# Transitions between Turbulent States in Rotating Rayleigh-Bénard Convection

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Weakly rotating turbulent Rayleigh-Bénard convection was studied experimentally and numerically. With increasing rotation and large enough Rayleigh number a supercritical bifurcation from a turbulent state with nearly rotation-independent heat transport to another with enhanced heat transfer is observed at a critical inverse Rossby number $1/\mathrm{Ro}_c \simeq 0.4$. The strength of the large-scale convection roll is either enhanced or essentially unmodified depending on parameters for $1/\mathrm{Ro} < 1/\mathrm{Ro}_c$, but the strength increasingly diminishes beyond $1/\mathrm{Ro}_c$ where it competes with Ekman vortices that cause vertical fluid transport and thus heat-transfer enhancement.

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Turbulence evolves either through a sequence of bifurcations, possibly passing through periodic and chaotic states [1] as in Rayleigh-Bénard (RB) convection [2] when the Rayleigh number Ra (to be defined below) is increased, or through subcritical bifurcations [3] as in pipe or Couette flow. Once the flow is turbulent, it usually is characterized by large random fluctuations in space and time and by a loss of temporal and spatial coherence. For the turbulent state common wisdom is that the large fluctuations ensure that the phase space is always fully explored by the dynamics, and that transitions between potentially different states that might be explored as a control parameter is changed are washed out.

Contrary to the above, we show that sharp transitions between distinct turbulent states can occur in RB convection [4] when the system is rotated about a vertical axis at an angular velocity $\Omega$. The angular velocity is given by the dimensionless inverse Rossby number $1/\mathrm{Ro} = 2\Omega/\sqrt{\beta g \Delta/L}$. Here $L$ is the height of a cylindrical sample, $\beta$ the thermal expansion coefficient, $\Delta$ the temperature difference between the bottom and top plate, and $g$ the gravitational acceleration. At relatively small Ra where the turbulence is not yet fully developed, we find that the system evolves smoothly as $1/\mathrm{Ro}$ is increased. However, when Ra is larger and the turbulent state of the nonrotating system is well established [5], we find that sharp transitions between different turbulent states occur, with different heat-transfer properties and different flow organizations. Sharp transitions between different states were reported also for turbulent flows in liquid sodium [6,7], where the increase of the magnetic Reynolds number beyond a certain threshold leads to bifurcations between different turbulent states of the magnetic field. Sharp transitions between turbulent states are found also in the rotating von Kármán experiment [8]. The transitions in RB convection are related to boundary-layer (BL) dynamics, whereas it is not known whether the transitions, e.g., in the dynamo experiment, are affected by boundaries. The influence of a possible transition between different states on the heat transport in RB convection is discussed in the context of a theoretical model in Ref. [9]. We note that in our case we have supercritical bifurcations, whereas all other cases are subcritical.

We present both experimental measurements and direct numerical simulations for a sample with diameter $D$ equal to $L$. They cover different but overlapping parameter ranges and thus complement each other. Where they overlap they agree very well. Without or with only weak rotation, it is known for this system that there are thermal BLs just below the top and above the bottom plate, with a temperature drop approximately equal to $\Delta/2$ across each. The bulk of the system contains vigorous fluctuations, and in the time average a large-scale circulation (LSC) that consists of a single convection roll with upflow and downflow opposite each other and near the sidewall.

The numerical scheme was already described in Refs. [10–13]. The apparatus also is well documented [13,14], and we give only a few relevant details. The sample cell had $D = L = 24.8$ cm, with Plexiglas sidewalls of thickness 0.32 cm and copper top and bottom plates kept at temperatures $T_t$ and $T_b$, respectively. The fluid was water. The Rayleigh number $\mathrm{Ra} \equiv \beta g \Delta L^3/(\kappa\nu)$ ($\nu$ and $\kappa$ are the kinematic viscosity and the thermal diffusivity, respectively), Prandtl number $\mathrm{Pr} \equiv \nu/\kappa$, and Ro were computed from the fluid properties at the mean temperature $T_m = (T_t + T_b)/2$. The Nusselt number $\mathrm{Nu} \equiv \lambda_{\mathrm{eff}}/\lambda$ was determined from the effective thermal conductivity $\lambda_{\mathrm{eff}} = QL/\Delta$ ($Q$ is the heat-current density) and the conductivity $\lambda(T_m)$ of the quiescent state. Eight thermistors, labeled $k = 0, \ldots, 7$, were imbedded in small holes drilled horizontally from the outside into but not penetrating the sidewall [15]. They were equally spaced around the circumference at the horizontal midplane ($z = 0$). A second and third set were located at $z = -L/4$ and $z = L/4$. Since the LSC carried warm (cold) fluid from the bottom (top) plate up (down) the sidewall, these thermistors detected the location of the upflow (downflow) of the LSC by indicating a relatively high (low) temperature. To determine the orientation and strength of the LSC, we fit the function

