

## DYNAMICS AND CONTROL OF A GAS-FIRED FURNACE

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**Abstract**—A non-linear model has been developed for a gas-fired furnace in which oil is heated. The model is applicable from minimum to maximum heat load of the furnace. The dynamics of the model have been compared to experimental results, which were obtained for a pilot-scale furnace. They are in good agreement. A cascade control with feed-forward action has been compared to single feed-back control. Proportional feed-forward action already gives much better results than the latter control.

An approximative expression is derived for the large time constant in the furnace dynamics.

### 1. INTRODUCTION

Development and application of mathematical models of processes for the design of apparatus and analysis of the dynamics and control of processes, is commonly used.

For boilers many models have already been developed [1, 2]; but only little work has been done on the dynamics and modelling of industrial furnaces. One of the reasons could be that there are many different types of furnaces, designed for the various purposes they are used for.

Dynamics and control of blast furnaces have been studied extensively [3, 4]. Also, much work has been done on the dynamics and control of room-heating systems, in which a gas-fired furnace is used [5-7].

In the oil and chemical industries special types of furnaces are used. One of these types is the furnace for thermal cracking or pyrolysis of hydrocarbons. The several types of cracking processes, and the application of different types of furnaces for these processes, are described by Davenport [8]. Another type is used for the partial evaporation of a liquid. In a third type of furnace only a fluid is heated.

We shall restrict ourselves to the last type. Work on the dynamics of a furnace in which oil is heated, has been done by Endtz *et al.* [9], while a simulation model of this type of furnace is given by Ryan [10]. However, this model is only applicable in a small region around the operating point, because the heat transfer coefficients are assumed to be constant. A second disadvantage of this model is the assumption that all surfaces have the emission- and absorption characteristics of black bodies, which is not very realistic. Furthermore the tube and oil are lumped in one section, which is a very rough approximation of

reality. The model described in this article is developed for the actual furnace and is applicable from minimum to maximum heat load of the furnace; the dependence of heat transfer coefficients on temperature has been included and all surfaces have grey-body absorption characteristics. The model is compared with experiments on this actual furnace, since only little experimental data are available in the literature. Before going into detail about the model a description of the experimental set-up will be given.

### 2. DESCRIPTION OF THE FURNACE

As can be seen from Fig. 1, the furnace is built up in two sections: a radiation- and a convection section. The dimensions of the radiation section are:

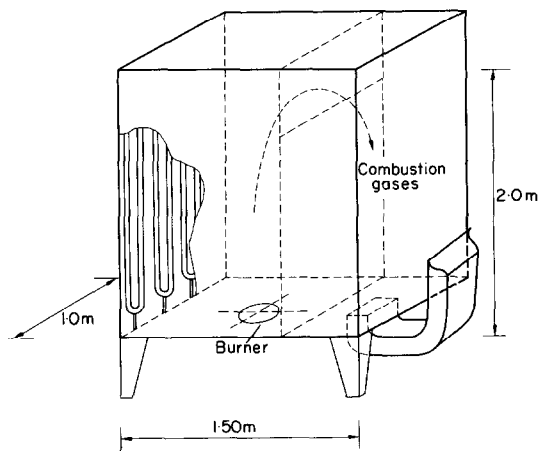


Fig. 1. Diagram of the furnace.

1.0 × 1.25 × 2.0 m; the dimensions of the convection section:  
1.0 × 0.25 × 2.0 m.

In both sections of the furnace a tube circuit is mounted vertically along the four walls; the total length of the tubes is about 150 m.

The walls of the furnace consist of fire-resistant brick stone and steel, between which an isolating material is mounted.

In the centre of the radiation section a burner is mounted in the bottom, to which gas and air are supplied under pressure. The maximum burner capacity is about 280 kJ/sec.

In Fig. 2 a diagram of the instrumentation is given. In the air supply to the burner a control valve is mounted. The gas supply is adjusted by the air supply pressure, because the air pressure adjusts the opening of a ratio control valve, which is mounted in the gas circuit. Tube skin temperatures and oil temperatures can be measured by thermocouples in different locations in the furnace.

3. ASSUMPTIONS MADE

In developing the model some assumptions and simplifications have been made:

1. The walls of the furnace can be lumped in three sections, corresponding to layers arranged from inside to outside, each having its own average temperature. The number of sections has been determined experimentally.
2. The tubes and the oil therein can be lumped in five sections, each having its own average temperature.
3. From steady-state heat calculations and meas-

urements it was found that there is a deposit inside the tubes, originating from the oil. We assume that this deposit has the same thickness and the same thermal properties over the total length of the tube.

4. The radiation in the convection section is ignored compared to the convection.
5. The hot gases are perfectly mixed, the average temperature in the radiation section is  $T_{gr}$ ; in the convection section  $T_{gc}$ . This assumption holds for small gas-fired furnaces; however, not for large oil-fired furnaces as can be seen from reference [11].

4. DESCRIPTION OF THE HEAT TRANSFER IN THE FURNACE

The situation in the furnace is schematically given in Fig. 3.

Heat is transferred from the hot gases to the furnace walls by radiation and convection:

$$Q_{gw} = A_w (\alpha_{gw} \epsilon_w + \alpha_c) (T_g - T_w) \quad (1)$$

in which:

- $Q_{gw}$  is the heat transferred from gas to wall
- $A_w$  is the total area of the wall inside the furnace
- $\alpha_{gw}$  is the heat transfer coefficient for radiation
- $\epsilon_w$  is the absorption coefficient of the wall
- $\alpha_c$  is the heat transfer coefficient for convection
- $T$  is the temperature

The amount of heat  $Q_{gw}$  is used for radiation to the tubes and for heat losses through the furnace walls.

