NUMERICAL SOLUTION OF THE TRANSVERSE RESISTIVITY OF SUPERCONDUCTING CABLES UNDER AC CONDITIONS

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Abstract

Interstrand coupling losses are mainly due to currents perpendicular to the strands. A numerical model for calculating the current distribution of a single strand carrying a transverse transport current is presented. Also the effective resistence felt by the transport current is computed.

Introduction

In this paper we will develop a numerical method for calculating the transverse resistivity of superconducting cables.
A superconducting cable, as treated in this paper, exists of a twisted bundle of strands with a nonconducting inner region. If such a cable is placed in an external magnetic field, the induced currents will not merely flow in the axial direction, but also around the centre, in the plane of the cross section [1] (see figure 1). These currents can saturate some parts of the cable. This may result in a much smaller value of the maximal transport current then expected, and hence small coupling losses. This is the mean reason why the transverse resistivity is so important for the use of superconducting cables in technical applications.

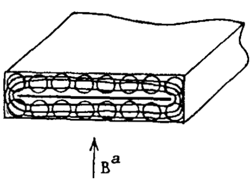

Fig. 1. Current patterns if a cable is placed in an applied field.

In previous papers [2,3,4] we have already computed a first order approximation of the current distribution of superconducting cables, where the applied field is a function of place and time. This was achieved by modelling a cable by a current sheet, using Carr's approximation [5] and a conductivity perpendicular to the direction of the wires. To improve these results a more accurate value of the effective conductivity is needed. We will model a strand by a rectangular slab with a layer of twisted wires (see figure 2). The strand carries an induced transverse current. Due to symmetry, only a quarter of the section is needed. A numerical method is presented to solve Maxwell's equations and the nonlinear relation between the electric field and the current density. For a number of situations the current distribution in the cross section is calculated, and some conclusions about the parameter dependence of the transverse resistivity will be given.

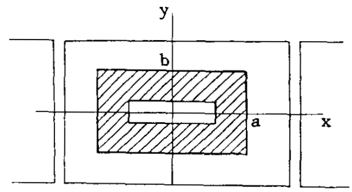

Fig. 2. A mathematical model of the cross section of a strand. A superconducting layer, embedded in normal conducting material.

Theory

The transverse resistivity of a single strand can be calculated by solving Maxwell's and the constitutive equations. To reduce the problem to a two dimensional one, we will not incorporate the twist of the wires. This means that along the axis of the cable the current distribution will not vary.
We distinguish three regions. The two normal conducting regions and the superconducting layer. Maxwell's equations in all three regions read:

$$\begin{aligned} \partial_x B_y - \partial_y B_x &= \mu_0 j_z, \\ \partial_y B_z &= \mu_0 j_x, \\ -\partial_x B_z &= \mu_0 j_y, \end{aligned} \qquad (1)$$

$$\begin{aligned} \partial_x E_y - \partial_y E_x &= -\partial_t B_z, \\ \partial_y E_z &= -\partial_t B_x, \\ -\partial_x E_z &= -\partial_t B_y, \end{aligned} \qquad (2)$$

$$\partial_x B_x + \partial_y B_y = 0, \qquad (3)$$

$$\partial_x j_x + \partial_y j_y = 0. \qquad (4)$$

One can easily see that equations (3) and (4) are fulfilled by equations (1) and (2).
The constitutive equation in the normal conducting regions are given by Ohm's law:

$$\bar{j} = \bar{\bar{\sigma}}\, \bar{E}, \qquad (5)$$

with $\sigma_{ii} = \sigma$, $\sigma_{ij} = 0$ if $i \neq j$.

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This equation (5) enables us to eliminate the electric field components of equation (1) and (2), resulting in the Helmholtz equations:

$$\begin{aligned} \partial_x^2 B_x + \partial_y^2 B_x &= \mu_0 \sigma \partial_t B_x , \\ \partial_x^2 B_y + \partial_y^2 B_y &= \mu_0 \sigma \partial_t B_y , \\ \partial_x^2 B_z + \partial_y^2 B_z &= \mu_0 \sigma \partial_t B_z . \end{aligned} \tag{6}$$

The constitutive equation in the superconducting layer is given by:

$$\bar{j} = \bar{\bar{\sigma}}\, \bar{E} + j_s \bar{e}_s , \tag{7}$$

with $\sigma_s = \sigma(1-\lambda)$,

$\sigma_n = \sigma(1+\lambda)/(1-\lambda)$,

$\sigma_n' = \sigma_n$.

where the index s denotes the direction of the filaments, n, n' the two directions perpendicular to the filaments.

