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The Simulation of Skin Temperature Distributions by Means of a Relaxation Method

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ABSTRACT. To solve the differential equation for the heat in a two-layer, rectangular piece of skin tissue, a relaxation method, based on a finite difference technique, is used. The temperature distributions on the skin surface are calculated. The results are used to derive a criterion for the resolution for an infrared thermograph in a specific situation. A major limitation on the resolution in medical thermography is given. As an example of the power of the model, the sensitivity of the temperature profiles for variations of the layer thickness is determined.

1. Introduction

In medical thermography the question of the kind of phenomena responsible for the measured temperature profile is of vital importance. Further, in drafting the specifications for thermographic instrumentation, some uncertainties exist about the required spatial and temperature resolution. These two problems are concerned with the questions: 'What is a normal thermal pattern? How well do we need to resolve excursions from this normal pattern? and Are we able to interpret them?' The lack of criteria for setting norms in this field was the motivation for an attempt to construct a useful model of the heat transfer through skin tissue to a known environment.

The resulting skin temperature profiles may provide an answer to the above questions in some specific situations and later in the article we give detailed examples. When the model is homogeneous, has infinite dimensions and concerns line or point source, the thermal pattern on the skin surface can be determined analytically (Draper and Boag 1971). However, when a piece of skin of limited size and composed of different layers is considered, analytical methods fail and it is necessary to revert to numerical procedures.

A numerical solution based on an optimized relaxation technique (Brouwers 1972) is used here for the actual calculations of the skin surface isotherms.

2. Physical considerations for heat transfer through the skin

Human skin tissue has a very complex structure and at first sight it is not possible to obtain a model of the heat transfer which is both simple and valid. Nevertheless one finds, in studying physiological handbooks, that the heat transfer coefficients λ of the various types of human tissue given by different researchers are of the same order of magnitude (Stacey, Williams, Worden and McMorris 1955, Crosbie, Hardy and Fessenden 1963, Gröber, Erk and Grigull

1963, Draper and Boag 1971); typical values are, for fat $\lambda_f \simeq 0{\cdot}20$ W m^{-1}K^{-1} and for wet tissue $\lambda_t \simeq 0{\cdot}25$–$0{\cdot}42$ W m^{-1}K^{-1}. To a first approximation, the passive skin tissue can be regarded as homogeneous with respect to the thermal steady-state properties. Problems arise when heat transport by blood is also taken into account. However, when the environmental temperature is below 19 °c the state of vasoconstriction of the skin vessels is such that the skin tissue cools as if it were passive (Stacey *et al.* 1955). The results of Mali (1969) and van der Staak, Brakkee and de Rijke-Hereweijer (1968) confirm these results in part, giving a conductivity value of $\lambda_t \simeq 0{\cdot}33$ W m^{-1}K^{-1}. The foregoing entails two constraints for the model used; firstly the environmental temperature is chosen to be 18 °c, and secondly the restriction is made that only the basal metabolism is taken into account as a heat source. This implies that in practice the patient must be completely at rest. It is now possible to design a simple model as shown in fig. 1.

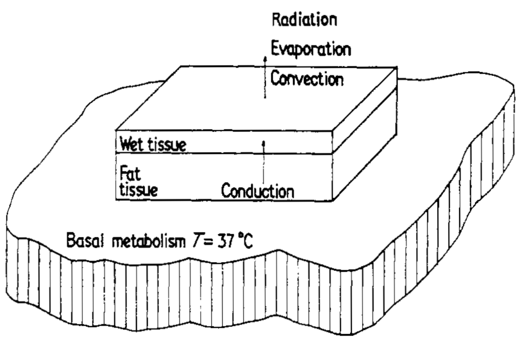

Fig. 1. Block-shaped model for heat transfer in skin.

Inside the body, heat transport is effected only by conduction; at the surface, radiation, convection and evaporation apply. For the nude human body 60% of the heat loss is by radiation (Stacey *et al.* 1955). The assumption is made that the heat exchange by respiration is as much as 20% of the total heat loss. Since this is a localized process the heat loss by radiation for an area of skin only, requires correction; it is chosen to be 75% of the total rather than 60% (Brouwers 1972).