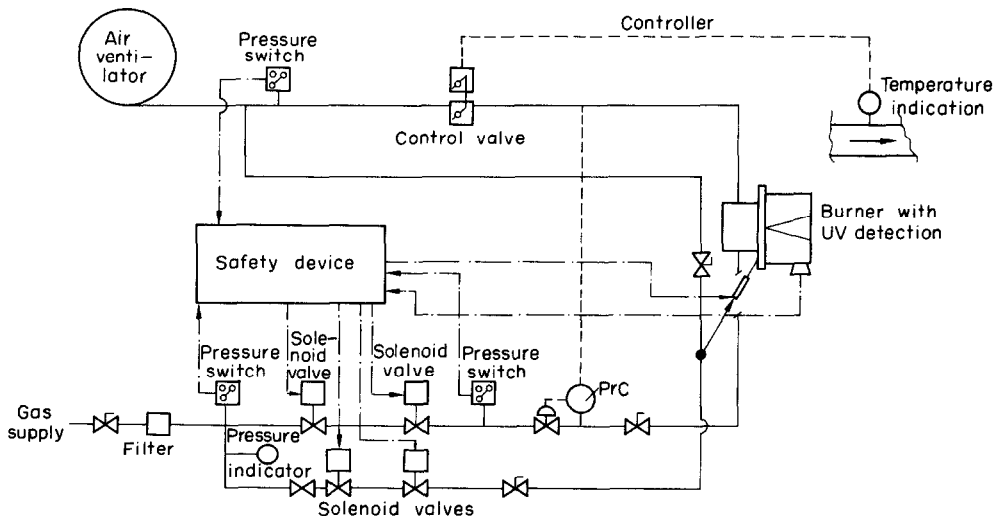


Fig. 2. Diagram of the instrumentation.

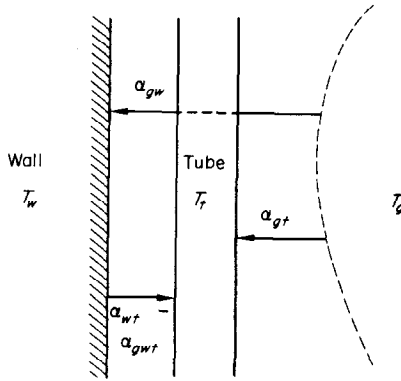


Fig. 3. Radiation heat transfer in the furnace.

$$Q_{gw} = Q_{wt} + Q_{loss} \tag{2}$$

For the estimation of heat exchange by radiation between furnace walls and tubes, we must know the emission coefficient  $\epsilon_{wt}$  which is given by [12]:

$$\epsilon_{wt} = \frac{1}{\frac{1}{\epsilon_t} + \frac{A_t}{A_w} \left( \frac{1}{\epsilon_w} - 1 \right)} \tag{3}$$

in which

- $\epsilon_t$  is the absorption coefficient of the tubes
- $A_t$  is the heat absorbing area of the tubes
- $A_w$  is the wall area emitting heat to the tubes

If the radiation from the walls is directly absorbed by the tubes, then

$$Q_{wt} = \alpha_{wt} \epsilon_{wt} A_t (T_w - T_t) \tag{4}$$

But a certain fraction of this heat is absorbed by the hot gases, and just that amount which would be radiated from the gas to the tubes, if the gas had the temperature of the furnace walls.

This amount of absorbed heat is:

$$Q_{abs} = \alpha_{gwt} \epsilon_w A_t (T_w - T_t) \tag{5}$$

in which  $\alpha_{gwt}$  is the heat transfer coefficient for radiation between a gas temperature, equal to the temperature of the furnace walls, and the temperature of the tubes.

Therefore the real amount of heat which is transferred from the walls to the tubes is given by:

$$Q_{wt} = A_t (\alpha_{wt} \epsilon_{wt} - \alpha_{gwt} \epsilon_w) (T_w - T_t) \tag{6}$$

The heat transfer by radiation and convection

between the combustion gases and the tubes is given by:

$$Q_{gt} = A_t (\alpha_{gt} \epsilon_t + \alpha_c) (T_g - T_t) \tag{7}$$

From Eqs. (1)–(5) we can see that for the total heat transfer five heat transfer coefficients are important:  $\alpha_{gt}$ ,  $\alpha_{gw}$ ,  $\alpha_{gwt}$ ,  $\alpha_{wt}$  and  $\alpha_c$ .

The heat transfer coefficients for gas radiation  $\alpha_{gt}$ ,  $\alpha_{gw}$  and  $\alpha_{gwt}$  are dependent on the nature of the gas, the excess of air, the temperature of the gases, the dimensions of the furnace, etc.

The gas radiation is dependent on the dimensions of the furnace, the temperature difference and the partial pressure of the radiating compounds  $CO_2$  and  $H_2O$ .

The partial pressure of carbon dioxide is equal to 0.09 bara; of water equal to 0.175 bara.

The amount of heat which is transferred by radiation from a flame with temperature  $T_g$  to a black body with a temperature of 0°K can, according to reference [13], be given by:

$$q_{H_2O} = 3.57 \times 10^{-3} \left( \frac{T_g}{100} \right)^{3.4} \text{ kJ/m}^2 \text{ sec} \tag{8}$$

$$q_{CO_2} = 5.88 \times 10^{-4} \left( \frac{T_g}{100} \right)^{4.0} \text{ kJ/m}^2 \text{ sec} \tag{9}$$

with  $T_g$  in °K.

