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Stochastic Differential Systems—Analysis and Filtering*

V. S. PUGACHEV and I. N. SINITSYN

Reviewer: A. BAGCHI
Faculty of Applied Mathematics, University of Twente, P.O. 217, NL-7500 AE Enschede, The Netherlands.

THIS BOOK is concerned with dynamical systems described by stochastic differential equations. The first part of the book deals with the analysis of such systems and the remainder of the book is on application to the filtering problem. The stochastic term driving the dynamical system is taken to be an arbitrary process with independent increments, so that the usual Wiener process model is considered as a special case.

There are excellent mathematical texts on filtering for stochastic differential systems (see, for example, Lipster and Shiryayev (1977), and Kallianpur (1980)). The book under review does not discuss filtering in that generality and stresses the engineering aspect throughout. In fact, the non-linear filtering problem is not solved in this book and only approximate non-linear filtering algorithms are developed. The present book is a combination of stochastic signal analysis and continuous-time filtering theory. There are, of course, numerous engineering texts on stochastic signals and systems (some recent publications include those of Gray and Davisson (1986), Mortensen (1987), and Stark and Woods (1986); see also Wong and Hajek (1985) for a more advanced treatment). There are also standard "engineering" books on filtering theory where continuous-time models are handled in detail (Maybeck, 1982; Jazwinski, 1970). None of these books cover both these topics in as much detail as the book under review.

Chapter 1 starts with preparatory material on deterministic dynamical systems, including concepts such as inputs, outputs and states of a system, characteristics of linear systems and results on linear differential equations. The chapter ends with an intuitive introduction to stochastic differential systems as extensions of deterministic differential systems with stochastic forcing terms.

Chapter 2 gives an introduction to stochastic processes. It starts with the definition of a stochastic process and its construction based on a family of finite-dimensional distributions. The second-order moment leads to covariance and cross-covariance functions and, in special cases, to white noise and Gaussian processes. This is followed by a section on orthogonal expansion of finite-dimensional densities of a stochastic process. This includes Hermite polynomial expansion, Edgeworth series and related orthogonal expansions. This section contains material not usually found on this topic. These expansions play an important role later in the book dealing with the filtering problem. The chapter ends with the mean-square calculus of stochastic processes.

Chapter 3 deals with stochastic integrals and stochastic differential equations. Stochastic integrals are first defined for non-random integrands with respect to an arbitrary process with uncorrelated increments. In this connection white noise is introduced as the derivative of a process with an uncorrelated increment. Stochastic measures are defined in order to define stochastic integrals for non-random functions of vector arguments. The authors then study linear stochastic differential equations. Stochastic integrals for random integrands are then defined, but with respect to processes with independent increments. The Wiener process and the Poisson process are introduced and an arbitrary process of independent increments is expressed in terms of a Wiener process and an integral involving a Poisson stochastic measure. Although the Itô integral plays a central role, Stratonovich and other stochastic integrals are also introduced. The next section deals with the Itô differentiation rule. The last section is concerned with non-linear stochastic differential equations. The treatment here is sketchy, as the authors do not use the mathematical machinery needed to study the existence and sample-path properties of solutions of stochastic differential equations. They concentrate, instead, on Itô's formula for processes satisfying stochastic differential equations and on the connection with other types of stochastic differential equations.

Chapter 4 is devoted to stationary stochastic processes. This is somewhat out of place and could have been studied earlier. The topics considered are standard and include spectral theory and linear operations on stationary stochastic processes.

Chapter 5 deals with what the authors call the theory of stochastic differential systems. It starts with the modelling question; that is, on transforming a system equation into a stochastic one. Then the moments of the state vector of a linear system are explicitly worked out. This is followed by a general theory of finite-dimensional distributions of the state vector of a non-linear system and the chapter ends with further consideration of the special case of finite-dimensional distributions of the state vector of a linear system.

Chapter 6 is a continuation of the theory of non-linear stochastic differential systems. First the case of deterministic systems with random initial data is considered. The rest of the chapter gives an exhaustive treatment of non-linear differential systems based on approximate solutions of the equations for the finite-dimensional distributions of the state vector. The first approach used is the normal approximation of the finite-dimensional distributions of the state vector which was derived by one of the authors (V. S. Pugachev) more than 40 years ago. This is followed by more modern approximation techniques used in other contexts in statistics: method of moments, semi-invariant methods and methods based on orthogonal expansions. Chapter 2 contains all the necessary preparatory material.

The last three chapters deal exclusively with the optimal filtering problem. Chapter 7 is on the general theory of optimal filtering, including the special case of the linear filter. In this situation, only the Wiener process forcing term for the noise model is considered, both for the state and the observation equations. The equation of evolution of the conditional density for any function of the state (within an appropriate class) is derived following the approach of Lipster and Shiryayev (1977). This is then used to derive the stochastic differential equation satisfied by the conditional mean. The linear case of the Kalman–Bucy filter is also considered. The idea of the "innovation process" in this context is discussed and the case of correlated noise is also studied.