The conductivity tensors in the two parts of the superconducting layer read:

$$\bar{\bar{\sigma}} = \begin{pmatrix} \sigma_s \sin^2\alpha + \sigma_n' \cos^2\alpha & 0 & (\sigma_s - \sigma_n')\sin\alpha\cos\alpha \\ 0 & \sigma_n & 0 \\ (\sigma_s - \sigma_n')\sin\alpha\cos\alpha & 0 & \sigma_s \cos^2\alpha + \sigma_n' \sin^2\alpha \end{pmatrix}$$

for that part of the superconducting layer parallel to the x-direction,

$$\bar{\bar{\sigma}} = \begin{pmatrix} \sigma_n & 0 & 0 \\ 0 & \sigma_s \sin^2\alpha + \sigma_n' \cos^2\alpha & (\sigma_s - \sigma_n')\sin\alpha\cos\alpha \\ 0 & (\sigma_s - \sigma_n')\sin\alpha\cos\alpha & \sigma_s \cos^2\alpha + \sigma_n' \sin^2\alpha \end{pmatrix}$$

for that part of the superconducting layer parallel to the y-direction,

$\cos\alpha = L_p/\sqrt{(L_p^2 + P^2)}$, $\sin\alpha = P/\sqrt{(L_p^2 + P^2)}$.

L_p the twistpitch, P the perimeter of the layer as function of the position.

The superconducting current j_s is a function of the electric field along the direction of the filmants given by:

$$\begin{aligned} j_s &= j_c E_s/\epsilon \qquad \text{for } |E_s| < \epsilon \\ j_s &= j_c \,\mathrm{sign}(E_s) \qquad \text{for } |E_s| \geq \epsilon \end{aligned} \tag{8}$$

$\epsilon = \frac{8}{3\pi} |\dot{B}_n| R_f$, [6] (see figure 3),

$|\dot{B}_n|$ the time derivative of the magnetic field in the plane perpendicular to the direction of the filaments,

R_f the radius of the filaments,

j_c the critical current.

Because equation (7) is a nonlinear relation between $\bar{E}$ and $\bar{j}$ it is not possible to eliminate the electric field components as we did in the normal conducting regions. Furthermore, we have to compute the six components of the electromagnetic field in an iterative way. The boundary conditions for the six components read:

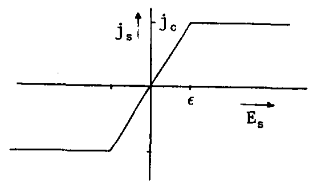


Fig. 3. The relation between the electric field component along the direction of the filaments E_s and the superconducting current j_s.

$$\begin{aligned} &\partial_x B_x = B_y = \partial_x B_z = 0, \\ &\partial_x E_x = E_y = \partial_x E_z = 0 \quad \text{at } x = 0,\ x = a, \end{aligned} \tag{9}$$

$$\begin{aligned} &\partial_y B_x = B_y = B_z = 0, \\ &\partial_y E_x = E_y = E_z = 0 \quad \text{at } y = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} &B_x \text{ and } B_y \text{ continuous}, \\ &B_z = \mu_0 I^A(t)/2 \qquad \text{at } y = b, \end{aligned} \tag{11}$$

with I^A the transverse current as function of time.

