Evaluation of Čerenkov Second Harmonic Generation in Planar Waveguides in the Fourier Domain

Hugo J. W. M. Hoekstra, J. Čtyroký, and L. Kotăcka

***Abstract*—Analytical expressions for Čerenkov second-harmonic-generation in planar waveguiding structures are derived, based on evaluation of the conversion in the Fourier domain, assuming no depletion of the fundamental guided beam. The derivation is much shorter than that in existing methods and allows for a relatively simple interpretation of the main features.**

***Index Terms*—Čerenkov radiation, conversion efficiency, optical planar waveguides, second harmonic generation.**

I. INTRODUCTION

THE observation, based on computations, that phase matching may play a dominant role in Čerenkov second harmonic generation (CSHG) [1] has recently initiated new activitities in the field of SHG [2]–[7]. Theoretical treatments of CSHG as presented in the literature are among others based on the beam propagation method (BPM) [1], [8], plane wave solutions corresponding to an infinitely long interaction length, or coupled mode theory (CMT) [5], [6], to study the complicated length dependence of the conversion rate. The latter two methods assume an undepleted mode at the fundamental frequency. The CMT for CSHG requires a lengthy and delicate treatment. For this reason the theory presented here has been developed. It allows for a short-hand derivation of the conversion rate, the case of TE-TE conversion is treated; other polarizations may be handled similarly. The approach presented here is based on the solution of the wave equation for the SH field using the Fourier transform as in [9] but the considerations are now much more detailed, which brings deeper physical understanding of the phenomenon.

The paper is organized as follows. In Section II, the theory is given. Section III presents a detailed discussion on the main features of the obtained expression for CSHG. In Section IV, a classification of different types of CSHG is given. In Section V, numerical examples are presented and discussed; the paper ends with conclusions.

II. THEORY

We will consider Čerenkov SH radiation assuming no depletion of the pump beam of a three-layer system in 2-D for

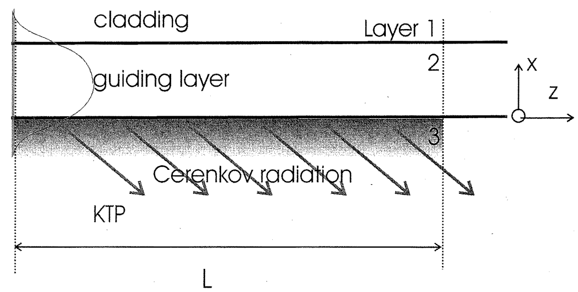

Fig. 1. Schematic picture of the considered three-layer system.

TE polarization (Fig. 1). It is also assumed that only the substrate is nonlinear and that the SH light is radiated mainly into the substrate. The latter will occur according to the presented theory for not too small interaction lengths if the index of the cladding is considerably smaller than that of the nonlinear substrate and than the effective index of the mode at the fundamental frequency. These assumptions are made to ease the discussion somewhat, but are not essential for the phenomena discussed below.

In this paper, time dependencies $\exp(i\omega t)$ and $\exp(i2\omega t)$ for the fundamental and SH beams, respectively, are assumed, but suppressed. The modal field at the fundamental frequency, running along the positive z-axis, is given by

$$E_f(x,z) = e_f(x)\exp(-i\beta_f z) \quad (1)$$

where β_f is the propagation constant. Here and in the rest of the paper parameters related to the fundamental frequency are indicated with a subscript f. The wave-equation for the SH field is

$$(\partial_{xx} + \partial_{zz} + k_0^2 n^2)E_s = -k_0^2 \chi_2 E_f^2 \quad (2)$$

where the term at the right hand side is due to the nonlinear polarization $P_{NL}^{2\omega} = \varepsilon_0 \chi_2 E_f^2$, with χ_2 the nonlinear susceptibility. Here E_f is the field of a guided mode at the fundamental frequency, k_0 $(= 2k_{0,f})$ is the wavenumber of the SH wave, and it is assumed that the nonlinearity is nonzero only in one of the outermost layers, say layer 3. Considering the fundamental field in layer three, we may write

$$E_{f,3}^2 = A_{+,f,3}^2 \exp(2\alpha_{f,3}x - i\beta_s z), 0 < z < L, x < 0 \quad (3)$$

with $A_{+,f,3}$ the amplitude of the fundamental modal field at $x = 0$, $\alpha_{f,3} \equiv \sqrt{\beta_f^2 - k_{0,f}^2 n_{f,3}^2}$, $\beta_s \equiv 2\beta_f$ $(\equiv k_0 N_f \equiv 2k_{0,f}N_f)$,

Manuscript received April 23, 2002; revised July 30, 2002. Part of this work was performed in the framework of project NWO 047-006.014.

