{U}{\omega\delta} \sim \sqrt{\frac{c_0}{\delta\omega}}\,, \tag{19}$$

where c_0 is the speed of sound in the gas. The velocity fluctuation predicted by this formula is significantly smaller than the previous estimate and in reasonable agreement with the experimental results reported in the paper. Furthermore, this conceptual picture suggests effective boundary conditions at the pipe's end of the form

$$p - p_0 \;=\; 0 \qquad \text{for outflow}, \tag{20}$$

$$p - p_0 \;=\; C\frac{\partial u}{\partial t} - \rho_0 u^2 \qquad \text{for inflow}, \tag{21}$$

where p_0, ρ_0 are the undisturbed pressure and density outside the pipe, C is proportional to the added mass, and u is the velocity at the pipe mouth.

While better than (18), the agreement of this theory with experiment was still incomplete and Leen kept working on the problem even as he was getting more and more into bubbly liquids. His experiments with Wormgoor [26] and his series of papers with his student Disselhorst [34–36] represent a major contribution to an intricate problem which had eluded a number of other investigators. The early experiments [26] revealed that, due to the large centrifugal force on the air flowing around the pipe edge during the inflow phase, a separation bubble was generated that was later ejected as a vortex ring during the subsequent outflow. This sequence of events was confirmed in a more extensive series of experiments [36] in which schlieren photography provided an accurate picture of the phenomenon.⋆ A direct confirmation of the importance of vortex shedding on the overall response of the standing wave system was provided by the difference between data obtained with round- and sharp-edged tubes. While a theory that neglects vortex shedding – essentially embodied in Equations (20) and (21) – was in good agreement with experiment in the former case, it did not match data in the second one.

⋆ In this paper [36], in addition to the usual knife-edge schlieren system, the authors used the variant with a Wollaston prism. This is one of the very rare color illustrations published by the *J. Fluid Mech.* before the present decade.

To attack the problem theoretically, Disselhorst and van Wijngaarden simulated numerically the ejection of discrete vortices from a two-dimensional duct in an otherwise irrotational flow, for a given velocity in the duct $-\hat{u}\sin\omega t$. They disregarded viscosity and the wall thickness and employed the Kutta condition (the applicability of which they had checked on the schlieren images) to determine the strength of the vortices. They were able to carry the calculation far enough to reach an approximately steady regime and in this way they derived the following effective boundary condition at the pipe exit:

$$p - p_0 = 0.6\,\rho_0\,\omega R\left(\frac{\hat{u}}{\omega R}\right)^{2/3} u, \tag{22}$$

where $\hat{u}$ is a suitably defined velocity amplitude and u is the instantaneous velocity. When used to express the energy balance in the tube, this equation gave results in excellent agreement with experiment for velocity amplitudes smaller than ωR.

6. Other Problems

Before concluding this brief review, I would like to touch upon a few other problems to which Leen has contributed with his customary physical insight and mathematical sophistication.

I still remember a lecture he gave at Caltech in 1975 on rotating flows. I think that his interest in the subject had been prompted by talk in those years of a European uranium enrichment plant. Although he never worked "officially" on rotating flows, his involvement was far greater than a casual one. His only paper on this topic [46] is an unusually lucid review focusing on the work of his Enschede colleagues Dijkstra, Zandbergen, and van Heijst (see Zandbergen and Dijkstra, 1987). In it he modestly states "I had the pleasure to see their work in progress and to take part in their discussions". Judging from what I recall of his Caltech lecture, and from this paper, his participation in those discussions must have been quite an active one.

By addressing the mechanism of cavitation noise with an eye to the important problem of its scaling, in the last few years Leen has returned to the earlier interests in flow cavitation that first blossomed in Wageningen. This is another area where the dynamics of a single bubble can only explain part of the data. The largest fraction of the noise can only be interpreted in terms of cooperative effects on which, after all, he is one of the world's leading experts. As usual, he has looked at a number of facets of the problem.

His student Omta (1987) considered the linear and moderately nonlinear oscillations of a spherical bubble cloud described by means of the averaged equations derived in paper [45] with Biesheuvel. Although Leen only appears as a co-author in a preliminary version of this study [47], his inspiration is patent. A linear theory of the problem had been presented earlier by d'Agostino and Brennen (1983; see also d'Agostino and Brennen, 1989; Kumar and Brennen, 1991), who explicitly acknowledge the influence of Leen's 1964/1966 paper [13] on their work.