$$T_{f,k}(z = 0) = T_{w,0} + \delta_0 \cos(k\pi/4 - \theta_0), \qquad k = 0, \ldots, 7 \tag{1}$$

separately at each time step, to the eight temperature readings $T_k(z = 0)$ obtained from the thermistors at $z = 0$. Similarly we obtained $\theta_t$, $\delta_t$, and $T_{w,t}$ for the top level at $z = L/4$. At $z = -L/4$ only the mean temperature $T_{w,b}$ was used in the current work.

In Ref. [13] we explored Nu as a function of Ra, Pr, and Ro in a large parameter regime, ranging towards strong rotation ($1/\text{Ro} \gg 1$) and from small to large Pr. Here we focus on $\text{Pr} \approx 4$–7 (typical of water) and weak rotation ($\text{Ro} \gtrsim 1$) to study the transition from the nonrotating state at $1/\text{Ro} = 0$ towards the rotating case for different Ra.

We start with numerical results for the relatively small $\text{Ra} = 4 \times 10^7$ which is not accessible with the current experimental apparatus because $L$ is too large. Those simulations where done on a grid of $65 \times 193 \times 129$ nodes in the radial, azimuthal, and vertical directions, respectively, allowing for a sufficient resolution of the small scales both inside the bulk of turbulence and in the BLs adjacent to the bottom and top plates where the grid-point density was enhanced [11,12]. The small Ra allowed for very long runs of 4000 dimensionless time units and thus excellent statistics. Figure 1 shows the ratio of $\text{Nu}(\Omega)$ in the presence of rotation to $\text{Nu}(\Omega = 0)$ as function of $1/\text{Ro}$. This ratio increases rather smoothly with increasing rotation. For the larger $\text{Ra} = 2.73 \times 10^8$ and $\text{Pr} = 6.26$ where the turbulence of the nonrotating system is well developed, both numerical and experimental findings are very different. In Fig. 2 one sees that now there is a critical inverse Rossby number $1/\text{Ro}_c \approx 0.38$ at which the heat-transfer enhancement suddenly sets in. For weaker rotation the data are consistent with no heat-transfer modification as compared to the nonrotating case. The experimental and numerical data (now based on a resolution of $129 \times 257 \times 257$, see [13]) agree extremely well. In Ref. [12] data from direct numerical simulations were reported on the relative Nusselt number for $\text{Ra} = 1 \times 10^9$ and $\text{Pr} = 6.4$, which show a similar transition also at $1/\text{Ro}_c \approx 0.4$.

The increase in Nusselt is thought to be due to the formation of the Ekman vortices which align vertically and suck up (down) hot (cold) fluid from the lower (upper) BLs (Ekman pumping) [12,13,16–20]. This is supported by the change in character of the kinetic BL near the bottom

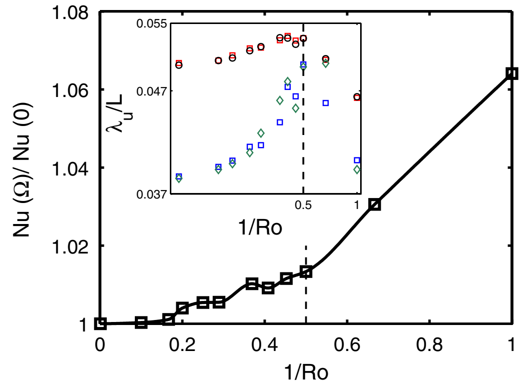


FIG. 1 (color online). The ratio $\text{Nu}(\Omega)/\text{Nu}(\Omega = 0)$ as a function of $1/\text{Ro}$ for $\text{Ra} = 4 \times 10^7$ and $\text{Pr} = 6.26$. Open black squares indicate the numerical results. The numerical error is approximately 0.2%, which is indicated by the size of the symbols. Inset: The thickness of the kinematic top and bottom BLs based on the maximum rms azimuthal [upper symbols: black circles (red squares) for top (bottom) BL] and radial [lower symbols: green diamonds (blue squares) for top (bottom) BL] velocities. The vertical dashed lines in both graphs represent $1/\text{Ro}_c$ and indicate the transition in boundary-layer character from Prandtl-Blasius (left) to Ekman (right) behavior.