Within the temperature range from 600–1300°K these equations fit the equations proposed by Bosse [14] for respectively  $CO_2$  and  $H_2O$  radiation within 10 per cent.

The amount of heat which is transferred by radiation from the flame to the tubes is therefore given by:

$$q_{H_2O,gt} = 3.57 \times 10^{-3} \left[ \left( \frac{T_g}{100} \right)^{3.4} - \left( \frac{T_t}{100} \right)^{3.4} \right] \text{ kJ/m}^2 \text{ sec} \tag{10}$$

$$q_{CO_2,gt} = 5.88 \times 10^{-4} \left[ \left( \frac{T_g}{100} \right)^{4.0} - \left( \frac{T_t}{100} \right)^{4.0} \right] \text{ kJ/m}^2 \text{ sec.} \tag{11}$$

Now

$$\alpha_{CO_2,gt} = \frac{q_{CO_2,gt}}{T_g - T_t} \tag{12}$$

and

$$\alpha_{H_2O,gt} = \frac{q_{H_2O,gt}}{T_g - T_t} \tag{13}$$

So

$$\alpha_{gt} = \alpha_{CO_2,gt} + \alpha_{H_2O,gt} \tag{14}$$

In the same way  $\alpha_{gw}$  and  $\alpha_{gwt}$  can be determined.

The heat transfer coefficient for radiation between the furnace walls and the tubes  $\alpha_{wt}$  only depends on the temperature of walls and tubes and can therefore be calculated by the Stefan-Boltzmann law:

$$\alpha_{wt} = 5.78 \times 10^{-3} \frac{\left(\frac{T_w}{100}\right)^4 - \left(\frac{T_t}{100}\right)^4}{T_w - T_t} \text{ kJ/m}^2 \text{ sec } ^\circ\text{K} \quad (15)$$

with  $T$  in  $^\circ\text{K}$ .

The value of  $\alpha_c$  depends on the gas velocity and is given by [9]:

$$\alpha_c = 6.3 \times 10^{-3} + 4.2 \times 10^{-3} v_{\text{gas}} \text{ kJ/m}^2 \text{ sec } ^\circ\text{K} \quad (16)$$

if  $v_{\text{gas}}$  is less than 5 m/sec.

For our furnace the value of  $\alpha_c$  varied between 0.0105 and 0.028 kJ/m<sup>2</sup> sec  $^\circ\text{K}$ .

5. DESCRIPTION OF THE DYNAMIC EQUATIONS

As already mentioned in section 3 the tubes with oil are divided into five sections. In some simulation runs this number has been chosen differently, but with five sections the calculation of the dynamics was sufficiently accurate (error less than 1 per cent in the dynamic variation).

A diagram of one section is given in Fig. 4.

The non-stationary energy balance for a section  $\Delta x$  of the tube is given by:

$$M_t c_{p,t} \Delta x \frac{d\bar{T}_t}{dt} = Q_{\text{to tube}} \Delta x - \alpha_0 A_i \Delta x (\bar{T}_t - \bar{T}_0) \quad (17)$$

in which  $Q_{\text{to tube}}$  is the sum of the heat transfer by

radiation and convection from gas and furnace walls to the tubes and  $\Delta x$  is the fractional distance.

The non-stationary energy balance for the oil to be heated is:

$$M_o c_{p,o} \Delta x \frac{d\bar{T}_o}{dt} = \alpha_0 A_i \Delta x (\bar{T}_t - \bar{T}_0) - \phi_{m,o} c_{p,o} (T_{0,x+\Delta x} - T_{0,x}) \quad (18)$$

The heat transfer coefficient inside the tube  $\alpha_0$  can be calculated from a  $Nu-Re-Pr$  relationship. Only at the entrance of the furnace the oil flow is laminar, but as temperature increases, the flow soon becomes turbulent. The total resistance for heat transfer from tube to oil is equal to the sum of the resistances in the oil film and oil deposit. The physical properties of the oil and walls of the furnace were taken as a function of temperature; they are given in Table 1. The other dynamic equations in the model are energy balances for the sections of the wall and an energy balance for the hot combustion gases.

The non-stationary energy balance for one section of the wall is:

$$M_w c_{p,w} \frac{dT_w}{dt} = Q_{g,w} - Q_{\text{loss}} - Q_{w,t} \quad (19)$$

The non-stationary energy balance for the hot gases is:

$$M_g c_{p,g} \frac{dT_g}{dt} = \phi_{\text{gas}} \Delta H - Q_{g,w} - Q_{g,t} - Q_{\text{out}} \quad (20)$$

in which  $Q_{\text{out}}$  is the heat transported by the gases which leave the furnace.

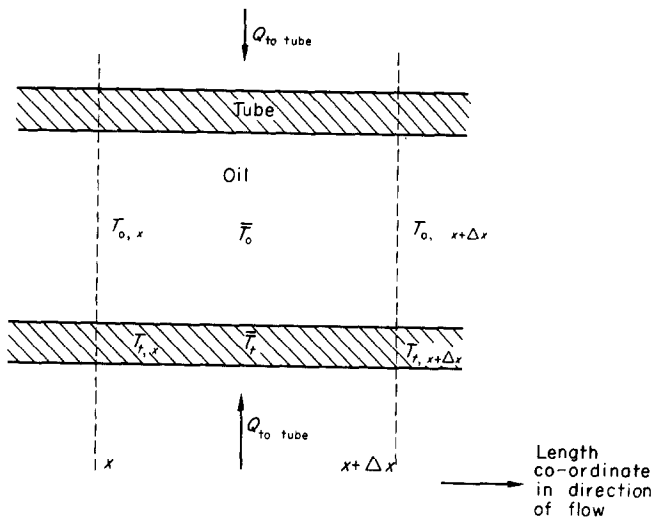


Fig. 4. Diagram of a tube section.