As is well known, there is no closed-form solution for the non-linear filtering problem and one has to take recourse to some suitable suboptimal scheme in practice. This is

**Stochastic Differential Systems—Analysis and Filtering* by V. S. Pugachev and I. N. Sinitsyn. Wiley, Chichester (1987). ISBN 0 471 91243 3. £49.50.

discussed in Chapter 8. The approximation techniques of the finite-dimensional distributions of the state vector discussed in Chapter 6 are used to derive various suboptimal filtering methods; for example, the method of normal approximation of conditional distribution, the method of moments, the method of semi-invariants and methods based on orthogonal expansions. The usual suboptimal filter techniques based on linearization; for example, the extended Kalman filter, the second-order filter and the Gauss filter are also discussed.

Chapter 9 is based entirely on some recent works of the authors on what they call the theory of conditionally optimal estimation. The idea is to look for the optimal estimate within a certain class of functions and obtain the best minimum mean square estimate within that class. Various extensions and related problems are studied in this framework. The last section is concerned with the discrete-time version of the conditionally optimal estimation.

This book has been written primarily for engineers and measure theory has been mostly avoided. This forced the authors to restrict themselves to the mean-square theory. But one cannot develop a mathematical theory pathwise in this framework. This fundamental difficulty is not explained clearly in the book and this may be misleading. However, within this limitation, the authors give a careful and correct treatment of the subject.

A considerable part of the book deals with the works of Professor Pugachev and his co-workers. Specifically, various approximations of the finite-dimensional distributions of the state vector applied to approximate non-linear filtering and the last chapter on conditionally optimal estimation are their own works. These are not widely familiar to systems scientists in the West and, therefore, are quite useful. The only problem is that, in the absence of any further analysis, these methods do not offer any advantage over other usual suboptimal filter algorithms. On the other hand, some important works on stochastic differential systems are either treated cursorily or not mentioned at all. The Wong–Zakai correction term is not treated in sufficient detail. The role of the innovation process in non-linear filtering, the martingale formulation, the Bayes approach to non-linear filtering and the smoothing problem are not treated in this book. Numerical aspects in implementing the filter equation, steady-state behaviour of the Kalman–Bucy filter and fast algorithms for computing filters are also not covered.

Engineers are in a position to read this book without being overwhelmed by mathematical machinery of stochastic differential equations. Unfortunately, however, this book does not cover many useful topics. It is, therefore, not particularly suitable as a textbook. On the other hand, there are numerous interesting problems at the end of each chapter which will be extremely useful for students and researchers alike. In this sense, it is a good reference book for teachers and for scientists interested in learning the subject on their own.

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About the reviewer

Professor A. Bagchi received the B.Sc. degree in mathematics and the M.Sc. degree in applied mathematics from Calcutta University in 1966 and 1968, and the M.S. and Ph.D. degrees in engineering from the University of California, Los Angeles, in 1970 and 1974. In 1971–1972, he worked as a senior research engineer at the Jet Propulsion Laboratory, Pasadena. In July 1974 he joined the Department of Applied Mathematics at the University of Twente, The Netherlands. Since then, except for a short stint as a Professor of Electrical and Computer Engineering at the Northeastern University, Boston, he has been affiliated with the University of Twente, where he is currently Professor of Stochastic Systems and Optimization Theory. He is the author of the monograph *Stackelberg Differential Games in Economic Models* (Springer Lecture Notes in Control and Information Sciences, Vol. 64) and the editor (together with Prof. H. Th. Jongen) of *Systems and Optimization: Proceedings of the Twente Workshop* (Springer Lecture Notes in Control and Information Sciences, Vol. 66).

Mobile Control of Distributed Parameter Systems*

A. G. BUTKOVSKIY and L. M. PUSTYL'NIKOV

Reviewer: SEPPO POHJOLAINEN
Department of Mathematics, Tampere University of Technology, P.O. Box 527, SF-33101 Tampere, Finland.

A RESEARCHER in the field of distributed parameter systems is sometimes asked about the usefulness of her or his theoretical results. This happens because controllers may often be realized only as finite-dimensional systems, and since distributed parameter systems in most cases can be approximated sufficiently well by finite-dimensional ones, so why take the trouble of studying and using partial differential equations and functional analysis instead of ordinary differential equations and linear algebra.

In their book Butkovskiy and Pustyl'nikov give a number of good and well-known reasons for using distributed parameter systems theory, starting from the fact that physical modelling naturally introduces partial differential equations, so that the substance of the parameters and variables is often easily understood. These systems cannot, in general, be approximated by finite-dimensional models without the loss of some essential qualitative features. Controllability, observability and stability of a finite-dimensional approximation do not necessarily imply that the corresponding properties hold for the infinite-dimensional "real" model.

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