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Digital Object Identifier 10.1109/JLT.2003.808658

and N_f is the mode index at ω. Here, the effect of a mode at ω only in the interval $z \in [0, L]$ is considered. Equation (3) may be rewritten in terms of its Fourier transform, B, as

$$E_{f,3}^2 = \int_{-\infty}^{\infty} B(k_z) \exp(-ik_z z) dk_z \frac{\exp(2\alpha_{f,3} x)}{\sqrt{2\pi}} \quad (4)$$

with

$$B(k_z) \equiv A_{+,f,3}^2 L \mathrm{Sinc}\left\{(k_z - \beta_s)\frac{L}{2}\right\} \frac{\exp\left\{i(k_z - \beta_s)\frac{L}{2}\right\}}{\sqrt{2\pi}}. \quad (5)$$

We may now calculate the SH field for each Fourier component corresponding to k_z, whereby we will consider only values $k_z \in [-K, K]$ ($K \equiv k_0 n_3$) (in this paper n_p denotes the index of layer p for the SH frequency), as only these correspond to outgoing plane waves contributing to the SH power.

The following Fourier expansion is used

$$E_s(x,z) = \frac{\int_{-K}^{K} G(k_z, x) \exp(-ik_z z) dk_z}{\sqrt{2\pi}}. \quad (6)$$

Substituting (6) and (4) into (2), and collecting terms with the same k_z, we arrive at

$$(\partial_{xx} - k_z^2 + k_0^2 n^2) G(k_z, x) = [-k_0^2 \chi_2 B(k_z) \exp(2\alpha_{f,3} x)]_{layer3}. \quad (7)$$

The solution of (7) in layer three, can be written as the sum of the homogeneous and the inhomogeneous solutions

$$G(k_z, x) = D_{+,3}(k_z) \exp(ik_{x,3} x) + C(k_z) \exp(2\alpha_{f,3} x). \quad (8)$$

Here, $k_{x,3} \equiv \sqrt{K^2 - k_z^2}$. By substitution (8) into (2) taking (6) into account it follows by equating terms with $\exp(2\alpha_{f,3} x)$ that

$$C(k_z) \equiv \frac{-k_0^2 \chi_2 B(k_z)}{|a|^2}, a \equiv 2\alpha_{f,3} + ik_{x,3}. \quad (9)$$

The field solutions in layer one should correspond to an evanescent or outgoing wave, and the continuity conditions at the interfaces should be fulfilled. As a consequence of this, the following should hold at the interface between layers two and three:

$$\frac{\partial_x G(k_z, x)}{G(k_z, x)} = \frac{ik_{x,3}(r_{31} - 1)}{(r_{31} + 1)}. \quad (10)$$

Here, both the reflection coefficient for a plane wave coming in on layers 3 to 1, r_{31} and $k_{x,3}$ have to be evaluated at k_z. Now requiring continuity of the electric field and its derivative at the interface between layers two and three it follows:

$$D_{+,3}(k_z) = \frac{-(a + r_{31} a^*) C(k_z)}{(2ik_{x,3})}. \quad (11)$$

The outgoing field in layer three can now be expressed as the backward transform

$$E_{+,3} = \frac{\int_{-K}^{K} D_{+,3}(k_z) \exp(-ik_z z + ik_{x,3} x) dk_z}{\sqrt{2\pi}}. \quad (12)$$