Table 1. Physical properties of oil and wall materials as a function of temperature

|           | $\lambda$<br>(J/m sec °K)       | $\rho$<br>(kg/m <sup>3</sup> ) | $c_p$<br>(kJ/Kg °C)          | $\nu$<br>(m <sup>2</sup> /sec) |
|-----------|---------------------------------|--------------------------------|------------------------------|--------------------------------|
| Oil       | $0.152-7 \times 10^{-5} T$      | $1049-0.62 T$                  | $2.895-4.0 \times 10^{-3} T$ | $5 \times 10^5 T^{-4.30}$      |
| Stone     | $0.750 + 5.8 \times 10^{-4} T$  | 2200                           | 1.176                        | —                              |
| Isolation | $-0.073 + 3.3 \times 10^{-3} T$ | 40                             | 0.840                        | —                              |
| Steel     | 58.3                            | 7800                           | 0.504                        | —                              |

In these equations  $T$  is the temperature expressed in °K.

6. EXPERIMENTAL WORK AND SIMULATION STUDY

As shown in Fig. 5, three input variables are important for the control of the furnace: the flow rate of gas and air, the flow rate of the oil and the inlet temperature of the oil. In many industrial furnaces the outlet temperature of the oil is controlled by the heat input to the furnace, the other inputs being sources of disturbances.

The Bode amplitude- and phase plots from gas supply to outlet temperature are derived from experiments. The developed model was simulated in CSMP on an IBM 360 computer and Bode plots are derived for the model too.

It is also possible to derive an analytical expression for the transfer function from gas supply to outlet temperature. Since the heat capacity of the walls is much larger than the heat capacity of the gases, we assume that the variation in heat flow from wall to tubes may be ignored compared to the variation in heat flow from gas to tubes.

Let us consider only small variations, hence we may assume the heat transfer coefficients being constant. Then transformation of Eqs. (17) and (18) into partial differential equations, followed by linearisation, and taking the Laplace transform with respect to time, results in:

$$\frac{T_{oil}}{T_{gas}} = \frac{1.075(1 - 0.627e^{-179s})}{(21.4s + 1)(397s + 1)} \quad (21)$$

where the time constants are given in sec.

The method of derivation of this equation is indicated by Harriott[15], while in Appendix 1 a modified application to our furnace is given.

The results of experiments, simulation and

calculation are given in Figs. 6 and 7. The Bode plots, derived from the experiments, were corrected for the time constant of the thermocouple. From the corrected plots two important time constants were found: a time constant of 18 sec, and a large time constant of 96 sec.

The time for the oil to run through the furnace is equal to 114 sec, hence the large time constant has the same order of magnitude as the residence time of the oil in the furnace. This is also found by Endtz *et al.* [9]. In Appendix 2 a derivation of an approximate expression for the estimation of the large time constant, that is found in heat exchanger and furnace analysis, is given. For a large furnace, in

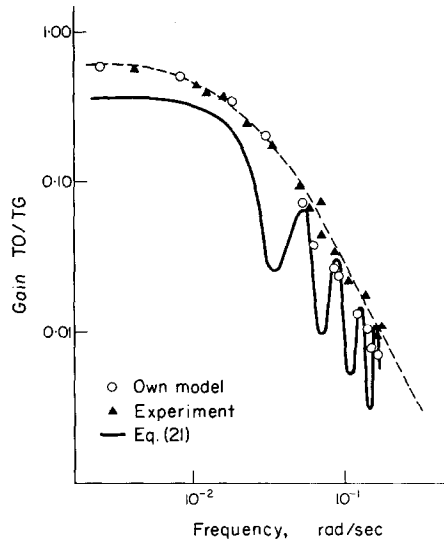


Fig. 6. Bode amplitude diagram from gas to oil.

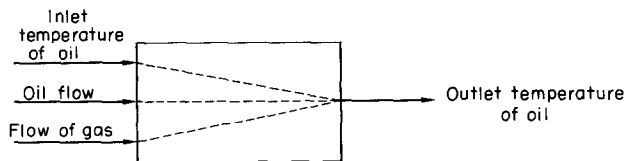


Fig. 5. Information diagram of the furnace.

