Brenner *et al.* Reply: The preceding Comment [1] criticizes our simplifications of the equations that describe the time evolution of shape perturbations in a periodically forced bubble. The heart of the critique is that the *boundary layer* (BL) *approximation* that we made to Prosperetti's formulas [2] is of the wrong form in the case of extremely low forcing. The summary of our reaction to this remark is as follows: (i) While it is of interest to look at the asymptotic form as the forcing goes to zero, this is far from the situation in which sonoluminescence (SL) occurs. (ii) Treating the full equations in the regime of interest is sufficiently difficult that approximations seem necessary. (iii) The intuitive, nonrigorous approximations that we made are in respectable, if not precise, agreement with experiments that were performed after our predictions were made. (iv) The numbers given in the Comment are based on Eq. (8) [containing the free shape oscillation frequency Ω] and *not* on the criticized Eq. (3) [containing the forcing frequency ω].

A point that makes it difficult to read the Comment [1] in conjunction with the original Letter [3] is that the authors refer only to the *Rayleigh-Taylor* (RT) *instability*, while in the Letter two types of instabilities were distinguished: RT, which is very quick, and *parametric*, which takes many cycles to have an important effect. For moderate levels of forcing the *parametric instability* is the one that is relevant.

The appearance of BLs is common in fluid systems that are periodically forced. A well-known example (due to Stokes) is the flow past an oscillating solid body (cf. [4]): In this case dissipation is primarily in an oscillatory BL of thickness $\sim\sqrt{\nu/\omega}$. Similar remarks apply to the classical Faraday instability (the planar version of the problem considered in [3,5]), whose character is also parametric: (i) It requires strong forcing to occur; and (ii) viscous BLs $\sim\sqrt{\nu/\omega}$ modify the damping rate; see e.g., [6].

References [3,5] utilize Prosperetti's elegant analysis [2] to include viscous effects on the shape stability of an oscillating bubble. He reduces the mathematical problem to a scalar advection diffusion equation with integrodifferential boundary conditions. Our strategy of exploring the entire parameter space accessible to a sonoluminescing bubble led us to construct a BL approximation, i.e., viscous effects were treated as acting only within a distance $\delta \sim \sqrt{\nu/\omega}$ from the bubble [7]. While our approximations may seem crude to some, and we would be very interested in seeing an analysis of the full equations in the SL regime, we don't expect the full equations to give exact agreement with experiments either, because, as pointed out in [3,5], potentially important physical effects have been neglected. Two such effects are heat transfer (which slightly shifts the phase diagrams of Refs. [3,5], as recently shown by Prosperetti) and surfactants.

Despite neglecting these effects and using a BL approximation, our results agree favorably with recent experiments [8,9]; see Fig. 1. One finds the following:

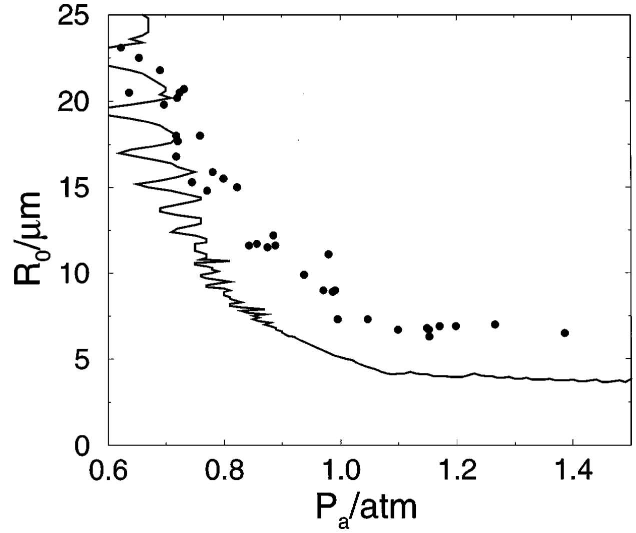

FIG. 1. The line shows the $n = 2$ parametric shape instability boundaring in the $P_a - R_0$ parameter space. The frequency is $\omega/2\pi = 20.6$ kHz, the water temperature 20 °C; there is no fitting parameter in this calculation. Circles indicate experimental data [8]. Not shown in this figure is the (short time scale) Rayleigh-Taylor shape instability [3,5]. It becomes important around $P_a = 1.4-1.5$ atm and presumably sets the upper P_a threshold of the single bubble SL domain.

(i) The shape of the two stability curves are similar, the threshold becoming almost independent of the driving pressure amplitude P_a at large P_a. (ii) For small P_a the quantitative agreement is also satisfactory [9]. (iii) Whether or not the underestimation at larger P_a is due to our BL approximation or originates from neglecting thermal or other damping effects in [3,5] remains to be seen. Finally, we note that our approximation has been used to explain the temperature dependence of single bubble SL and to make predictions for upscaling SL [10].

Note added in proof.—Meanwhle the fill PDES of Ref. [2] have numerically been solved by Prosperetti and Hao [11] confirming the results of the boundary layer approximation [3,5] after an initial transient of a few cycles. The origin of the deviations between experiment and theory in Fig. 1 is found to be the neglect of heat transfer effects, as speculated above.

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