A few years later, with his student Buist, he studied the flow of an unbounded mass of liquid separated from a wavy surface by a bubbly layer [54]. The sinusoidal waviness of the wall represents one Fourier mode of more complex and realistic flows.* The situation is superficially similar to one studied by d'Agostino et al. (1988) in whose case, however, there was no clear-liquid region. The concern of Leen's paper is with the mechanism by which a layer of cavitation bubbles next to a surface radiates sound into the clear liquid far away. Such a flow, when modelled by averaged equations, appears steady and therefore it is incapable of acoustic radiation. The authors observe however that sound generation becomes possible if one takes statistical fluctuations into account. While one may envisage several agents responsible for such fluctuations, the particular one studied here is the relative motion between the bubbles and the liquid. The picture underlying this mechanism is that the bubbly-liquid/clear-liquid interface fluctuates with a random normal velocity v' and, in so doing, radiates an acoustic power $\overline{p'v'}$, where p' is the disturbance pressure and the overline indicates a (time, space, or ensemble) average. Since the pressure disturbance can be related to v', the calculation of the acoustic spectrum is reduced to that of the correlation function of the fluctuating velocity. The result of this calculation gave noise levels about 50 dB's lower than measured ones. The mechanism considered, therefore, even if present, cannot be the chief source of cavitation noise.

For this reason, Leen went back to bubble cloud noise with the following picture in mind (see, e.g., [57]). It is well known that steady sheet cavitation gives rise to a paradox in that the condition of uniform pressure in the cavity is in conflict with the presence of a stagnation point where the cavity re-attaches to the body. The resolution of the paradox by means of a reentrant jet that conveniently disappears onto another Riemann sheet is classic (see, e.g., Birkhoff and Zarantonello, 1957, p. 56). Remarkably, a reentrant jet does exist but, because of it, the situation is not steady. Cyclically, the jet flows upstream along the solid surface, strikes the cavity near its inception point and, in so doing, causes the detachment of a large fraction of it. Associated with this loss of simple connectivity of the liquid volume is the appearance of a strong vortex that entrains the large bubble cloud generated by the fragmentation of the detached cavity. The subsequent collapse of the cloud is most likely responsible for the strong acoustic emission in the frequency range around 1 kHz.

This picture is based on a conspicuous number of experimental observations, including those of one of Leen's Master's students, R.H.M. van der Stegen (1993). A quantification of this scenario requires a clear understanding of the unsteady reentrant jet phenomenon, a problem on which Leen's last Ph.D. student, F. de Lange, has made considerable progress. I am very pleased indeed that my Depart-

* In a 1962 paper, Leen had studied magnetohydrodynamic flows in a similar geometry [5].

ment at Johns Hopkins has played an albeit small role in this development as de Lange spent about two months with us closely interacting with Dr. H.N. Oğuz.

7. Conclusions

Upon assuming the chair of Fluid Mechanics in the Department of Mechanical Engineering at the University of Twente in 1966, Leen delivered an inaugural lecture by the title "On the practice of fluid mechanics" [11]. The ideas that he expressed in that circumstance are as fresh now as they were then.

After mentioning several great Dutch scientists – first and foremost Burgers – he reflects on the nature of Fluid Mechanics looking at the "where, why, and how" of the discipline. He crisply perceives the need for both pure and applied research and draws a very balanced picture of the natural symbiosis between industry and university in this area. He uses Seneca's (re-worded) dictum "Non scholae sed vitae discimus" (we learn for life, not for the school) to stress the desirability of an education that, while based on fundamentals, keeps the applied side of the discipline in clear view of the students.

Equally balanced is his view of the "how" of Fluid Mechanics. He puts a great emphasis on experiment quoting Einstein: "All knowledge of reality starts with experiment and ends in it". At the same time he points to the essential role of theory and of its chief tool, applied mathematics. Remarkably, he is quite clear even about the impact that the bursting of computers on the scene would soon have. Nevertheless, he says, "every now and then I try to find analytical solutions to the problems that come my way. Those hours may not seem to have been spent efficiently, but they are spent in patience and dedication".

Leen concludes his lecture with a simile drawn from an area for which he always nurtured a lively interest: "As in literature many minor authors are necessary for one Shakespeare, great researchers step out of thick rows of lesser colleagues". One can easily imagine that the young author of these words (Leen was 34 at the time) was hopeful to be one of the few who would, one day, "step out".

And – sure enough – he did.

Acknowledgments

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Appendix
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