Table 2. List of numerical parameter values for the simulation at a total heat input in the furnace of 203 kJ/sec

|   |                                  |
|---|----------------------------------|
| A. Values of the total area in the furnace (m <sup>2</sup> ):   |                                  |
| Convection area of tubes in radiation section   | 7.7                              |
| Radiation area of tubes in radiation section  | 5.0                              |
| Total wall area in radiation section  | 10.0                             |
| Wall area for radiation in radiation section  | 8.0                              |
| Total wall area in convection section   | 3.3                              |
| Convection area of tubes in convection area   | 2.2                              |
| B. Tube diameter, inside/outside  |                                  |
|   | 17/20 mm                         |
| C. Average values of the temperatures for the fourth section (°K)                                     |                                  |
| Gas temperature in radiation section  | 845                              |
| Oil temperature   | 464                              |
| Tube temperature  | 559                              |
| Stone temperature   | 721                              |
| D. Average values for the heat transfer coefficients in the fourth section (kJ/m <sup>2</sup> sec °K) |                                  |
| $\alpha_{gw}$   | 0.0322                           |
| $\alpha_{gwt}$  | 0.0217                           |
| $\alpha_{wt}$   | 0.0637                           |
| $\alpha_{gt}$   | 0.0259                           |
| $\alpha_{oil}$  | 0.847                            |
| $\alpha_c$  | 0.014                            |
| E. Average values of the absorption coefficients  |                                  |
| $\epsilon_{wt}$   | 0.777                            |
| $\epsilon_w$  | 0.820                            |
| $\epsilon_t$  | 0.850                            |
| F. Numerical values for Eq. (34)  |                                  |
| $M_t$   | 81.6 (kg)                        |
| $c_t$   | 0.504 (kJ/kg °K)                 |
| $A_t$   | 6.40 (m <sup>2</sup> )           |
| $\alpha'_0$   | 0.147 (kJ/m <sup>2</sup> sec °K) |
| G. Parameter values for Eq. (57)  |                                  |
| $\gamma$  | 1.075                            |
| $T_2$   | 175 (sec)                        |
| $T_1$   | 48.8 (sec)                       |
| $T_{12}$  | 43.7 (sec)                       |
| $\tau_{res}$  | 114 (sec)                        |
| $\bar{T}_g$   | 814 (°K)                         |
| $T_{o,out}$   | 450 (°K)                         |
| $T_{o,in}$  | 293 (°K)                         |

which the heat capacity of the tubes is small compared to the oil capacity, the large time constant can be expressed as:

$$\tau = \tau_{res} \cdot \frac{(\gamma - 1)\bar{T}_g - (T_{o,out} - \bar{T}_0)}{(T_{o,out} - T_{o,in})} \quad (22)$$

in which  $\tau_{res}$  is the time for the oil to flow through the tubes,  $\bar{T}_g$  is the average temperature of the combustion gases,  $\bar{T}_0$  is the log-mean temperature of the oil and  $\gamma$  is a constant, dependent on the radiation and convection in the furnace. For our particular furnace  $\gamma$  is equal to 1.075 and the large time constant is found to be smaller than the oil

residence time. In general for a large furnace  $\gamma \gg 1$ , and the large time constant will be larger than the residence time. From Eq. (22) it can be seen that the large time constant that will be found in the dynamic analysis, is not *a priori* equal to the residence time of the oil in the tubes but also depends on furnace geometry and temperatures in the furnace. From Fig. 6 we see that for low frequencies the value of  $\delta T_o / \delta T_g$  is higher for the points, calculated from our model, than for the points, calculated from Eq. (21). This effect is due to the simplification that has been made in the derivation of Eq. (21), in which the wall temperature was supposed to be constant.

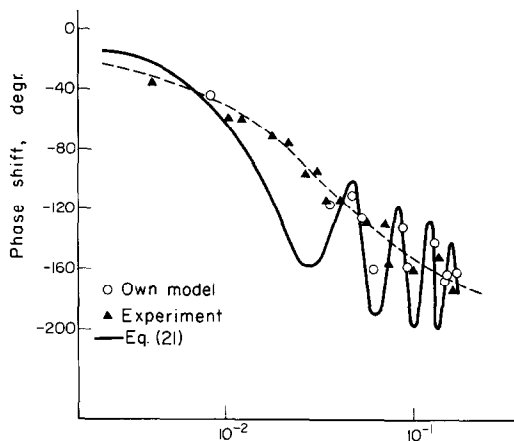


Fig. 7. Bode phase diagram from gas to oil.

In Fig. 8 the temperature of the product at the furnace outlet is plotted vs time, after a step-disturbance in the burner-pressure. Since the walls of the furnace have a large heat capacity, it takes a long time to reach a new steady-state. For the run plotted in Fig. 8 this time was approximately equal to 1080 sec.

In Fig. 9 the ratio of heat transfer by radiation and convection from the gas to the tubes is given as a function of the gas temperature. From this figure it can be seen that the model is non-linear, as also can be seen from section 4. For a higher temperature, radiation increases more than convection, hence the ratio is increasing.

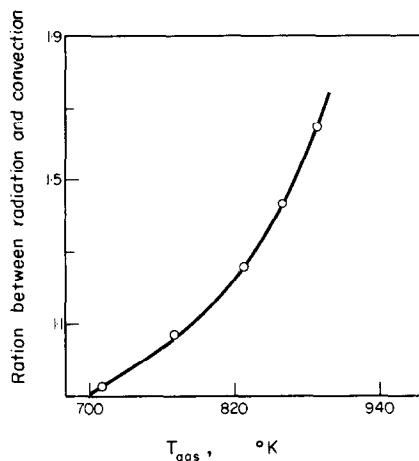


Fig. 9. Relation between the ratio of radiation and convection from gas to tubes and the gas temperature.

7. CONTROL STUDY

After the dynamics had been studied, different possibilities for control were checked. The master-slave control loop with feed-forward action, as shown in Fig. 10, was compared to a normal PID control loop, in which the outlet temperature of the

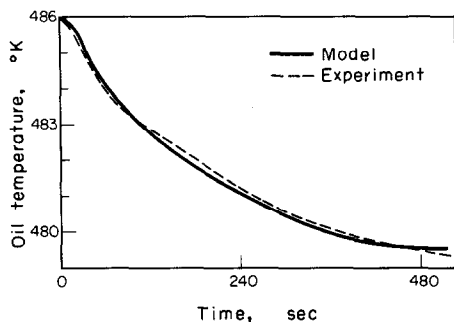


Fig. 8. Dynamic response of the outlet temperature after a step-disturbance in the burner pressure of 10 per cent.

