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**Parametric Optimization: Singularities, Pathfollowing and Jumps.** By J. Guddat and F. Guerra Vasquez with H. Th. Jongen. B. G. Teubner, Stuttgart, John Wiley & Sons, Chichester, 1990. viii + 191 pp. \$85.00, cloth. ISBN 3-519-02112-9 (Teubner). ISBN 0-471-92807-0 (Wiley).

A general strategy in nonlinear optimization is to advance along a one-dimensional path towards a local or global optimum. The hill climbing technique with line search is based on this principle and has led to powerful numerical methods. However, even if theoretically global convergence can be expected, the hill climber may get lost somewhere in the  $n$ -dimensional landscape. An explanation is that he has only local information available.

As an alternative with more information of global type, continuation methods have been proposed. They are the subject of this monograph. The original problem is embedded in a family of problems  $P(t)$  which depend continuously on a real parameter  $t$ . Again a one-dimensional path is followed, which now consists of solutions  $x(t)$  of the parameterized problem. If everything works as expected the path leads from the known solution  $x(0)$  of a simple problem  $P(0)$  to the required solution  $x(1)$  of the given problem  $P(1)$ . When the continuation principle is applied to optimization it is appropriate to take as  $x(t)$  the critical points of the parameterized problem. They form a bifurcation diagram; that means there can be singular points where the path splits or turns backward. It is even possible that  $x(0)$  and  $x(1)$  lie in distinct connected components. Ordinary path following will not succeed in such a case.

The book starts with an analysis of the possible types of critical points, due to Jongen, Jonker, and Twilt. Five types of unavoidable critical points are classified. Points of type 1 are nondegenerate, the others represent different kinds of singularities. For example, type 2 means that an active constraint becomes inactive or vice versa. More complicated singularities are also possible, but avoidable by means of small perturbations.

This theory forms the basis for computational methods. A predictor-corrector technique is used to follow nondegenerate parts of the path. Special techniques are employed to identify singularities and proceed further. A jumping process makes it possible to overcome the problem of disconnectedness, at least in certain cases. If at a singularity the path turns backward it is no longer possible to advance via critical points in the direction of increasing  $t$ . But at singularities  $x(t_0)$  of type 2, 3, and a subtype of 4, a direction of descent can be found for the problem  $P(t_0)$  with fixed  $t_0$ . Then a descent method can be used to leave the path of critical points and jump to another branch of the bifurcation diagram. A second jumping process is described which can serve to detect as many branches as possible.

Of course the choice of a parameterization influences the working of a continuation method. Several possibilities for this choice are discussed. The book concludes with remarks about global and multiobjective optimization.

A lot of work has been done to implement the methods and make numerical experiments. Although most of the reported examples are relatively small, the power of the continuation method as a tool to solve nonconvex optimization problems is clearly demonstrated. All in all this well-written monograph is a valuable addition to the literature on optimization methods.

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**Interfacial Transport Phenomena.** By John C. Slattery. Springer-Verlag, New York, 1990. xvi + 1159 pp. \$89.00. ISBN 0-387-97387-7.

This monograph of over 1000 pages is an encyclopedic work that everyone involved in the area of interfacial phenomena will want on their bookshelf. Both the publisher and the author are to be congratulated for producing this book at a price that will allow individuals to acquire a copy. The style is terse and the pace is rapid, as one can tell by reading the first section of Chapter 1. The concept of a material particle is introduced on page 3, the Cauchy–Green tensors are presented on page 8, and on page 9 one