The outgoing power in layer three follows from the Poynting vector along $-x$:

$$P_{2\omega}(L) = \int_{-\infty}^{\infty} \frac{\mathrm{Re}\{E_{+,3}(i\partial_x E_{+,3})^*\}}{(2k_0 Z_0)} dz \quad (13)$$

$$= \frac{\int_{-K}^{K} k_{x,3} |D_{+,3}|^2 dk_z}{(2k_0 Z_0)} \quad (14)$$

$$= C_1 L^2 \int_{-K}^{K} \frac{|H|^2 \mathrm{Sinc}^2\left\{(k_z - \beta_s)\frac{L}{2}\right\}}{k_{x,3}} dk_z \quad (15)$$

where $Z_0 \equiv \sqrt{\mu_0/\varepsilon_0}$ and we have used for (14) that $\int_{-\infty}^{\infty} \exp(ikz) dz = 2\pi\delta(k)$. For (15), we have used (5), (9), (11), and

$$C_1 \equiv \frac{A_{+,f,3}^4 k_0^3 \chi_2^2}{(16\pi Z_0 \alpha_{f,3}^2)}, \; H \equiv \frac{\alpha_{f,3}(a + r_{31} a^*)}{|a|^2}. \quad (16)$$

Note that (15) is the exact expression for the generated SH power, except that we have neglected the power radiated into layer one, which may in principle be expressed with a similar formalism, but which is negligible if the index of layer one is much lower than that of layer three and the mode index N_f. In (16), H is chosen to contain the term $a_{f,3}$. As will be shown, $|H|$ remains finite if $\alpha_{f,3} \to 0$, i.e., if the mode at the fundamental frequency approaches cutoff. Note that C_1 is also finite in that case, as follows from standard waveguide theory: if $\alpha_{f,3} \to 0$, $A_{+,f,3}^2/\alpha_{f,3} \to 4P_f Z_0/N_f$, with P_f the modal power.

For numerical evaluation, it is convenient to remove a possible singularity related to $k_{x,3} \to 0$ by changing the integration variable [5] k_z into $k_{x,3}$. Using that $k_{x,3} = \sqrt{K^2 - k_z^2}$ and neglecting the small contribution in (15) for the integral over $k_z \in [-K, 0]$ it follows:

$$P_{2\omega} \approx \frac{C_1 L^2 \int_0^K |H|^2 \mathrm{Sinc}^2\left\{(k_z - \beta_s)\frac{L}{2}\right\} dk_{x,3}}{\beta_s} \quad (17)$$

where k_z takes only positive values, and where we have replaced k_z in the denominator of the integrand by β_s, as only values of $k_z \approx \beta_s$ contributes significantly to the integral (15) in most practical cases, for not too small values of the interaction length, L.

III. Discussion on the Presented Theory

The theory above leading to the final expression (15) is a short hand derivation of [5, eq. (27)]. The latter is identical to (15) if in [5, eq. (27)] the following substitutions are made: d_{eff} by $\chi_2/2$ and k_z by $k_0 N_f$, where the symbols except d_{eff} are in the notation of this paper.

We remark that in a similar way as above expressions can be obtained for any layer system, with a nonlinearity in any of the layers. If more than one layer is nonlinear the resulting nonlinear fields (as given here by $E_{+,3}$ and $D_{+,3}(k_z)$) have to be added for evaluation of the generated power, like in (14). Also the case of z-dependent nonlinear effects, like in periodically poled materials, can be treated by adapting the Fourier transform of $\chi_2 E_f^2$ accordingly.

Although this treatment, leading to (15), differs from that of coupled mode theory the same parameters are found to play a role such as, $1/k_{x,3}$ which is a measure for the density of substrate radiation modes at 2ω and the overlap in the nonlinear region

$$\begin{aligned} H =&\alpha_{f,3} \\ &\cdot\int_{-\infty}^{0} \exp(2\alpha_{f,3}x)\{\exp(-ik_{x,3}x)+r_{31}\exp(ik_{x,3}x)\}dx \\ =&\alpha_{f,3}\frac{\{(2\alpha_{f,3}+ik_{x,3})+r_{31}(2\alpha_{f,3}-ik_{x,3})\}}{(4\alpha_{f,3}^2+k_{x,3}^2)}. \end{aligned} \quad (18)$$

This follows from the definition of H ((16)) by substitution of a defined in (9).