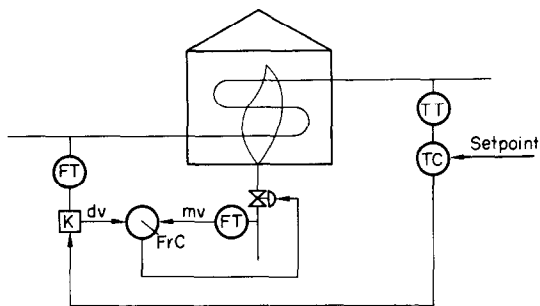


Fig. 10. Cascade control of the furnace with anticipatory action.

oil is measured and the gas supply to the furnace is controlled.

In the control scheme of Fig. 10, the outlet temperature is also PID-controlled, but the output of the controller determines the ratio setting of a flow-ratio controller between oil and gas. The control mode of the ratio controller is PI. The gain and integration time were determined experimentally.

If there is any difference in the transfer function from gas temperature to outlet temperature and oil flow to outlet temperature, ideal feed-forward action should contain a compensation for this difference. The transfer function from oil flow to oil temperature was therefore determined and could be approximated by a second order transfer function with time constants of 22 and 53 sec.

In order not to complicate the control scheme too much, at first control was checked without compensation for the difference in transfer function and this

result was so excellent that there was no more need for compensation.

In Figs. 11 and 12 some experimental runs are given.

In Fig. 11 the response of the outlet temperature is shown after a disturbance in the gas supply of 50 per cent. The maximum deviation with the single control loop is 8°K, whereas for the same case the maximum deviation for the master-slave controller is only 0.2°K. This result is only due to flow-ratio control.

The effect of feed forward action is shown in Fig. 12, where the response of the outlet temperature is given after a disturbance in the oil flow equal to 10 per cent. The maximum deviation for single loop control was 9°K; for cascade control with feed-forward action 1°K. Hence we see that the latter type of control action gives better results than single loop control.

### CONCLUSIONS

Although a relatively simple model was made, the agreement between experiments and model was rather good. The simplification of the detailed model

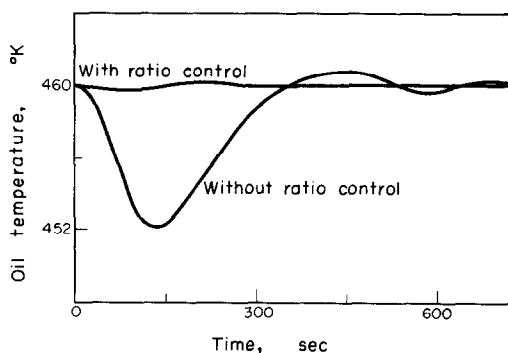


Fig. 11. Response of oil temperature at the furnace outlet after a step-disturbance in the burner pressure of -50 per cent.

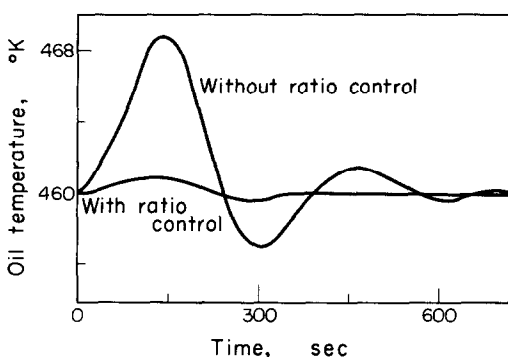


Fig. 12. Response of oil temperature at the furnace outlet after a step-disturbance in the oil flow of -10 per cent.

to a second order or second order plus delay time is satisfactory. For a large furnace the large time constant in the responses of the outlet temperature can be approximated by:

$$\tau = \tau_{res} \cdot \frac{(\gamma - 1)\bar{T}_g - (T_{0,out} - \bar{T}_0)}{(T_{0,out} - T_{0,in})}$$

while the tube wall capacity contributes the second time constant.

For the control of the furnace master-slave control with feedforward action from the oil flow gave much better results than single loop control of the outlet temperature by fuel supply.

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### NOTATION

- $A$  area,  $m^2$
- $D$  diameter,  $m$
- $\Delta H$  heat of combustion,  $kJ/kg$
- $M$  mass per unit length,  $kg/m$
- $Q$  heat flow,  $kJ/sec$
- $T$  temperature,  $^{\circ}K$
- $V$  volume,  $m^3$
- $c_p$  specific heat,  $kJ/kg^{\circ}K$
- $q$  heat flow density,  $kJ/m^2 sec$
- $t$  time,  $sec$
- $\Delta x$  fractional length,  $m/m$
- $\alpha$  heat transfer coefficient,  $kJ/m^2 sec^{\circ}K$
- $\gamma$  constant
- $\epsilon$  emission- or absorption coefficient
- $\rho$  density,  $kg/m^3$
- $\lambda$  heat conductivity,  $J/m sec^{\circ}K$
- $\nu$  kinematic viscosity,  $m^2/sec$
- $\phi_m$  mass flow,  $kg/sec$

### Indices

- $c$  convection
- $g$  gas
- $i$  inside
- $gwt$  gas at  $T_w$  to tubes
- $gw$  from gas to wall
- $gt$  from gas to tube
- $ot$  outside
- $o$  oil
- $t$  tube
- $w$  wall
- $wt$  from wall to tube

### Defined ratios's

- $T_1 = M_o c_{p,o} / \alpha_o A_i$
- $T_2 = M_i c_{p,i} / \alpha_g A_{ot}$
- $T_{12} = M_i c_{p,i} / \alpha_o A_i$



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## APPENDIX 1

Derivation of Eq. (21).

