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Ceccherini-Silberstein, Tullio (I-SAN-EN)
Growth and ergodicity of context-free languages. II. The linear case. (English summary)
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For an infinite language L over an alphabet Σ, the "growth" of L is $\gamma(L) = \limsup_{n\to\infty}(\#\{w \in L \mid |w| = n\})^{1/n}$, where $|w|$ is the length of the word w and $\#X$ denotes the cardinality of the finite set X. This growth is exponential if $\gamma(L) > 1$ and sub-exponential otherwise (i.e., if $\gamma(L) = 1$).

For each finite nontrivial language F over Σ (i.e., $0 < \#(F\cap\Sigma^+) < \infty$), L^F is defined by $L^F = \{w \in L \mid \text{no } v \text{ from } F \text{ is a subword of } w\}$. Then L is called "growth-sensitive" if $\gamma(L^F) < \gamma(L)$ for any such F.

For a context-free grammar $G = (N, \Sigma, P, S)$, the dependency graph of G is the directed graph $D(G) = (N, E)$ with $E = \{(A, B) \mid A, B \in N; (A, w) \in P; B \text{ occurs in } w\}$. G is called "ergodic" if (1) $D(G)$ is strongly connected—i.e., if there is directed path from A to B in $D(G)$ for each pair of points $A, B \in N$—and (2) $\#E \geq 1$. If G is linear the author requires in addition that (3) each $w \in L(G)$ occurs as a proper subword of some word generated by a so-called essential nonterminal. A (linear) context-free language is ergodic if it is generated by a reduced ergodic (linear) context-free grammar.

Ergodic regular languages of exponential growth are growth-sensitive [T. G. Ceccherini-Silberstein, A. Machì and F. Scarabotti, Theoret. Comput. Sci. **307** (2003), no. 1, 93–102; MR2022842 (2004k:68083)], and in Part I [Trans. Amer. Math. Soc. **354** (2002), no. 11, 4597–4625 (electronic); MR1926891 (2003g:68067)] the author and W. Woess proved a similar result for unambiguous nonlinear context-free languages. The main result of the present paper establishes that ergodic unambiguous linear context-free languages of exponential growth are growth-sensitive as well.

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References

1. M. R. Bridson and R. H. Gilman, *Context–free languages of sub-exponential growth*, J. Comput. System Sci. **64** (2002), no. 2, 308–310. MR1906807 (2003d:68132)
2. T. Ceccherini–Silberstein, *On the growth of linear languages*, Adv. in Appl. Math. **35** (2005), no. 3, 243–253. MR2164918 MR2164918 (2006e:68080)
3. T. Ceccherini–Silberstein, F. Fiorenzi and F. Scarabotti, *The Garden of Eden theorem for cellular automata and for symbolic dynamical systems*, In "Random walks and geometry",Vadim A. Kaimanovich ed., Walter de Gruyter, Berlin, 2004. MR2087780 (2005g:37013)
4. T. Ceccherini–Silberstein, A. Mach'ıand F. Scarabotti, *On the entropy of regular languages*, Theoret. Comput. Sci. **307** (2003) 93-102. MR2022842 (2004k:68083)
5. T. Ceccherini–Silberstein and W. Woess, *Growth and ergodicity of context-free languages*,

Trans. Amer. Math. Soc. **354** (2002) 4597–4625. MR1926891 (2003g:68067)
6. T. Ceccherini–Silberstein and W. Woess, *Growth-sensitivity of context-free languages*, Theoret. Comput. Sci. **307** (2003) 103-116. MR2022843 (2005h:68066)
7. N. Chomsky and P. M. Sch"utzenberger, *The algebraic theory of context-free languages*, Computer Programming and Formal Systems (P. Braffort and D. Hirschberg, eds.) North-Holland, Amsterdam (1963) 118–161. MR0152391 (27 #2371)
8. S. Eilenberg, *Automata, languages, and machines*, Vol. A, Pure and Applied Mathematics, Vol. 58, Academic Press, New York, 1974. MR0530382 (58 #26604a)
9. S. Ginsburg, *The Mathematical Theory of Context Free Languages*, McGraw-Hill Book Co., New York-London-Sydney, 1966. MR0211815 (35 #2692)
10. R.I. Grigorchuk and P. de la Harpe, *On problems related to growth, entropy and spectrum in group theory*, J. Dynam. Control Systems **3** (1997) 51-89. MR1436550 (98d:20039)
11. M. Harrison, *Introduction to formal language theory*, Addison-Wesley Publishing Co., Reading, Mass., 1978. MR0526397 (80h:68060)
12. J. E. Hopcroft and J. D. Ullman, *Introduction to automata theory, languages, and computation*, Addison-Wesley Series in Computer Science. Addison-Wesley Publishing Co., Reading, Mass., 1979. MR0645539 (83j:68002)
13. R. Incitti, *The growth function of context–free languages*, Theoret. Comput. Sci. **255** (1999) 601–605. MR1819093 (2001m:68098)
14. W. Kuich, *On the entropy of context-free languages*, Information and Control **16** (1970) 173–200. MR0269447 (42 #4343)
15. W. Kuich, *Semirings and formal power series: their relevance to formal languages and automata*, Handbook of Formal Languages, Vol. 1, 609–677, Springer, Berlin, 1997. MR1470001 (98m:68152)
16. D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, Cambridge, 1995. MR1369092 (97a:58050)
17. W. Ogden, *A helpful result for proving inherent ambiguity*, Math. Systems Theory **2** (2) (1968) 191-194. MR0233645 (38 #1966)
18. F. Scarabotti, *On a lemma of Gromov and the entropy of a graph*, European J. Combinatorics **23** (2002) 631-633. MR1931946 (2003h:37010)
19. E. Seneta, *Non-negative Matrices and Markov Chains*, 2nd edition, Springer, New York, 1981. MR2209438
20. V. I. Trofimov, *Growth functions of some classes of languages*, Kibernetika **1981** (1981), no. 6, i, 9–12, 149 [English translation: Cybernetics **17** (1982) 727–731]. MR0689421 (84d:68088)
21. V. A. Ufnarovskii, *A growth criterion for graphs and algebras defined by words*, Mat. Zametki **31** (1982), no. 3, 465–472, 476 [English translation: Math. Notes **31** (1982), no. 3, 238–241]. MR0652851 (83f:05026)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.