The human skin surface is assumed to be a black body radiator in the wavelength between 2 and 20 μm (Hardy 1934, 1939, Steketee 1973). Heat loss by radiation dQ/dt (in W), through a surface A (in m^2), is therefore described by the formula

$$\frac{dQ}{dt} = A\sigma(T_0{}^4 - T_\infty{}^4) \tag{1}$$

where σ is the Stefan Boltzman constant ($5{\cdot}67 \times 10^{-8}$ W m^{-2}K^{-4}), T_0 is the radiator temperature (in K) and T_∞ is the ambient temperature (in K). It is easy to show that eqn (1) may be linearized with no greater relative error than 0·5% when the surface temperature changes less than 1 K along the surface

($T_0 - T_\infty$ being 16 K). Then eqn (1) transforms to

$$\frac{dQ}{dt} = A\varepsilon(T_0 - T_\infty) \tag{2}$$

with

$$\varepsilon = \sigma(T_0^3 + T_0^2 T_\infty + T_0 T_\infty^2 + T_\infty^3).$$

The other cooling effects, being small, are substantially linear and therefore the final heat loss formula can be approximated by

$$\frac{dQ}{dt} \simeq A\sigma'(T_0 - T_\infty). \tag{3}$$

This is the so-called Newtonian cooling law.

The value of σ' is supposed to be somewhat higher than ε to account for all the heat loss effects (in this case 75% radiation, 25% evaporation and convection). Therefore

$$\sigma' = \frac{100}{75}\varepsilon$$

when

$$T_\infty = 291\ \mathrm{K}, \quad T_0 = 307\ \mathrm{K} \quad \text{and} \quad \sigma = 5{\cdot}67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}.$$

The value of σ' is $8\ \mathrm{W\,m^{-2}\,K^{-4}}$. For the thermal model, the Newtonian cooling law acts as a boundary condition on the skin surface, the inside heat transport being controlled by the laws of potential flow (Laplace)

$$\nabla^2 T = 0. \tag{4}$$

Because we are considering a piece of skin of limited size, there are two more boundary conditions:

(1) The deepest boundary is in contact with tissue at 37 °C.

(2) The side planes have zero temperature gradients.

To take into account the different skin layers a multilayer model is introduced. In the first model two layers are chosen (fig. 1), one representing wet tissue with $\lambda_1 = 0{\cdot}33\ \mathrm{W\,m^{-1}\,K^{-1}}$ (Mali 1969). The fat tissue layer is considered to be built up in a homogenous way of 50% pure fat and 50% wet tissue; λ_2 is therefore taken to be $0{\cdot}27\ \mathrm{W\,m^{-1}\,K^{-1}}$. Regions of constant temperature, representing the anatomy and/or pathology of the tissue, are used as boundary conditions and excitation sources in this model. They are introduced for two reasons:

(1) It seems apparent that, when blood emanates from the deep 37 °C boundary, a vessel containing this blood flow may be considered as a temperature source.

(2) It is planned to check the validity of the model in the future by means of subcutaneous measurements: temperature is easier to measure than heat production.

Even in situations where heat sources are more likely to apply it is possible to replace the heat source by a constant temperature region having the same temperature as the heat source, taking into account the complementary structure of the equations.

3. The solution of the problem

The model described by eqn (4) and the above boundary conditions is an elliptic partial differential equation with first, second and third boundary conditions. To use the model we must solve eqn (4) in a specific situation. As stated in the Introduction, it is not possible to obtain an analytical solution; it is therefore necessary to revert to numerical procedures to get a sufficiently accurate approximation.

It is possible to use a relaxation method to get an approximate solution of eqn (4). The piece of skin is considered to be divided into elementary cubes by means of a three-dimensional grid of equidistant points (unit distance = h). Eqn (4) can be approximated by

$$T(x,y,z) = \tfrac{1}{6}[T(x+h,y,z)+T(x-h,y,z)+T(x,y+h,z) + T(x,y-h,z)+T(x,y,z+h)+T(x,y,z-h)]. \quad (5)$$

Thus the temperature of the point (x,y,z) is the average of the surrounding points each at a distance h.