The boundary conditions at y = b imply that we have to solve the Helmholtz equations (6) for B_x and B_y outside the strand, where of course the conductivity σ is zero. The general solutions of B_x and B_y satisfying equations (6) *outside* the cable, with respect to the periodicity in the x-direction, and that B_x, B_y decreases to zero for large values of y, can be given by:

$$B_x(x,y) = \sum_{n=0}^{\infty} c_n \cos\left(\frac{n\pi x}{a}\right) e^{-\left(\frac{n\pi y}{a}\right)}, \tag{12}$$

$$B_y(x,y) = \sum_{n=0}^{\infty} - c_n \sin\left(\frac{n\pi x}{a}\right) e^{-\left(\frac{n\pi y}{a}\right)} \qquad 0 \leq x \leq a,\ y \geq b.$$

At y = b we may say that:

$$\int_0^a B_x(x,y)\cos\left(\frac{n\pi x}{a}\right)dx + \int_0^a B_y(x,y)\sin\left(\frac{n\pi x}{a}\right)dx = 0,$$

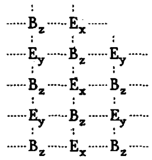
due to Fourier's theory.

Numerical model

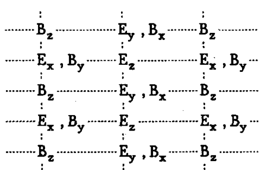
We will solve Maxwell's and the constitutive equations by a discretization of equations (1), (2) and (6). Therefore, we need a grid. Rem [7] introduced a staggered grid for the six components of the electric field based on equations (1) and (2). We may divide the six components into two sets namely:

1) E_x, E_y and B_z,
2) B_x, B_y and E_z.

The first equation of (2) and the last two equations of (1) state, that if we want the discretization matrix to be positive definite, we have to stagger E_x, E_y and E_z in the following way:



One can easily see that this staggering holds for the second set B_x, B_y and E_z. There are different ways to put those grids on top of each other, but due to our geometry, E_z has to be put between E_x and E_y:



The reason for this is that j_s is a function of E_s. If we put E_z on top of E_x for example, we cannot calculate j_s properly in the other part of the superconducting layer, where E_s is a linear combination of E_z and E_y.
In the normal conducting regions we will hold on the staggering, exept that we do not calculate the electric field (see figure 4). Our discretization in space is second order due to the staggering. For the iteration in time however, it is very important to have a good prediction because of the nonlinear relation between E_s and j_s. Therefore, a second order time discretization is required to establish the convergence. We have chosen for a implicit three point backward discretization, except for the first timestep where we have used the Crank-Nicolson scheme. We will not write down the discretized versions of equation (1), (2) and (6), because this is rather lengthy, and these can be found in every textbook on

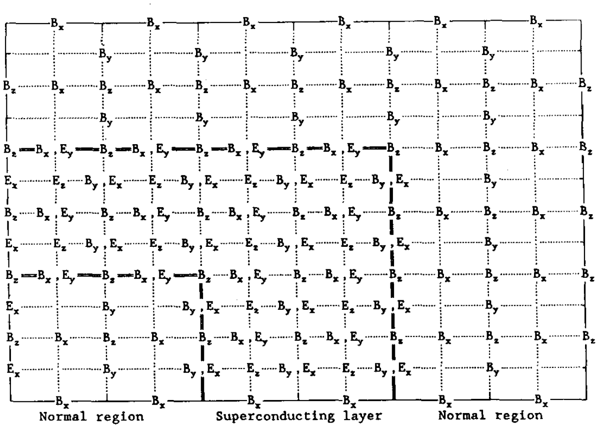


Fig. 4. The staggered grid used for the calculations.

numerical mathematics. A point of interest is the nonlinear relation between j_s and E_s. This is an implicit function of the local time derivitive of the induced magnetic field. ϵ determines the timestep. For very small values of $|\partial_t B_n|$ however, ϵ can be smaller than the greatest discretization error. This means that discretization errors can determine whether a part of the superconducting layer is saturated or not, resulting in bad convergence. To avoid this we have to make ϵ equal to the maximum of $8|\partial_t B_n|R_f/(3\pi)$ and ϵ', where ϵ' denotes the largest discretization error.