As can be seen from (15) the magnitude and L-dependence of the conversion efficiency depends on all structural parameters. A full discussion on this item would be too involved. Instead, the main features will be discussed; some of these will be illustrated with numerical examples Section V. We consider first H, in Section IV the effect of the interaction length and the value of β_s will be discussed.

As a first step the reflection coefficient, r_{31}, will be rewritten for the region $k_0n_1 < k_z < K$, assuming that $n_1 < n_3 < n_2$. From standard reflection laws it follows:

$$r_{31} = \frac{\{-r_{23}+\exp(i\psi)\}}{\{1-r_{23}\exp(i\psi)\}} \equiv \frac{z_1}{z_1^*} \quad (19)$$

with

$$z_1 \equiv 1-r_{23}+i(1+r_{23})\tan\left(\frac{\psi}{2}\right) \text{ and } \psi \equiv \varphi_{21}-2k_{x,2}t_2 \quad (20)$$

where φ_{21} is the phase shift on reflection at the interface between layer two and one, t_2 is the thickness of layer two, $k_{x,2} \equiv \sqrt{k_0^2n_2^2-k_z^2}$ and r_{23} is the (real) coefficient of reflection for a plane wave incident on the interface between layers two and three

$$r_{23} = \frac{(k_{x,2}-k_{x,3})}{(k_{x,2}+k_{x,3})}. \quad (21)$$

For the following discussion, it is important to note that, using the definition of ψ in (20) and that $r_{23}(k_{x,3}=0)=1$ it follows that

$$\psi(k_{x,3}=0) = -2m\pi, m=0,1,\ldots \quad (22)$$

corresponds to a guided mode at cutoff. Only in that case $r_{31}(k_{x,3}\to 0)=1$, in all other cases $r_{31}(k_{x,3}=0)=-1$. If (22) holds approximately, i.e., for small values of $\tan(\psi/2)$ at $k_{x,3}=0$, r_{31} changes rapidly from -1 at $k_{x,3}=0$ to nearly 1 at $k_{x,3} >\sim 0$. This can be seen as follows. For small values of $k_{x,3}$ it follows from (20)

$$\tan\left(\frac{\psi}{2}\right) \sim T_0 - T_1k_{x,3}^2 \quad (23)$$

with $T_0 \equiv \tan(\psi/2)$ at $k_{x,3}^2=0$, $T_1 \equiv (1/c_1+t_2)/(2c_2)$, $c_1 \equiv \sqrt{K^2-k_0^2n_1^2}$, and $c_2 \equiv \sqrt{k_0^2n_2^2-K^2}$. Now using also a Taylor expansion for $r_{23}(\approx 1-2k_{x,3}/k_{x,2})$ it follows that, if T_0 is small, for $k_{x,3} >> k_{x,2}T_0$ that $|\mathrm{Re}(z_1)| >> |\mathrm{Im}(z_1)|$. As a consequence it follows with (20) that the complex phase of r_{31} approaches 0, and so r_{31} approaches 1. The latter corresponds to a relatively large value of $|H|$, i.e., $|H| <\sim 1$ for $r_{31}\approx 1$ and small values of $k_{x,3}$.

In order to discuss the behavior of $|H|^2$ (18) is rewritten, using (19)–(21) into

$$|H|^2 = h_1h_2 \quad (24)$$

with $h_1 \equiv k_{x,3}^2/\{k_{x,3}^2+k_{x,2}^2\tan^2(\psi/2)\}$ and $h_2 \equiv \alpha_{f,3}^2\{4\alpha_{f,3}+2k_{x,2}\tan(\psi/2)\}^2/(4\alpha_{f,3}^2+k_{x,3}^2)^2$.

Considering (24) the following can be stated.

