

FILTERING AND IDENTIFICATION OF STOCHASTIC VOLATILITY FOR PARABOLIC TYPE FACTOR MODELS

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ABSTRACT. *We consider the dynamics of forward rate process which is modeled by a parabolic type infinite-dimensional factor model with stochastic volatility. The parameters included in the stochastic volatility dynamics are estimated from the factor process as the observation data. Based on the maximum likelihood technique, we propose the off-line identification scheme and provide some numerical examples .*

Keywords: Factor model, Stochastic volatility, Nonlinear filtering, M.L.E.

1. **Introduction.** The stochastic volatility model has been introduced to a stock model to capture the complexity of a real stock market, i.g., Heston's model [1] and [2]. For the factor model the stochastic volatility also has been already proposed in [3]. Usually in the option pricing problem, the implied volatility observed in the market based option is used for the volatility process. However it seems natural to use the historical data of the stock and bond prices to estimate the volatility process. Hence in the case of using the stochastic volatility dynamics, we need to construct the estimation algorithm of the volatility process and identification of the included parameters from the historical data.

First we consider the simple example of stochastic volatility case. The forward rate $f(t, x)$ is a solution of ¹

$$df(t, x) = \{\text{Drift term}\}dt + \sqrt{\sigma(t)}dw(t, x), \quad (1)$$

where $w(t, x)$ denotes an infinite-dimensional Brownian motion process with $E\{w(t, x)w(t, y)\} = q(x, y)t$ and the stochastic volatility process $\sigma(t)$ is given by

$$d\sigma(t) = \alpha(m - \sigma(t))dt + k\sqrt{\sigma(t)}dv(t) \quad (2)$$

where $v(t)$ is a R^1 -valued Brownian motion process.

¹The exact form of the forward rate $f(t, x)$ is given in Appendix.