Eqn (5) is excellently suited for numerical solution by a digital computer. Programming eqn (5) together with the boundary conditions provides a means of calculating the temperature at each point of the grid, sequentially over the whole field. This procedure is then repeated, starting with the previously calculated temperatures, until a predetermined accuracy criterion generates a stop command.

Using this technique rigorously requires excessive calculation time. Such long computation times are a result of the poor rate of convergence of the basic relaxation technique. To improve matters, this technique may be written as

$$T_n(x,y,z) = T_{n-1}(x,y,z)+R \quad (6)$$

where R is the difference between the new temperature value T_n and the old value T_{n-1}. When R is weighed by a factor ω with $1<\omega<2$ the so-called Successive Over Relaxation (SOR) technique is produced:

$$T_n(x,y,z) = T_{n-1}(x,y,z)+\omega R. \quad (7)$$

When ω is properly chosen an enormous gain in the rate of convergence can be obtained. However, this gain is very sensitive to the value of ω. Therefore a procedure is developed to optimize this ω-value as follows:

(1) Calculate the whole field 6 times by means of the SOR procedure with an estimation of ω.
(2) Compute a measure for the speed of convergence.
(3) Make a new estimation for ω.
(4) Repeat (1), (2) and (3) until the difference between two successive values of ω is less than a predetermined value. In practice it turned out that a difference smaller than 5×10^{-3} was not succeeded by a gain in convergence larger than 1 relaxation.

(5) The SOR method is then continued with the optimal ω until the predetermined criterion is met.

This procedure is called SORACO (Successive Over Relaxation with Automatic Convergence Speed Optimization; Brouwers 1972). Using SORACO in the computation of a three-dimensional, 8000 point model, a total of 46 iteration steps and a computer time of (typically) 8 min on an IBM 360 machine was required for a 0·01 K accuracy criterion. Finally a routine is used to determine isotherms on the skin surface: these can be drawn by calling in a digital plotter outline.

4. Results

Using this model it is possible in principle to make an atlas of isotherm patterns in relation to their medically relevant temperature sources, but this was beyond the scope of the present investigation. Some interesting cases are given below to illustrate the power of the method.

Example 1

Consider a temperature doublet of two constant temperature cubes A and B at 37 °C positioned as in fig. 2. The doublet position (i, j, k) is given by:

Cube A $(i, 17, 2)$, $(i, 17, 1)$, $(i, 18, 2)$, $(i, 18, 1)$
$(i+1, 17, 2)$, $(i+1, 17, 1)$, $(i+1, 18, 2)$, $(i+1, 18, 1)$

Cube B $(i, 17, -2)$, $(i, 17, -1)$, $(i, 18, -2)$, $(i, 18, -1)$
$(i+1, 17, -2)$, $(i+1, 17, -1)$, $(i+1, 18, -2)$, $(i+1, 18, -1)$
with $1 \leqslant i \leqslant 8$.

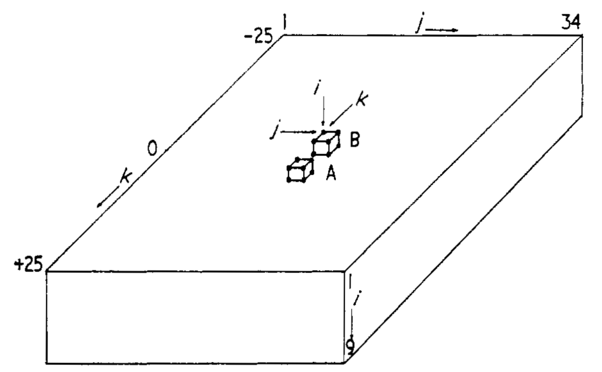

Fig. 2. Location of a double source.