- In order to have some idea of the maximum attainable by $|H|^2$, the quantify ψ is treated as an independent parameter. Then, $|H|^2$ has a maximum as a function of ψ

$$|H|^2 = \frac{4\alpha_{f,3}^2}{(4\alpha_{f,3}^2+k_{x,3}^2)}, \text{ if } \tan\left(\frac{\psi}{2}\right) = \frac{k_{x,3}^2}{(2\alpha_{f,3}k_{x,2})}. \quad (25)$$

  Note that always $|H|^2 \le 1$, and that $|H|^2=1$ for $k_{x,3}=0$, if $\tan(\psi/2)=0$ for $k_{x,3}=0$.
- Absolute minima may occur as follows:

$$|H|^2 = 0 \text{ if } \tan\left(\frac{\psi}{2}\right) = \frac{-2\alpha_{f,3}}{k_{x,2}}. \quad (26)$$

- At $k_{x,3}^2=0$, $|H|^2=0$, unless [5] $\tan(\psi/2)=0$ and so, $r_{31}=1$, then

$$|H|^2 = 1. \quad (27)$$

- As can be anticipated from the above (see also [5]) $|H|^2$ may take relatively high values if both $k_{x,3}^2$ and $\tan(\psi/2)$ are small compared to $\alpha_{f,3}$. The region around $k_{x,3}^2 >\approx 0$ will be investigated by studying h_1, defined in (24) and neglecting the effect on $|H|^2$ of h_2 (~ 1 for small values of $k_{x,3}$ and $\tan(\psi/2)$), which is a slowly changing function of $k_{x,3}$. Using that $k_{x,2}^2 \equiv k_0^2n_2^2-k_z^2 = c_2^2+k_{x,3}^2$, where the latter equality follows from the definitions of c_2 and $k_{x,3}$ above, and (23) it follows for $k_{x,3}=0$

$$\frac{\partial|H|^2}{\partial k_{x,3}^2} \approx c_3, c_3 \equiv \frac{1}{(c_2^2T_0^2)} \text{ if } T_0 \ne 0 \quad (28)$$

  and

$$\approx -c_4, c_4 \equiv \frac{1}{(c_2^2T_1^2)} \qquad \text{if } T_0 = 0. \quad (29)$$

  The latter corresponds to a guided mode at cutoff and a boundary maximum of $|H|=1$. For small values of T_0 it follows from (28) that $|H|^2$ is rapidly growing as a function of $k_{x,3}^2$, and values of $|H|^2 \sim< 1$ are attained for small values of $k_{x,3}^2$, as discussed below (23).

IV. Classification of CSHG

The function $|H(k_{x,3})|^2$ (or equivalently $|H(k_z)|^2$) and the value of β_s relative to K determines the conversion efficiency and the length dependence thereof. The following cases can be discerned.

A. $(\beta_s - K)L >> 1$

This is the region for conversion to guided SH modes, which will not be considered here. Also CSHG is in principle possible, but in the absence of phase matching the conversion efficiency may be expected to be low. Replacing in (15) k_z in the denominator of the sinc-function by K it follows:

$$P_{2\omega}(L) \approx \frac{4C_1 \int_{-K}^{K} \frac{|H|^2 \sin^2\left\{(\beta_s - k_z)\frac{L}{2}\right\}}{k_{x,3}} dk_z}{(\beta_s - K)^2} \propto C \quad (30)$$

with C a constant. Here we neglected the effect of the sine-function as for large L its period will become much smaller than that of the oscillations in $|H|^2/k_{x,3}$, so that the squared sine function can be replaced by its average value. The above means that for small L, phase-matching is not yet important and some conversion will take place. For large L $P_{2\omega}$ is constant, apparently due to the fact that, on increasing L with a small step, the power of converted light (at k_z-values corresponding to an increase of the sinc-function) is compensated by quenching at other k_z-values. In Section V an example of the above is presented.

