



## Book Reviews

Chain-scattering approach to  $H^\infty$ -control

H. Kimura

*Birkhäuser, Boston, 1997, 246 pages. ISBN 0-8176-3787-7*

Some eight years ago at a workshop in Como, Italy, Bill Helton perplexed many in the audience when he redirected some arrows in a feedback configuration and claimed this to be helpful in the analysis of the  $H_\infty$  control problem. Suddenly outputs were called inputs and the other way around. Helton was about to explain the use of chain-scattering descriptions for  $H_\infty$  control, although I am not sure he called it that.

From an input/output viewpoint, a chain-scattering representation is indeed simply a representation of a system where the role of some inputs and outputs are interchanged. It is well known in classical circuit theory Anderson and Vongpanitlerd, 1973. As I recall, the audience in Como did not feel at ease with what seemed as a trick, and this uneasiness may be one of the reasons that a chain-scattering approach to  $H_\infty$  is relatively unknown, but it is an important one and an elegant one. The approach when combined with the theory of  $J$ -spectral factorization, arguably provides the most compact theory for  $H_\infty$  control (see besides the book under review for example also Ball et al. (1990, 1991), Green (1992), Meinsma and Green (1992) and Weiland et al. (1997)).

Kimura's contribution to this field is extensive, and he is inventor of the transfer matrix method called *J-lossless conjugation*, a method that in his approach is an essential building block in the chain-scattering theory of  $H_\infty$  control. There are also other approaches to  $H_\infty$  control that use chain-scattering and  $J$ -losslessness — see e.g. the references listed above — but in the book the author has chosen for a (quote) “compilation of the author's recent work.” Some other approaches are briefly mentioned at the end of each chapter.

Here then is a self-contained book that in less than 250 pages provides an exposition of chain-scattering and its application to  $H_\infty$  control. The making of an excellent book one might think, but, regrettably, such is not the case. It seems to me that the author did not find the time to verify his own writing, how else can one comprehend the presence of so many mistakes? Besides that, the

organization of the material is somewhat uneven: In the beginning of the book standard material is discussed thoroughly, while more involved problems further on in the book are glossed over.

It starts off promising. After an introductory chapter the book continues in Ch. 2 with “elements of linear system theory.” Controllability and stability and such things are carefully described and the proofs are clear. The section on state feedback and output injection is compact but accessible.

Ch. 3 begins with definitions of signal spaces and norms. There the classic (but usually harmless) mistake is made with the claim that  $\lim_{t \rightarrow \infty} f(t) = 0$  for all  $f \in L_p$ ,  $1 \leq p < \infty$ . The presentation deteriorates from this point onwards. One might agree with the author that singular values are so ubiquitous that they need no definition, but since when is a function called *bounded* in norm only if it is bounded in norm by 1? By the way, the alternative characterization (3.30) of boundedness is not fully correct.

Section 3.2 on Hamiltonians and Riccati equations is fine, although it requires some knowledge of eigenspaces, which, though used, is not defined. Section 3.3 reviews some results on spectral factorization. The results, as the author mentions, “are all standard, but the derivations are nonstandard with emphasis on clarity and simplicity”. As it stands, however, Theorem 3.9, Corollary 3.11, Theorems 3.12–3.14 are not correct. They all need the assumption that the  $A$ -matrix is stable, and Theorems 3.13, 3.14 in addition need controllability and observability, respectively. It is not clear to me why the author does not include these essential assumptions. After all, minimality and observability are so clearly defined in the preceding chapter.

With the necessary details added, the material of Ch. 3 provides a compact overview of Riccati equations, the bounded real lemma and spectral factorization, with all proofs included.

In Ch. 4 properties of chain-scattering representations are reviewed. The relation between the usual i/o representation and the chain-scattering representation is one of swapping some inputs and outputs. Algebraically this

amounts to taking certain inverses. This chapter documents many formulae of chain-scattering representations. One of the nicest properties surely is that linear fractional transformations reduce to simple matrix multiplications when represented in chain-scattering form. This has some advantages.

Ch. 4 also discusses state-space characterizations of losslessness and  $J$ -losslessness, this with the aim to use these techniques in the  $H_\infty$  theory of later chapters. Losslessness and  $J$ -losslessness are essentially the same thing except that one applies to the system in standard i/o form and the other in chain-scattering form. The idea to bring together all these properties in one chapter is fine I think. I did not understand the need to include some comments on coprime factorizations (a notion left undefined in the book). And then there are the mistakes. Theorems 4.5 without some minimality condition is wrong, and its proof is incomplete. For the same reason Theorem 4.11 is wrong. The proofs of Lemma 4.7 and Theorem 4.15 are inadequate.