and top walls based on the maximum root-mean-square (rms) velocities in the azimuthal (and radial) direction. For $1/\text{Ro} \lesssim 1/\text{Ro}_c$ the BL thickness (based on the rms azimuthal velocity) is roughly constant or even slightly increases. In contrast, for $1/\text{Ro} \gtrsim 1/\text{Ro}_c$ it behaves according to Ekman's theory and decreases with increasing rotation rate; see the inset in Fig. 1 ($1/\text{Ro}_c \approx 0.5$) and Fig. 2 ($1/\text{Ro}_c \approx 0.38$), and see data for $\text{Ra} = 1 \times 10^9$ and $\text{Pr} = 6.4$ in Ref. [21]. The scaling with rotation rate is in agreement with Ekman BL theory $\lambda_u/L \sim \text{Ro}^{1/2}$, whereas the constant BL thickness is consistent with the presence of the LSC and the Prandtl-Blasius BL. Furthermore, the numeri-

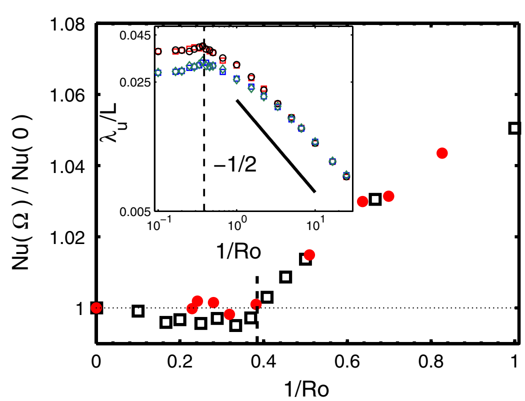


FIG. 2 (color online). $\text{Nu}(\Omega)/\text{Nu}(\Omega = 0)$ for $\text{Ra} = 2.73 \times 10^8$ and $\text{Pr} = 6.26$. Red solid circles: experimental data ($T_m = 24\,^\circ\text{C}$ and $\Delta = 1.00$ K). Open black squares: numerical results. The experimental error coincides approximately with the symbol size, and the numerical error is approximately 0.5%. Inset: Thickness of the kinetic BL. For dashed vertical lines and inset, see Fig. 1.

cal results [22] confirm the presence of Ekman vortices at the edge of the thermal BL above onset which are not present below onset. Furthermore, particle image velocimetry measurements have also shown that vortices are present close to the plates when rotation is applied [19,20]. In addition these measurements show that the number of vortices increases with the rotation rate.

To further characterize the flow field, we numerically calculated the rms velocities averaged over horizontal planes and over the entire volume, respectively. For $1/\mathrm{Ro} > 1/\mathrm{Ro}_c$ the normalized (by the value without rotation) volume-averaged vertical velocity fluctuations $w_{\mathrm{rms}}$ strongly decrease, indicating that the LSC becomes weaker (see Fig. 3). The decrease in normalized volume-averaged vertical velocity fluctuations coincides with a significant increase of the horizontal average at the edge of the thermal BLs, indicating enhanced Ekman transport (see also insets in Figs. 1 and 2). These averages provide additional support for the mechanism of the sudden transition seen in Nu and indicate an abrupt change from a LSC-dominated flow structure for $1/\mathrm{Ro} < 1/\mathrm{Ro}_c$ to a regime where Ekman pumping plays a progressively important role as $1/\mathrm{Ro}$ increases.

Our interpretation for the two regimes is as follows: Once the vertical vortices organize so that Ekman pumping sucks in the detaching plumes from the BLs, those plumes are no longer available to feed the LSC which consequently diminishes in intensity. A transition between the two regimes should occur once the buoyancy force, causing the LSC, and the Coriolis force, causing Ekman pumping, balance. The ratio of the respective velocity scales is the Rossby number. For $\mathrm{Ro} \gg 1$ the buoyancy-driven LSC is dominant, whereas for $\mathrm{Ro} \ll 1$ the Coriolis force and thus Ekman pumping is stronger. The transition between the two regimes should occur at $\mathrm{Ro} = \mathcal{O}(1)$, consistent with the observed $\mathrm{Ro}_c \approx 2.6$.