For this derivation we only consider small variations around the steady-state and assume therefore the heat transfer coefficients being constant.

For the heat flux to the tubes we may write:

$$Q_{\text{to tube}} = Q_{\text{wr}} + Q_{\text{gt}} \quad (23)$$

Substitution of Eqs. (10)–(14) in (6) together with the values for  $A_1$ ,  $\epsilon_{\text{wr}}$  and  $\epsilon_w$  from Table 2 results in:

$$Q_{\text{wr}} = 2.01 \times 10^{-10} [T_w^{4.0} - T_i^{4.0}] - 23.2 \times 10^{-10} [T_w^{3.4} - T_i^{3.4}] \quad (24)$$

Since the heat capacity of the wall of the furnace is large, we assume that the dynamic variation in the wall temperature may be ignored, so we may write for the variation in heat flux from wall to tubes:

$$\delta Q_{\text{wr}} = -8.02 \times 10^{-10} \bar{T}_i^3 \delta T_i + 78.9 \times 10^{-10} \bar{T}_i^{2.4} \delta T_i \quad (25)$$

For a typical run the average value of  $T_i$  is 519.3°K, hence this equation may be written as:

$$\delta Q_{\text{wr}} = -0.086 \delta T_i \quad (26)$$

The heat transfer from gas to tubes is built up in two parts: radiation and convection. The radiation is given by:

$$Q_{\text{gt1}} = \alpha_{\text{gt}} \epsilon_r A_{r1} (T_g - T_i) \quad (27)$$

Substitution of the expression for  $\alpha_{\text{gt}}$  from (10)–(14) with the values for  $A_{r1}$  and  $\epsilon_r$  results in:

$$Q_{\text{gt1}} = 0.241 \times 10^{-8} [T_g^{3.4} - T_i^{3.4}] + 0.250 \times 10^{-10} [T_g^{4.0} - T_i^{4.0}] \quad (28)$$

For a variation in the heat flux we can write:

$$\delta Q_{\text{gt1}} = 0.818 \times 10^{-8} [\bar{T}_g^{2.4} \delta T_g - \bar{T}_i^{2.4} \delta T_i] + 1.00 \times 10^{-10} [\bar{T}_g^3 \delta T_g - \bar{T}_i^3 \delta T_i] \quad (29)$$

With  $\bar{T}_g = 841^\circ\text{K}$  and  $\bar{T}_i = 519.3^\circ\text{K}$  this equation can be simplified to

$$\delta Q_{\text{gt1}} = 0.1451 \delta T_g - 0.0409 \delta T_i \quad (30)$$

The convection from gas to tubes is given by:

$$Q_{\text{gt2}} = \alpha_c A_{r2} (T_g - T_i) \quad (31)$$

The values of the parameters are:  $\alpha_c = 0.014 \text{ kJ/m}^2 \text{ sec } ^\circ\text{K}$  and  $A_{r2} = 7.70 \text{ m}^2$ . So we may write for a variation in the convection heat flux:

$$\delta Q_{\text{gt2}} = 0.1081 (\delta T_g - \delta T_i) \quad (32)$$

For the variation in total heat flux to the tubes we combine Eqs. (26), (30) and (32):

$$\delta Q_{\text{to tube}} = 0.2532 \delta T_g - 0.2354 \delta T_i \quad (33)$$

Equation (17) can be written as:

$$M_i c_i s \delta T_i = \delta Q_{\text{to tube}} - \alpha'_0 A_i (\delta T_i - \delta T_0) \quad (34)$$

In this equation  $\alpha'_0$  is the net heat transfer coefficient inside the tubes, calculated from:

$$\frac{1}{\alpha'_0} = \frac{1}{\alpha_0} + \left( \frac{d}{\lambda} \right)_{\text{deposit}} \quad (35)$$

With the parameter values from Table 2 we can write for Eq. (34):

$$41.15 s \delta T_i = 0.2532 \delta T_g - 0.2354 \delta T_i - 0.9433 (\delta T_i - \delta T_0) \quad (36)$$

or in Harriott's terminology[15]:

$$T_2 \left( \frac{\partial T_i}{\partial t} \right) = \gamma T_g - T_i - \frac{T_2}{T_{12}} (T_i - T_0) \quad (37)$$

Equation (18) can be written as:

$$T_1 \left( \frac{\partial T_0}{\partial t} \right) + v T_1 \left( \frac{\partial T_0}{\partial x} \right) = T_i - T_0 \quad (38)$$

The solution of Eqs. (37) and (38) is:

$$\frac{T_0}{T_g} = \frac{b}{a} (1 - e^{-a\tau_{\text{res}}}) \quad (39)$$

in which

$$a = \frac{(T_1 s + 1)(T_2 T_{12} s + T_2 + T_{12}) - T_2}{T_1 (T_2 T_{12} s + T_2 + T_{12})} \quad (40)$$

and

$$\frac{b}{a} = \frac{\gamma}{T_1 T_2 s^2 + (T_1 + T_2 + T_1 T_2 / T_{12})s + 1}. \quad (41)$$