Ch. 5 is the most interesting chapter. Here the nice idea of  $J$ -lossless conjugation is explained, which I consider to be Kimura's main contribution to this field. The central result is Theorem 5.2 that relates  $J$ -lossless conjugation with the existence of a stabilizing solution of a Riccati equation. The proof is technical but the theorem is interesting and quite powerful. With it the one-sided Nevanlinna–Pick interpolation problem is solved. For correctness Theorem 5.2 should assume (anti) stabilizability of the realization.

In Ch. 6 conjugation is used in the analysis of  $J$ -spectral factorization and  $J$ -lossless factorization. This is an important topic and includes for example ordinary spectral factorization as a special case. That ordinary spectral factorization plays an important role in the theory of LQG and  $H_2$  control, is well documented. Similarly,  $J$ -spectral factorization plays an important role in the theory of  $H_\infty$  control.

Theorems 6.13 and 6.14 need the assumption that the realization is detectable and stabilizable. On the other hand, the central results (Theorems 6.5, 6.6, 6.10 and 6.12) assume minimal realizations even though it is known that under the weaker assumptions of detectability/stabilizability they hold as well.

By now enough machinery is available to solve the standard  $H_\infty$  control problem. At this stage this can be solved completely in terms of transfer matrices, i.e., in the frequency domain. This is done in Ch. 7. The bottleneck in probably any theory of  $H_\infty$  control, is to show that existence of a solution *implies* the two famous Riccati equations to be solvable. The equivalent frequency domain result is that the existence of a solution implies a certain  $J$ -spectral factorization to exist. In the book this sort of, is Lemma 7.3. I am afraid to say, however, that the proof of that lemma is wrong and the proof cannot be fixed. Lemma 7.3 is nevertheless correct, which can be

deduced from e.g. (Ball et al., 1991; Green, 1992). With Lemma 7.3 established, there are various ways to proceed with the solution of the standard  $H_\infty$  control problem. The author takes the approach, also taken in Ball et al. (1991), to augment the system so that there are as many exogenous disturbance inputs  $w$  as there are observation outputs  $y$ . This allows to represent the system in chain-scattering representation for which the problem was solved earlier. What sort of augmentation one should take and what effect augmentation has on closed-loop stability is not considered. In the notes to this chapter briefly some other approaches of  $J$ -spectral factorization to  $H_\infty$  control are mentioned, among them is Green (1992). To my opinion the work Green (1992) is somewhat more elegant. It does not require augmentations and is less technical.

In the final chapter, Ch. 8, the frequency domain solution of the standard  $H_\infty$  control problem from the previous chapter, is translated into state-space formulae, i.e., the two famous Riccati equations. Here we see an advantage of frequency domain/chain-scattering/ $J$ -spectral factorization methods over the usual state-space approaches. In state-space approaches it is common to assume certain matrices to be zero or mutually orthogonal, which is needed to keep the formulae manageable. In frequency domain the formulae are manageable from start to finish and the state-space formulae can be derived from it algorithmically, and the resulting formulae (p. 193) come out simpler than those in, e.g., Zhou's book (Zhou, 1996, p. 451). The final state space formulae are collected in Theorem 8.11, but the author seems to have been a bit too excited as conditions (i) and (ii) of that theorem are not fully correct.

In summary, this is a book which — despite the high error density — may be useful for those who are interested in  $H_\infty$  control, and for those who have an interest in representations other than the usual i/o representations. I think that chain scattering has potential. This is exemplified by the fact that chain scattering is used elsewhere. Besides in circuit theory, I know of applications in behavioral control and infinite-dimensional system theory, and there are undoubtedly other applications. I would urge the author to make an errata list available via internet. With the errata the book can be a handy reference for  $J$ -spectral factorization and chain-scattering formulae. A short course on  $H_\infty$  control can be based on this book, but some other books have the advantage of having more exercises or of having an index (Green and Limebeer, 1995; Zhou et al., 1996). In the book there are some design examples, but these are academic examples. The author takes a “matrix manipulation” approach to  $H_\infty$  control. By that I mean that signal interpretations are avoided and that the transfer matrix is seen as the main object, an object that is to be manipulated with. Such an approach is not to everyone's taste, but it has the

benefit that a good understanding of matrix calculus is enough to follow most of the arguments.