The isotherm patterns were calculated for different values of i. A doublet was chosen to study resolution by investigating the depth at which a thermograph can detect a doublet of two separate sources. Figs 3, 4 and 5 give thermograms for different i-values, the source temperature being 37 °C. Considering

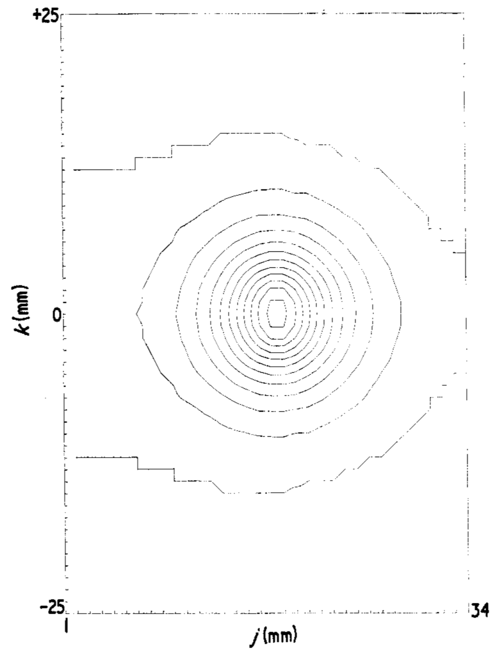

Fig. 3. Lowest isotherm 32·7 °c, isotherm increment 0·1 °c, highest isotherm 33·8 °c. Doublet on $i = 4, 5$.

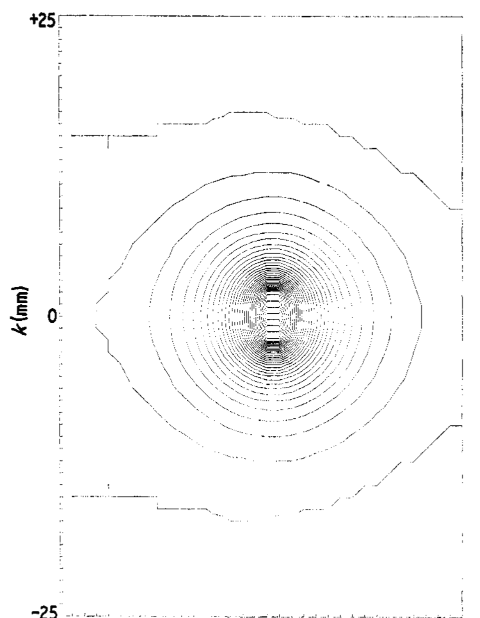

Fig. 4. Lowest isotherm 32·7 °c, isotherm increment 0·1 °c, highest isotherm 35·8 °c. Doublet on $i = 2, 3$.

the separation between wet and fat tissue,

for $i = 1, 2, 3,$ λ_t was taken to be $0{\cdot}33\ W\,m^{-1}K^{-1}$

for $i = 4, 5, 6, 7, 8$ and 9, λ_f was taken to be $0{\cdot}27\ W\,m^{-1}K^{-1}$.

So the separation depth $D = 2$ mm.

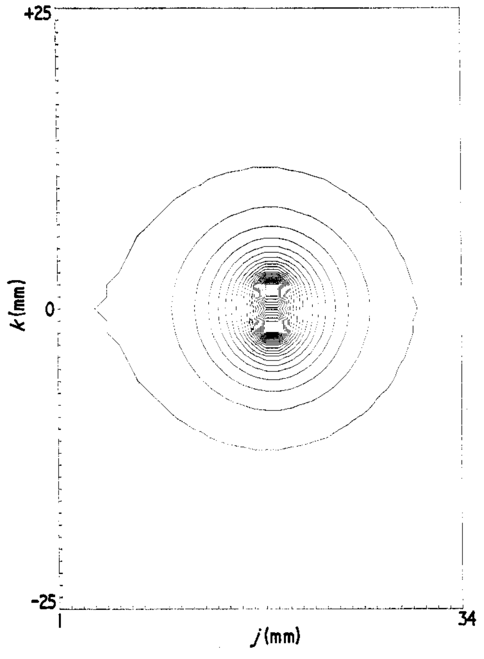

Fig. 5. Lowest isotherm 32·8 °C, isotherm increment 0·2 °C, highest isotherm 37·0 °C. Doublet on $i = 1, 2$.

5. Conclusions

With regard to this model we may state:

(1) It is almost impossible to detect the presence of temperature regions of these dimensions when they are situated deeper than 5–6 mm ($i = 6, 7$).