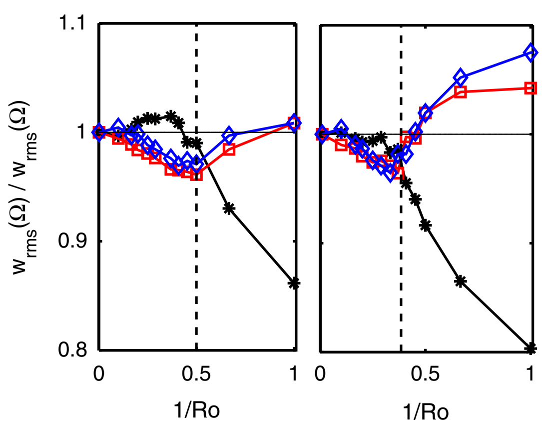


FIG. 3 (color online). The normalized averaged rms vertical velocities $w_{\mathrm{rms}}$ for $\mathrm{Ra} = 4 \times 10^7$ (left) and $\mathrm{Ra} = 2.73 \times 10^8$ (right) as a function of $1/\mathrm{Ro}$. The black line (with asterisks) indicates the normalized volume-averaged value of $w_{\mathrm{rms}}$. The red line (with squares) and blue line (with diamonds) indicate the normalized horizontally averaged $w_{\mathrm{rms}}$ at the edge of the thermal BL based on the slope at, respectively, the lower and upper plate. The vertical dashed lines again indicate the position of $1/\mathrm{Ro}_c$.

One wonders of course why the transition between the two regimes is sudden (in Nu) for $\mathrm{Ra} = 2.73 \times 10^8$ and less abrupt for the smaller $\mathrm{Ra} = 4 \times 10^7$ shown in Fig. 1. We do not know the answer. We speculate that below onset at the lower Ra the main effect is the thinning of the thermal BL through the rotation which is less pronounced at larger Ra, as there the BL is already thinner anyhow, thanks to the stronger LSC. The Nu vs Ra scaling at a fixed Ro number changes due to the transition, because $\mathrm{Nu}(\Omega)/\mathrm{Nu}(\Omega = 0)$ decreases with increasing Ra [13].

At even higher $\mathrm{Ra} = 9.0 \times 10^9$ (where $\Delta$ is larger and temperature amplitudes can thus more easily be measured) and $\mathrm{Pr} = 4.38$, an even more complex situation is revealed, as seen in Fig. 4 (here a direct numerical simulation is not available because it would be too time-consuming). We find that now $\mathrm{Nu}(\Omega)/\mathrm{Nu}(\Omega = 0)$ [Fig. 4(a)], after a slight

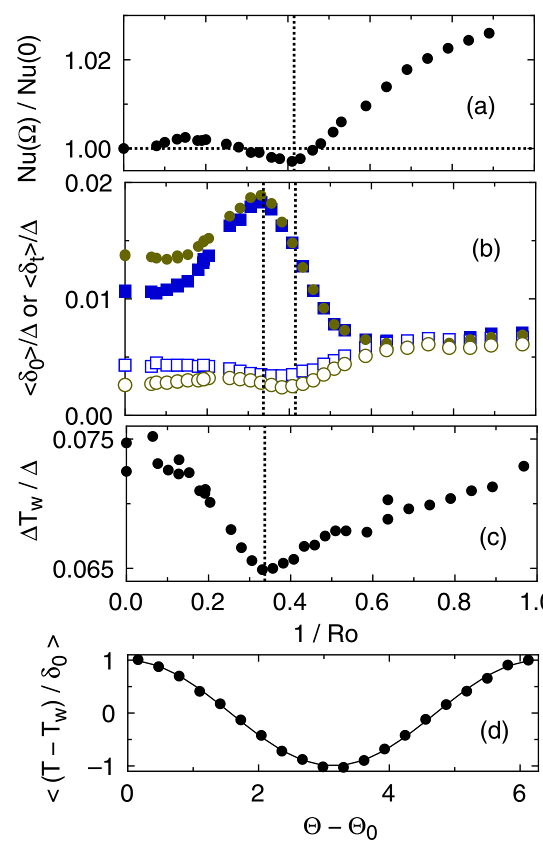


FIG. 4 (color online). Results for $\mathrm{Ra} = 9.0 \times 10^9$ and $\mathrm{Pr} = 4.38$ ($T_m = 40.00\,^\circ\mathrm{C}$, $\Delta = 16.00$ K). (a) $\mathrm{Nu}(\Omega)/\mathrm{Nu}(\Omega = 0)$ vs $1/\mathrm{Ro}$. The error bar is smaller than the size of the symbols. (b) Solid symbols: time-averaged LSC amplitudes $\langle\delta_0\rangle/\Delta$ ($z = 0$, circles) and $\langle\delta_t\rangle/\Delta$ ($z = L/4$, squares) as a function of $1/\mathrm{Ro}$. Open symbols: rms fluctuations about the cosine fit [Eq. (1)] to the temperature data. (c) Vertical temperature variation $\Delta T_w/\Delta$ along the sidewall. (d) Circles: time-averaged normalized sidewall-temperature profile $\langle[T(\theta) - T_w]/\delta_0\rangle$ at the horizontal midplane for $1/\mathrm{Ro} = 1$ determined as in [15]. Solid line: $\cos(\Theta - \Theta_0)$.