Substitution of the numerical values of the parameters from Table 2 results in:

$$\frac{b}{a} = \frac{1.075}{(21.42s + 1)(397.4s + 1)} \quad (42)$$

and

$$a = \frac{(48.78s + 1)(127.56s + 3.648) - 2.92}{48.78(127.56s + 3.648)} \approx 0.408 \times 10^{-2} + 1.568s. \quad (43)$$

Substitution in Eq. (39) results in:

$$\frac{T_0}{T_g} = \frac{1.075(1 - 0.627e^{-179s})}{(21.4s + 1)(397s + 1)}. \quad (44)$$

APPENDIX 2

Derivation of an expression for the large time constant in the furnace

In Eq. (40) *a* is given as a function of *s*. We can split *a* in a dynamic and a static part.

The static part (for *s*=0) is given by:

$$\alpha_{\text{stat}} = \frac{T_{12}}{T_1(T_2 + T_{12})}. \quad (45)$$

From the energy balances (37) and (38) we can derive that:

$$e^{-\alpha_{\text{stat}}\tau_{\text{res}}} = \frac{\gamma \bar{T}_g - T_{0,\text{out}}}{\gamma \bar{T}_g - T_{0,\text{in}}} \quad (46)$$

as a solution of a first order differential equation. Equation (39) can now be written as:

$$\frac{T_0}{T_g} = \frac{b}{a} \left( 1 - \frac{\gamma \bar{T}_g - T_{0,\text{out}}}{\gamma \bar{T}_g - T_{0,\text{in}}} \cdot e^{-\alpha_{\text{dyn}}\tau_{\text{res}}} \right). \quad (47)$$

The dynamic part of *a* is given by:

$$a_{\text{dyn}} = a - \frac{T_{12}}{T_1 T_2 + T_1 T_{12}}. \quad (48)$$

The  $e^{-\alpha_{\text{dyn}}\tau_{\text{res}}}$  is developed into a Taylor series in *s*. The first order term can be found from:

$$-\frac{\partial}{\partial s} (e^{-\alpha_{\text{dyn}}\tau_{\text{res}}})_{s=0} = \left[ \tau_{\text{res}} e^{-\alpha_{\text{dyn}}\tau_{\text{res}}} \left( \frac{\partial a_{\text{dyn}}}{\partial s} \right) \right]_{s=0}. \quad (49)$$

The term  $(\partial a_{\text{dyn}}/\partial s)$  may be derived from (48) and is equal to

$$\left( \frac{\partial a_{\text{dyn}}}{\partial s} \right)_{s=0} = \frac{(T_{12} + 2T_2 T_{12} + T_2^2 + T_2^2 T_{12}^2 / T_1)}{(T_2 + T_{12})^2}. \quad (50)$$

Then the factor between brackets in Eq. (47) can be given by:

$$\frac{T_{0,\text{out}} - T_{0,\text{in}}}{\gamma \bar{T}_g - T_{0,\text{in}}} + \frac{\gamma \bar{T}_g - T_{0,\text{out}}}{\gamma \bar{T}_g - T_{0,\text{in}}} \times \left[ \frac{T_{12} + 2T_2 T_{12} + T_2^2 + T_2^2 T_{12}^2 / T_1}{(T_2 + T_{12})^2} \right] \tau_{\text{res}} \cdot s. \quad (51)$$

The factor *b/a* of Eq. (47) is given by Eq. (41). The time constant of this factor for low frequencies is given by:

$$T_1 + T_2 + \frac{T_1 T_2}{T_{12}}. \quad (52)$$

The large time constant that will be found in the analysis of the dynamic behaviour of the furnace, can now be given by the difference of the time constant in Eqs. (51) and (52)

$$\tau = \frac{\gamma \bar{T}_g - T_{0,\text{out}}}{T_{0,\text{out}} - T_{0,\text{in}}} \times \left[ \frac{T_{12} + 2T_2 T_{12} + T_2^2 + T_2^2 T_{12}^2 / T_1}{(T_2 + T_{12})^2} \right] \tau_{\text{res}} - \left[ T_1 + T_2 + \frac{T_1 T_2}{T_{12}} \right]. \quad (53)$$

With the parameter values from Table 2 we find for  $\tau$  a value of 99.6 sec. The experimental value of the large time constant is 96.0 sec, so the agreement is rather good. For a large furnace we may assume

$$M_0 \gg M_1 \text{ and } \alpha_g > \alpha_0.$$

So  $T_2$  is negligible and  $T_1$  is generally large. For this case Eq. (53) can be simplified to:

$$\tau = \frac{\gamma \bar{T}_g - T_{0,\text{out}}}{T_{0,\text{out}} - T_{0,\text{in}}} \cdot \tau_{\text{res}} - T_1 \left( 1 + \frac{T_2}{T_{12}} \right). \quad (54)$$

Elimination of the last term by means of (45) and (46) results in the final equation:

$$\tau = \tau_{\text{res}} \cdot \frac{(\gamma - 1)\bar{T}_g - (T_{0,\text{out}} - \bar{T}_0)}{(T_{0,\text{out}} - T_{0,\text{in}})} \quad (55)$$

where

$$\bar{T}_0 = \bar{T}_g - \frac{T_{0,\text{out}} - T_{0,\text{in}}}{\ln \frac{\gamma/\bar{T}_g - T_{0,\text{in}}}{\gamma/\bar{T}_g - T_{0,\text{out}}}}. \quad (56)$$