Results

We have calculated the current distribution as function of the applied field for two reasons. First, the distribution tells us whether the transport current saturates some filaments in the superconducting layer. If so, it reduces the maxmimal value of the longitudinal transport current of the cable. Secondly, once the distribution is known, we can easily calculate the transverse resistivity.
Figure 5 shows a typical distribution of the current for some timepoints, where the transverse transport current is linear increasing in time. We see that most of the current flows in the superconducting layer along the x-direction. As the current reaches the right side of the strand it will leave the filaments and flows into the normal zone.

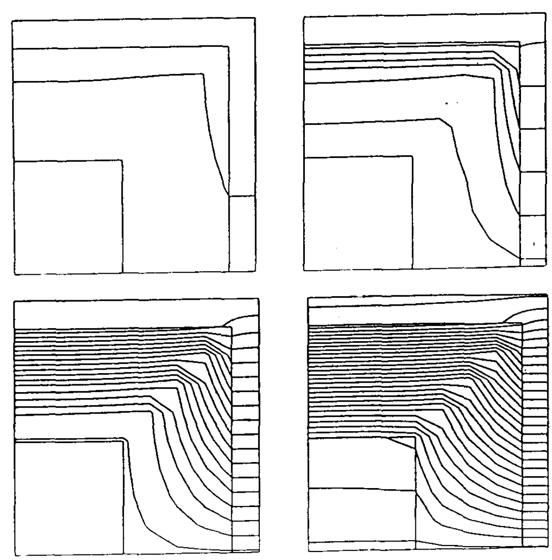


Fig. 5. Streamlines of the current in a quarter of the cross section of the strand for four timepoints, as $\partial_t I$ is constant.

Figure 6 shows the voltage-current for two values of $\partial_t I$. We can distinguish two regions. The first region, for small values of I, where the current will mainlyflow in the filaments. Secondly, for large values of I, where most of the filaments are saturated, so the current most flow through the matrix resulting in a greater slope and hence, a higher value of the transverse resistivity. The upper line corresponds with a greater value of $\partial_t I$.

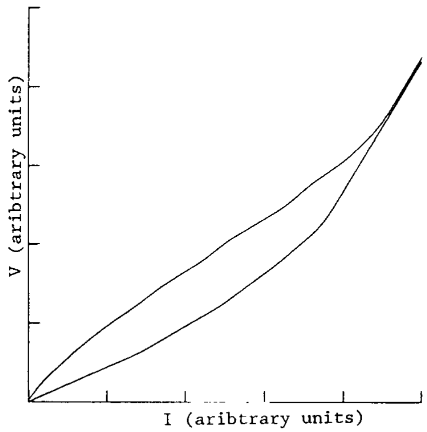


Fig. 6. The voltage-current relation for linearly increasing currents $\partial_t I$.

Figure 7 shows the hysteresis for $I(t) = \sin(\omega t)$. The value of the current where the two lines meet, is where most of the filaments are saturated. From that point the voltage-current relation has an ohmic nature, without any hysteresis.

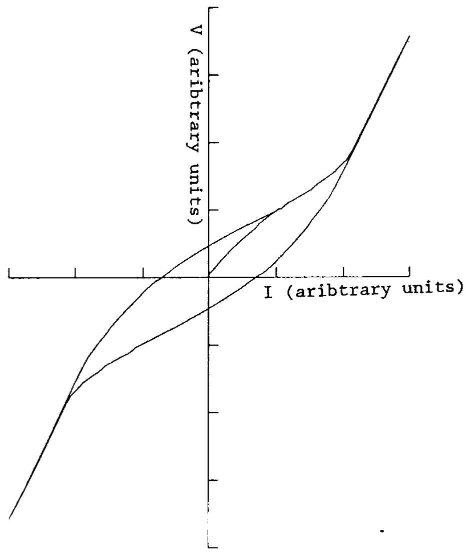


Fig. 7. The voltage-current relation, where the applied current is given by: $I(t)=\sin(\omega t)$.

Conclusions

The transverse transport current, which is induced by external fields acting on the cable, can saturate most of the filaments of the superconducting layer. This results in a smaller maximal value of a longitudinal transport current, and small coupling losses. In our model however, we did not, besides the induced currents, take any applied field into account. One can easily see, that in case of an external applied field in the axial direction, the remaining unsaturated filaments also may saturate.

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