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Erratum

Corrigendum to “Representations for the extreme zeros of orthogonal polynomials” [J. Comput. Appl. Math. 233 (2009) 847–851]

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ABSTRACT

We correct representations for the endpoints of the true interval of orthogonality of a sequence of orthogonal polynomials that were stated by us in the Journal of Computational and Applied Mathematics 233 (2009) 847–851.

In [1, Theorem 1] representations are given for the smallest zero x_{n1} and the largest zero x_{nn} of the polynomial P_n, $n > 0$, for when these polynomials satisfy a three-term recurrence relation of the type

$$\begin{aligned} &P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x), \quad n > 1, \\ &P_0(x) = 1, \qquad P_1(x) = x - c_1, \end{aligned} \tag{1}$$

where c_n is real and $\lambda_n > 0$, and therefore constitute a sequence of orthogonal polynomials. Since the smallest point ξ_1 and largest point η_1 of the true interval of orthogonality for these polynomials are the limits as $n \to \infty$ of x_{n1} and x_{nn}, respectively, the representations for x_{n1} and x_{nn} lead to representations for ξ_1 and η_1. However, an unjustified step in the limiting procedure has led to two incorrect statements in [1, Corollary 2]. Specifically, the second representation for ξ_1 is not correct and should be replaced by

$$\xi_1 = \lim_{n\to\infty} \min_{\boldsymbol{a}>\boldsymbol{0}} \left\{ \max_{1\le i\le n} \left\{ c_i - a_{i+1} - \frac{\lambda_i}{a_i} + \delta_{in} a_{n+1} \right\} \right\}, \tag{2}$$

where δ_{in} denotes Kronecker’s delta and $\boldsymbol{a} \equiv (a_1, a_2, \ldots)$. Also, the second representation for η_1 is not correct and should be replaced by

$$\eta_1 = \lim_{n\to\infty} \max_{\boldsymbol{a}>\boldsymbol{0}} \left\{ \min_{1\le i\le n} \left\{ c_i + a_{i+1} + \frac{\lambda_i}{a_i} - \delta_{in} a_{n+1} \right\} \right\}. \tag{3}$$

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These corrections have consequences for the applications in [1, Section 4]. Thus the second representation for the decay parameter δ of a nonergodic birth–death process with killing in [1, Theorem 3] should be replaced by

$$\delta = \lim_{n\to\infty} \min_{\boldsymbol{a}>\boldsymbol{0}} \left\{ \max_{0\le i\le n} \left\{ \alpha_i + \beta_i + \gamma_i - a_{i+1} - \frac{\alpha_{i-1}\beta_i}{a_i} + \delta_{in} a_{n+1} \right\} \right\}, \tag{4}$$

and the second representation for the decay parameter δ of a ergodic birth–death process in [1, Theorem 4] should be replaced by

$$\delta = \lim_{n\to\infty} \min_{\boldsymbol{a}>\boldsymbol{0}} \left\{ \max_{0\le i\le n} \left\{ \alpha_i + \beta_{i+1} - a_{i+1} - \frac{\alpha_i\beta_i}{a_i} + \delta_{in} a_{n+1} \right\} \right\}. \tag{5}$$

Here α_i, β_i and γ_i are, respectively, the birth, death and killing rate of the process in state i.

The hitch in the argument leading to the erroneous representation for ξ_1 in [1, Corollary 2] was caused by neglecting the requirement $a_{n+1} = 0$ when taking limits as $n \to \infty$ in [1, Eq. (11)], that is, in the inequalities

$$\min_{1\le i\le n} \left\{ c_i - a_{i+1} - \frac{\lambda}{a_i} \right\} \le x_{n1} \le \max_{1\le i\le n} \left\{ c_i - a_{i+1} - \frac{\lambda}{a_i} \right\}. \tag{6}$$

This oversight invalidates the resulting upper bound for ξ_1 but not the lower bound, and therefore affects the second representation for ξ_1 but not the first. Similar remarks pertain to the representations for η_1.

One can easily see that the second representation for δ in [1, Theorem 3], and hence the second representation for ξ_1 in [1, Corollary 2], cannot be correct by considering a transient, pure birth–death process with $\gamma_0 = 0$, and noting that, on choosing $a_i = \alpha_{i-1}$, this representation leads to the conclusion $\delta \le 0$, and hence $\delta = 0$, which is well known to be false in general.

References

[1] E.A. van Doorn, N.D. van Foreest, A.I. Zeifman, Representations for the extreme zeros of orthogonal polynomials, J. Comput. Appl. Math. 233 (2009) 847–851.