(2) The recognition of the doublet is only possible when the cubes are at a depth of not more than 1–2 mm ($i = 2, 3$).

Further statements about the resolution requirements are possible:

(3) In fig. 6 the curves A to E show the temperature profiles along the $j = 17$ line of figs 3–5 respectively. It is clear that to detect the presence of the small dicrotia of the curve D a temperature resolution of at least 0·1 K is necessary, the spatial resolution being better than 0·5 mm (Nyquist criterion).

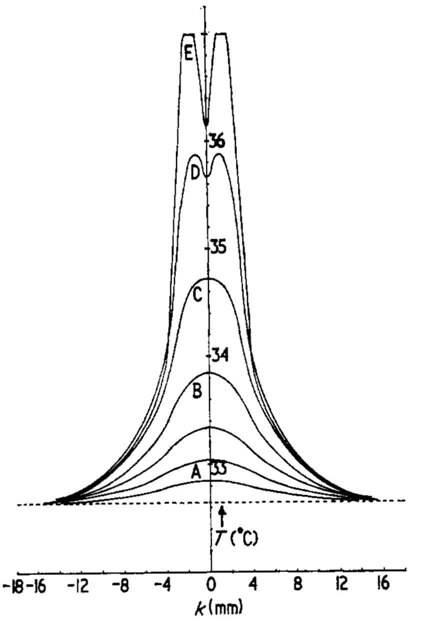

Fig. 6. Temperature profiles along the $j = 17$ line for different depths of the cube sources.

Example 2

Since it would be very interesting to investigate the influence of the depth of the separation between wet and fat tissue on the simulated temperature profiles, the skin temperature patterns were calculated for different values of the separation depth D. Fig. 7 gives four different classes of profile I, II, III and IV. I and IV represent extreme situations, I showing the profiles when the tissue is totally wet while IV is totally for fatty tissue. II and III give the temperature patterns for $D = 2$ and $D = 4$ mm respectively. The source configurations are the same as those of fig. 6, curves B and D, and the II curves are identical with fig. 6, B and D. Curves II and III occupy positions intermediate between I and IV. Fig. 8 shows the difference ΔT_{top} of the temperature T_{top} just above the source doublet (on $k = 0$) and the surface temperature T_{sa} at a distance large in relation to the source depth z, for different values of D.

Thus we also conclude that:

(4) The temperature profiles above the doublet are not strongly determined by the mutual proportion of the layers. In general the profiles in fat tissue are somewhat steeper than in wet tissue, but this is less certain for sources situated deeper under the skin surface. Thus a measurable relationship to depth D exists only for sources just under the skin surface: however, the profiles pertaining to this class of source are so distinctive that detection is certain with half a degree centigrade resolution. As is shown in fig. 7 the shape of the temperature profiles is hardly influenced by the inhomogeneity of the tissue. We therefore conclude that the resolution criterion formulated in Example 1 is not affected by the value of D.

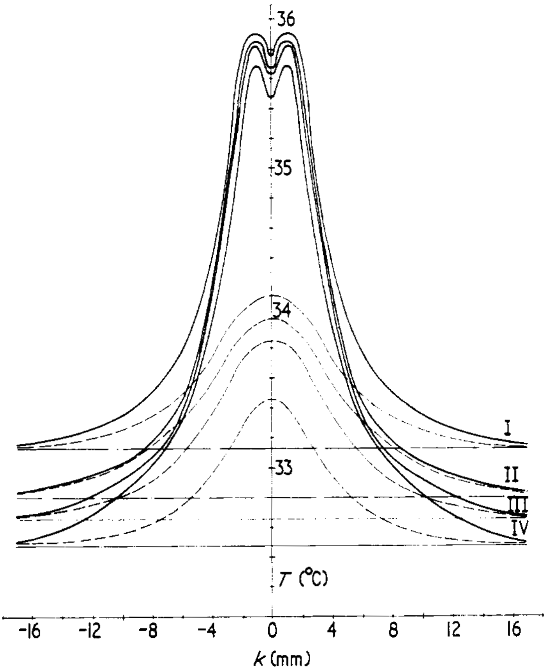

Fig. 7. Skin temperature profiles just above the sources for different depths of the separation between wet and fat tissue.