G. Meinsma  
Faculty of Applied Mathematics,  
University of Twente,  
P.O. Box 217,  
7500 AE, Enschede, The Netherlands.

## References

- Anderson, B. D. O., & Vongpanitlerd, S. (1973). *Network analysis and synthesis*. Englewood Cliffs, NJ: Prentice-Hall.
- Ball, J. A., Gohberg, I., & Rodman, L. (1990). *Interpolation of rational matrix functions, 45, of operator theory: advances and applications*, Basel: Birkhäuser Verlag.
- Ball, J. A., Helten, J. W., & Verma, M. (1991). A factorization principle for stabilization of linear control systems. *International Journal of Robust and Nonlinear Control*, 1, 229–294.
- Green, M. (1992).  $H_\infty$  controller synthesis by  $J$ -lossless coprime factorization. *SIAM Journal Control and Optimization*, 30, 522–547.
- Green, M., & Limebeer, D. *Linear robust control*. Englewood Cliffs, NJ: Prentice-Hall.
- Zhou, K., Glover, K., & Doyle, J. C. (1996). *Robust and optimal control*. Englewood Cliffs, NJ: Prentice-Hall.
- Meinsma, G., & Green, M. (1992). On strict passivity and its application to interpolation and  $H_\infty$  control. *Proceedings 31-st CDC, Tucson, Arizona*.
- Trentelman, H. L., & Willems, J. C. (1998).  $H_\infty$  control in a behavioral context: The full information case. *IEEE Transactions on Automatic Control*, accepted for publication.
- Weiland, S., Stoorvogel, A. A., & de Jager, B. (1997). A behavioral approach to the  $H_\infty$  optimal control problem, system and control theory in the behavioral framework. *Systems and Control Letters*, 32, (5), 323–334.

### About the reviewer

**Gjerrit Meinsma** was born in Opeinde, The Netherlands. He received his Ph.D. degree at the University of Twente in 1993 and after that held a three-year postdoc position at the University of Newcastle, Australia. From January 1997 he held a postdoc position with the Faculty of Applied Mathematics at the University of Twente. In 1998 the postdoc position finally became a permanent position.

© 1999 Published by Elsevier Science Ltd. All rights reserved  
PII: S 0005-1098(99)00028-X

# Eigenstructure Assignment for Control System Design

G.P. Liu, R.J. Patton

*Wiley, Chichester, ISBN 0 471 97549 4*

Following the early contribution of Wonham (1967) which states that the closed-loop eigenvalues of a multi-input controllable system can be assigned arbitrarily, many studies on pole assignability in linear systems with state and output feedbacks have been published. In a multi-input system the solution to this problem is generally nonunique. Hence, many papers have considered how, using the available degrees of freedom, different control objectives can be achieved simultaneously. The extension of analytical studies and design techniques in the framework of eigenstructure assignment, from the continuous state-space model to single- and multi-rate sampled-data systems, and to continuous and discrete time singular systems, becomes naturally a subject for further studies in the time-invariant linear system theory.

Hence, the far-reaching implication of the concept of eigenstructure assignment yields the need for a book which concentrates on this particular topic and its relevant applications. In this regard the contribution of the book under consideration is important. This book demonstrates the wide potential and the varieties of applications based on eigenstructure assignment techniques.

The authors contribution to the field of eigenstructure assignment in its wide sense, is significant. The book

contains many results that follow from the authors' research papers as well as from other studies. As stated by the authors in their preface, the purpose of the book is to spread familiarity with the techniques of eigenstructure assignment techniques among control researchers and control engineers and to present the major techniques and research directions associated with this concept for the design of MIMO linear systems.

The book consists of a preface, 11 chapters, an appendix, and an extensive and useful bibliography. Beginning with an overview of eigenstructure assignment, the authors discuss various topics in the framework of control laws designed via eigenstructure assignment. In Chapter 2 the fundamental theory of eigenstructure assignment is presented. Basic concepts like the freedom for eigenstructure assignment, the allowable eigenvector subspace and a design compromise between the selected eigenvalues and eigenvectors, are examined.

Chapter 3 deals with the parametric solutions to the eigenstructure assignment problem under some conditions. Parametric expressions for the state feedback are derived for the case when the closed-loop eigenvalues are distinct, and for more complicated structures which allow multiple eigenvalues of the closed-loop system. Chapter 4 is devoted to the problem of eigenvalue