B. $(K - \beta_s)L >> 1$

This is the region for the "classical" CSHG; a conversion rate proportional to L can be expected in general. Approximating in a region around $k_z \approx \beta_s$

$$\frac{|H|^2}{k_{x,3}} \approx a_0 + a_1(k_z - \beta_s) + a_2(k_z - \beta_s)^2 \quad (31)$$

it follows, by substituting $y \equiv (k_z - \beta_s)L/2$, from (15) that if $a_0 \neq 0$

$$P_{2\omega}(L) \approx 2C_1 L \int_{-(K+\beta_s)L/2}^{(K-\beta_s)L/2} a_0 \mathrm{Sinc}^2(y) dy \propto L. \quad (32)$$

If $a_0 = 0$ it follows as $|H|^2/k_{x,3} \geq 0$, that also $a_1 = 0$ and that

$$P_{2\omega}(L) \approx 4C_1 \int_{\beta_s - \Delta_1}^{\beta_s + \Delta_2} a_2 \sin^2\left\{(k_z - \beta_s)\frac{L}{2}\right\} dk_z \propto C \quad (33)$$

where $[\beta_s - \Delta_1, \beta_s + \Delta_2]$ is the region where (31) holds with. Equation (33) holds if (26) occurs for $k_z = \beta_s$.

C. $(\beta_s - K)L <\sim 0$

A special case is [5] if both $\beta_s = K$ and $T_0 = 0$ (see (27)). This situation may occur only for one specific λ, t_2 combination in a three-layer system [5] if dispersion of the refractive indices is assumed. Then, using $\beta_s = K$ and substituting $y \equiv (K - k_z)L/2$ and $|H|^2 \approx 1 - c_4 k_{x,3}^2$, according to (29), into (15), also using that $K + k_z \approx 2K$ it follows that [5]:

$$\begin{aligned} P_{2\omega}(L) \approx & 2C_1 L \\ & \cdot \int_0^{KL} \left\{1 - \frac{c_4 4yK}{L}\right\} \sqrt{\frac{L}{(4yK)}} \frac{\sin^2(y)}{y^2} dy \\ \approx & \frac{C_1 L^{1.5}}{\sqrt{K}} \int_0^{\infty} \frac{\sin^2(y)}{(y^2\sqrt{y})} dy = 2C_1 L^{1.5} \frac{\sqrt{\frac{\pi}{K}}}{3} \end{aligned} \quad (34)$$

TABLE I
INDICES AND THICKNESS OF THE STRUCTURE USED IN MOST OF THE EXAMPLES. THE INDICES CORRESPOND TO A WAVELENGTH OF $\lambda_f = 1063.53$ nm. THE INDEX OF THE ZERO-ORDER MODE AT THE FUNDAMENTAL FREQUENCY FOR $t_2 = 476$ nm IS $N_f = 1.887\,176$. THE NONLINEARITY OF THE SUBSTRATE (LAYER 3) IS $\chi_2 = 10$ pm/V

Layer number	n_f	n	Thickness/nm
1, SiO_2	1.4515	1.46042	-
2, Si_3N_4	1.9859	2.0267	476; variable
3, substrate KTP	1.8297	1.8873	-

for not too small values of L. In the above, we have used that the final integral can be rewritten as $\int_{-\infty}^{\infty} \mathrm{Sinc}^2(s^2) ds = 4\sqrt{\pi}/3$ [10]. The peculiar behavior described above is related to the coincidence of the maximum of the sinc^2-function, which narrows for increasing L the zero of the denominator $k_{x,3} = \sqrt{K^2 - k_z^2}$ and the fact that $|H|$ is nonzero for $k_z = K$. For $\beta_s <\sim K$ and $|T_0| >\sim 0$ similar L dependence can be expected as long as both K is close to the maximum of the sinc^2-function, i.e., if

$$\left|(K - \beta_s)\frac{L}{2}\right| << \pi. \quad (35)$$

For larger L a linear length dependence will be found, based on arguments similar to that leading to (32), unless $\beta_s = K$ (and still $|T_0| >\sim 0$). In this case it follows with (15) and $|H|^2 \approx c_3 k_{x,3}^2$ according to (28) and substitution of $y \equiv (K - k_z)L/2$ that

$$P_{2\omega}(L) \approx 4C_1 c_3 \sqrt{LK} \int_0^{KL} \mathrm{Sinc}^2(y)\sqrt{y} dy \approx 4C_1 c_3 \sqrt{\pi K L}. \quad (36)$$

The integral in (36) has been replaced by [10] $2\int_0^{\infty} \sin^2(s^2)/s^2 ds = \sqrt{\pi}$.