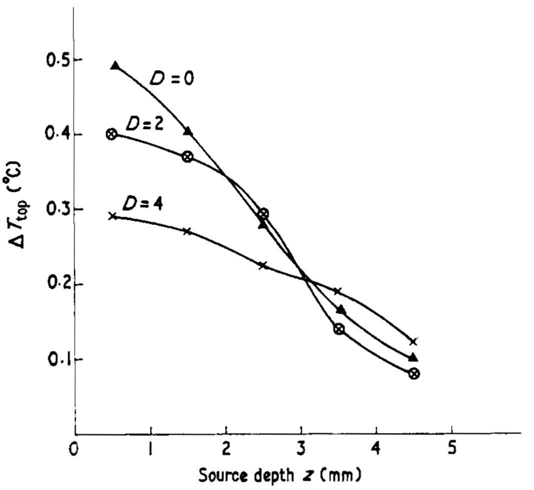

Fig. 8. Maximum relative top temperature ΔT_{top} as a function of the source depth z for different values of D (in mm).

6. Discussion

The main point of this work was to present a method giving some quantitative insight into the resolution one can expect of a medical thermograph. The results of the calculations are only as good as the model and the results of this investigation must be applied with caution. Nevertheless, this model can be helpful in estimating the sensitivity of temperature profiles for variations in the determining parameters. This is illustrated in Example 2.

Work is in progress studying spherical models, in order to investigate limitations of the resolution in the detection of small mammary carcinoma. We must point out that the use of this model in practical problems is limited by the number of matrix points; this determines the required computer memory and time for the job. Depending on the shape of the model, mathematical numerical problems of convergence and accuracy sometimes arise, and are not easy to solve. At the time of writing, the block-shaped model described above is optimized and operational. A two-dimensional circular model is almost completed. Work is in progress on a three-dimensional spherical model.

I would like to thank Professor D. Bosman for reading this manuscript and giving some valuable advice. I also thank Dr. P. Wesseling for help in solving some numerical problems.

Résumé

La simulation des distributions de la température de la peau au moyen d'une méthode de relaxation

Afin de resoudre l'équation différentielle pour la chaleur dans un morceau rectangulaire, consistant de deux couches, du tissu de peau, on a employé une méthode de relaxation, basée sur une technique de différence finie. On calcule les distributions de température sur la surface de la peau. Les résultats obtenus sont employés pour la déduction d'un critère pour la résolution en cas d'un thermographe infrarouge dans une situation spécifique. On donne une limitation importante de la résolution en thermographie médicale. Comme un exemple du pouvoir du modèle, on détermine la sensibilité des profiles de température aux variations de l'épaisseur de la couche.

Zusammenfassung

Nachbildung von Hauttemperaturverteilungen mittels eines Relaxationsverfahrens

Um die Differentialgleichung für die Wärme in einem rechteckigen Doppelschicht-Hautgewebestück zu lösen, ist ein auf einer endlichen Differenztechnik beruhendes Relaxationsverfahren angewandt worden. Es werden die Temperaturverteilungen auf der Hautoberfläche berechnet. Die erhaltenen Ergebnisse werden verwendet, um ein Kriterium für die Auflösung für einen infraroten Thermograph in einer spezifischen Lage abzuleiten. Es wird eine wichtige Begrenzung für die Auflösung in der medizinischen Thermographie angegeben. Als ein Beispiel der Leistung des Modells, wird die Empfindlichkeit der Temperaturprofile auf Veränderungen der Schichtdicke bestimmt.

Резюме

Моделирование распределения кожной температуры путем метода релаксации

Для решения дифференциального уравнения для тепла в двухслойном прямоугольном куске кожной ткани применяется метод релаксации, основанный на приеме конечной разности. Расчитываются распределения температуры на поверхности кожи. Полученные результаты применяются для вывода критерия для разрешающей способностн для инфракрасного термографа в конкретном положении. Дается главное ограничение разрешающеи способности в медицинской термографии. В качестве примера производительности модели определяется чувствительность профилей температуры к изменениям толщины слоя.

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