increase, first decreases, but these changes are only a small fraction of a percent. Then Nu undergoes a sharp transition at $1/\mathrm{Ro}_{c,2} = 0.415$ [vertical dotted line in Fig. 4(a)] and beyond it increases due to Ekman pumping. Comparison with Figs. 1 and 2 shows that the transition of Nu is not strictly at a constant $1/\mathrm{Ro}_c$, but that $\mathrm{Ro}_c$ depends weakly on Ra and/or Pr.

The LSC amplitudes $\delta_0$ and $\delta_t$ determined from fits of Eq. (1) to the sidewall-thermometer readings are shown in Fig. 4(b) as solid symbols. Consistent with the results reported in Ref. [15], $\delta_t < \delta_0$ when there is no rotation ($1/\mathrm{Ro} = 0$). This inequality disappears as $1/\mathrm{Ro}$ increases. Both amplitudes first increase by nearly a factor of 2. At $1/\mathrm{Ro}_{c,1} \simeq 0.337$, where the two amplitudes have just become equal to each other, they begin to decrease quite suddenly and remain equal to each other up to the largest $1/\mathrm{Ro}$. The transition at $1/\mathrm{Ro}_{c,1}$ is indicated by the leftmost vertical dotted line in Figs. 4(b) and 4(c). At that point there also is a transition revealed by the vertical temperature difference $\Delta T_w = 2 \times [T_{w,b} - T_{w,t}]$ along the sidewall as seen in Fig. 4(c) which shows $\Delta T_w/\Delta$ as a function of $1/\mathrm{Ro}$. Consistent with the initially enhanced LSC amplitudes $\delta_0$ and $\delta_t$, these results first show a reduction of the thermal gradient as the LSC becomes more vigorous, but then reveal an increase due to enhanced plume and/or vortex activity above $1/\mathrm{Ro}_{c,1}$.

Also of interest are the rms fluctuations $\delta T/\Delta = \langle [T_k(z=0) - T_{f,k}(z=0)]^2 \rangle^{1/2}/\Delta$ about the fit of Eq. (1) to the temperature measurements at the horizontal midplane ($z = 0$), and similarly at $z = L/4$. They are shown as open symbols in Fig. 4(b). These fluctuations begin to rise at $1/\mathrm{Ro}_{c,2}$ rather than at $1/\mathrm{Ro}_{c,1}$. Then they soon become comparable to $\delta_0$ and $\delta_t$, suggesting that the LSC becomes more and more hidden in a fluctuating environment. Nonetheless, remnants of the LSC survive and can be found when the fluctuations are averaged away, as shown in Fig. 4(d). There we see that even for $1/\mathrm{Ro} = 1.0$ the time average $\langle [T_k(z=0) - T_{w,0}]/\delta_0 \rangle$ of the deviation from the mean temperature $T_{w,0}$ retains a near-perfect cosine shape.

From these measurements we infer that the establishment of the Ekman-pumping mechanism is a three-stage process. First, up to $1/\mathrm{Ro}_{c,1}$, the time-averaged LSC amplitudes, such as $\langle \delta_0 \rangle/\Delta$, nearly double in value [see Fig. 4(b)] and thereby reduce the vertical thermal gradient along the wall [see Fig. 4(c)]. Beyond $1/\mathrm{Ro}_{c,1}$ there is an enhanced accumulation of plumes and vortices, which coincides with an increase of the BL thickness near onset as shown by the simulations at lower Ra (see insets in Figs. 1 and 2). This accumulation detracts from the driving of the LSC, but the flow is not yet organized into effective Ekman vortices. This organization sets in at $1/\mathrm{Ro}_{c,2}$, leads to Ekman pumping, and enhances Nu and reduces the strength of the LSC as supported by the volume average of $w_{\mathrm{rms}}$ [see Fig. 3 (for lower Ra)]. This sequence of events is altered as Ra (and presumably also Pr) is changed, but it is remarkable that for fully developed turbulent RB convection sharp supercritical bifurcations occur.

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