Supplementary materials for "Vertically bounded double diffusive convection in the finger regime: comparing no-slip vs free-slip boundary conditions"

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Here we provide the details of our numerical simulations. We solve the incompressible Navier-Stokes equations with two scalar fields, namely,

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \partial_j^2 u_i + g\delta_{i3}(\beta_T \theta - \beta_S s), \tag{1a}$$

$$\partial_t \theta + u_j \partial_j \theta = \kappa_T \partial_j^2 \theta, \tag{1b}$$

$$\partial_t s + u_j \partial_j s = \kappa_S \partial_j^2 s, \tag{1c}$$

$$\partial_i u_i = 0. \tag{1d}$$

We employ the Oberbeck-Boussinesq approximation, which assumes that the fluid density depends linearly on both scalar fields, i.e. $\rho = \rho_0(1 - \beta_T \theta + \beta_S s)$ with ρ_0 being a reference density, θ and s the temperature and salinity relative to some reference values, respectively. In the equations u_i with $i = 1, 2, 3$ are the velocity components, p is the pressure, ν is the kinematic viscosity, g is the constant acceleration of gravity, β is the positive expansion coefficient, and κ is the molecular diffusivity. δ_{ij} is the Kronecker delta. Three global balance relations can be derived from the governing eqution (1) as [22]

$$\epsilon_\theta \equiv \left\langle \kappa_T [\partial_i \theta]^2 \right\rangle_V = \kappa_T \, (\Delta_T)^2 \, L^{-2} \, Nu_T, \tag{2a}$$

$$\epsilon_s \equiv \left\langle \kappa_S [\partial_i s]^2 \right\rangle_V = -\kappa_S \, (\Delta_S)^2 \, L^{-2} \, Nu_S, \tag{2b}$$

$$\epsilon_u \equiv \left\langle \nu [\partial_i u_j]^2 \right\rangle_V = \nu^3 L^{-4} \, \left[Ra_S \, Pr_S^{-2} (Nu_S - 1) - Ra_T \, Pr_T^{-2} (Nu_T - 1) \right], \tag{2c}$$

where L is the height of the fluid layer and Δ_T and Δ_S are the temperature and salinity differences across the layer. These relations will be used in our numerical simulations.

The fluid is bounded by two parallel plates which are perpendicular to the direction of gravity and separated by a distance L. The horizontal size of the domain is much larger than the finger width so that the periodic boundary conditions can be applied to both horizontal directions. At two plates both temperature and salinity are constant and the top plate has higher temperature and salinity. Thus the temperature difference drives the flow and the salinity difference stabilises the flow. For velocity either no-slip or free-slip boundary conditions are applied for each set of the control parameters. For the initial conditions, the fluid is at rest, the temperature field has a linear profile and the salinity field is uniform and equals to the mean of the values at the two plates. These initial distributions are the same as those in the experiments of Ref. 22. Small random perturbations, with a magnitude of 0.1% of the total difference between two plates, are superposed onto the initial temperature and salinity fields in order to accelerate the flow development.

It is well known that small molecular diffusivity poses a great challenge for direct numerical simulations and requires a very fine resolution. This is exactly the case in our numerical work, especially for the salinity field which has a Prandtl number of $Pr_S = 700$. In order to overcome this difficulty we developed a multiple resolution code which is described in details in Ref. 27. Our code has been validated by a one-to-one comparison with experiments [22]. In the current study, to ensure the flow is adequately resolved, we always check the following three criteria.

First, in the multiple resolution method we use a base mesh for the momentum and temperature and a refined mesh for the salinity. The grid size h_b of the base mesh satisfies $h_b \leq \pi\eta = \pi(\nu^3/\varepsilon_u)^{1/4}$, where η is the viscous length scale and ε_u is the momentum dissipation, respectively. Correspondingly, the grid size h_r of the refined mesh satisfies $h_r \leq \pi\eta_S = \pi(\kappa_S^3/\varepsilon_u)^{1/4}$. Second, the two Nusselt numbers Nu_S and Nu_T are calculated independently by two different methods. Namely, one can calculate the convective fluxes based on the definition and take average over the whole volume. Meanwhile, one can also calculate the global dissipation rates and use relations (2a) and (2b) to deduce the two Nusselt numbers. In our simulations the difference between the two methods is smaller than 1% for $Ra_S < 10^{10}$. For higher Ra_S the difference is slightly bigger but always less than 3%. The consistency between the values obtained from different methods also indicate the flow is well resolved. Third, by using the relation (2c) we also ensure the global balance between the convection term and the dissipation term for all cases.