Finally, if $|T_0| >> 0$ it follows with the same arguments as above that $P_{2\omega}(L) \propto L$, unless $\beta_s = K$, then $P_{2\omega}(L) \propto \sqrt{L}$.

D. $(\beta_s - K)L >\sim 0$

For relatively small interaction lengths, if (35) holds, conversion rates similar to that corresponding to $\beta_s <\sim K$ can be expected. For larger lengths low conversion rates and a length-dependence according to (30) is anticipated.

From the above it can be concluded that the highest conversion rates can be achieved if a structure is designed such that for a certain wavelength the maxima of the sinc^2-function and $|H|^2$ occur for $k_z = K$, i.e., if both $T_0 = 0$ and $\beta_s = K$. For $\beta_s <\sim K$ and $|T_0| >\sim 0$ almost equally high conversion rates can be found as long as both (35) holds and $|H|^2$ takes a relatively large value at $k_{x,3}$ corresponding to β_s [see also the discussion below (23)].

V. NUMERICAL EXAMPLES AND DISCUSSION

For the computations, the same structure has been used as in [5]; parameters are given in Table I. The structure is excited with the zeroth-order mode at the fundamental wavelength. The parameters are chosen such that (22) holds approximately, i.e., the thickness of the guiding layer is such that the first-order

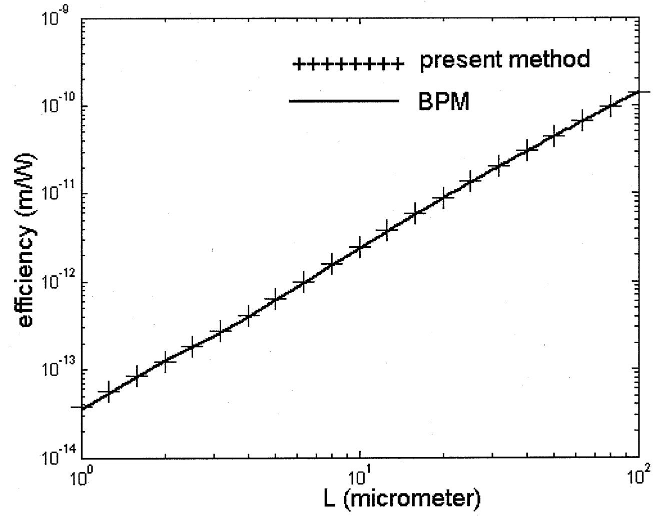

Fig. 2. Comparison of calculations of the conversion efficiency according to the present method with those according to BPM. The parameters are given in Table I.

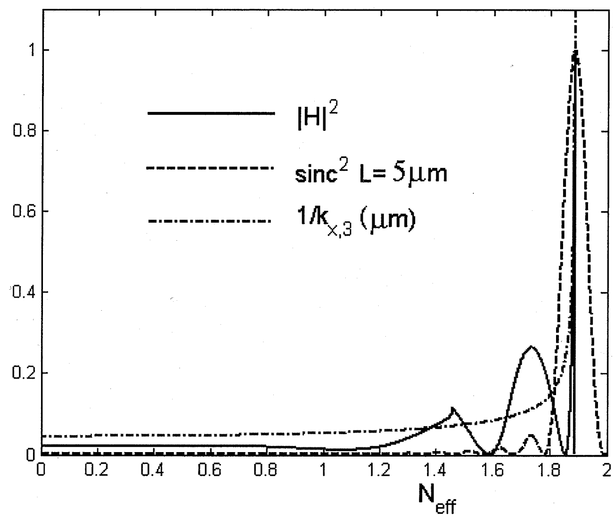

Fig. 3. Calculated values of $1/k_{x,3}$, $|H|^2$ and the sinc^2-function for $L = 5\ \mu$m defined in (15) as a function of N_{eff} ($\equiv k_z/k_0$). The parameters are according to Table I.

mode at the SH wavelength is just below cutoff. Also, it holds that $\beta_s <\sim K$ (or equivalently $N_f <\sim n_3$), which means that the structure will behave as described below (34).