The cases with no-slip boundary conditions used in this study are part of a larger dataset which has been reported in details in Ref. [24]. Here in Table I we provide the numerical details of the free-slip cases.

TABLE I. Numerical details for the cases with free-slip boundary conditions. Columns from left to right: Ra_S, Ra_T, aspect ratio Γ of the domain (width/height), resolution of the base mesh, resolution of the refined mesh, time duration for statistics sampling, Nu_T, Nu_S, Re, λ_S, and ϵ_b. The domain width and the resolution are the same in the two horizontal directions.

Ra_S	Ra_T	Γ	$N_{1,2}^b \times N_3^b$	$N_{1,2}^r \times N_3^r$	$T_{average}$	Nu_T	Nu_S	Re	λ_S	ϵ_b
1.0×10^6	1.0×10^5	4.0	216×144	432×144	600	1.010	15.37	0.1961	4.40×10^{-2}	5.38×10^{-4}
2.0×10^6	1.0×10^5	4.0	192×144	576×144	500	1.025	19.87	0.3179	3.42×10^{-2}	5.50×10^{-4}
5.0×10^6	1.0×10^5	4.0	240×144	720×288	400	1.080	28.40	0.6025	2.24×10^{-2}	5.99×10^{-4}
8.0×10^6	1.0×10^5	4.0	240×192	960×384	240	1.141	32.93	0.8128	1.87×10^{-2}	5.77×10^{-4}
1.0×10^7	1.0×10^5	4.0	240×192	960×384	200	1.188	34.24	0.9037	1.79×10^{-2}	5.20×10^{-4}
1.0×10^7	1.0×10^6	2.4	192×144	576×288	600	1.023	32.28	0.5002	2.01×10^{-2}	4.06×10^{-4}
2.0×10^7	1.0×10^6	2.4	240×192	720×384	500	1.057	43.23	0.8362	1.51×10^{-2}	4.43×10^{-4}
5.0×10^7	1.0×10^6	2.4	240×240	960×480	300	1.173	56.74	1.482	1.11×10^{-2}	3.90×10^{-4}
8.0×10^7	1.0×10^6	2.4	288×288	1152×576	200	1.313	67.15	2.031	9.10×10^{-3}	3.88×10^{-4}
1.0×10^8	1.0×10^6	2.4	320×288	1280×576	200	1.413	72.90	2.363	8.28×10^{-3}	3.85×10^{-4}
1.0×10^8	1.0×10^7	1.2	216×240	648×480	500	1.050	66.36	1.259	9.50×10^{-3}	2.86×10^{-4}
2.0×10^8	1.0×10^7	1.2	240×288	720×576	400	1.123	87.87	2.093	7.36×10^{-3}	3.02×10^{-4}
5.0×10^8	1.0×10^7	1.2	288×288	1152×864	240	1.347	111.2	3.692	5.48×10^{-3}	2.51×10^{-4}
8.0×10^8	1.0×10^7	1.6	432×288	1728×864	200	1.600	125.7	4.876	4.73×10^{-3}	2.23×10^{-4}
1.0×10^9	1.0×10^7	1.6	480×320	1920×960	160	1.828	142.9	5.851	4.15×10^{-3}	2.40×10^{-4}
1.0×10^{10}	1.6×10^8	0.5	384×768	1536×3072	200	1.919	290.0	14.18	2.06×10^{-3}	1.69×10^{-4}
1.0×10^{11}	1.6×10^9	0.3	576×1728	1728×5184	105	2.686	582.5	36.43	9.81×10^{-4}	1.16×10^{-4}
1.0×10^{12}	1.6×10^{10}	0.2	576×2304	1728×9216	70	4.067	1109.0	90.97	4.70×10^{-4}	7.20×10^{-5}