In order to get confidence in the presented method we have compared, in Fig. 2, calculated conversion efficiency, defined by, $\eta \equiv P_{2\omega}/P_f^2$ as a function of the length with that of BPM [8]. For the latter an input power of $P_f = 10^7$ W/m has been chosen, so that depletion can be neglected. The agreement between the two methods is fairly well.

As anticipated from the above the conversion efficiency shows a length-dependence, in the considered region, of approximately $\eta \propto L^{1.5}$. This behavior is explained below (34) and can be understood from (15) and Figs. 3 and 4, both showing the function $|H(N_{eff})|^2$, with $N_{eff} \equiv k_z/k_0$, and the sinc-function, occurring in the integrand of (15). For not too large values of L the difference between n_3 and N_f ($= 1.24 \cdot 10^{-4}$) can be neglected and, also neglecting structure in $|H(N_{eff})|^2$, the above length dependence is expected. Here not too large values of L can be quantified by (35), which leads for the above case to: $L << 2 \cdot 10^3\ \mu$m.

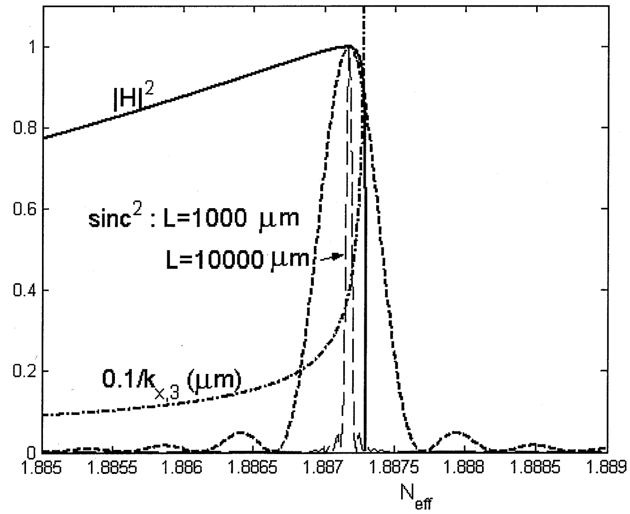

Fig. 4. Detail of Fig. 3, the sinc-functions are evaluated for the indicated lengths.

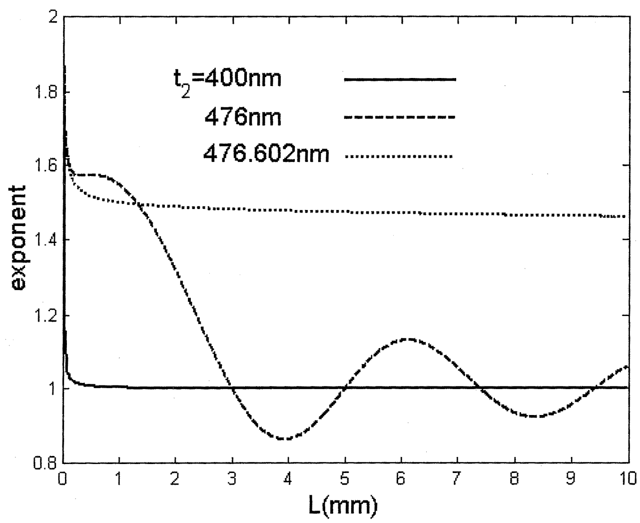

Fig. 5. Numerically evaluated exponent of L occurring in the length dependence of the conversion efficiency, η as a function of L. Parameters, except the thickness of the guiding layer are given in Table I.

Fig. 5 shows the calculated exponent of the length dependence, ϕ, defined by

$$\phi(L_{m+1/2}) \equiv \frac{\ln\left(\frac{\eta_{m+1}}{\eta_m}\right)}{\ln\left(\frac{L_{m+1}}{L_m}\right)} \qquad (37)$$

over a longer range of L. In (37) the subscripts indicate different evaluations of (17), with $L_m = m\Delta L$, and for which a sufficiently small stepsize ΔL has been used. Indeed, the exponent ϕ changes from ~ 1.5 to a lower value at $L \sim 1$ mm, for larger lengths the power seems to converge to a value of $\psi = 1$, as it should.

For comparison Fig. 5 shows also the exponent ϕ for thicknesses $t_2 = 476.602$ nm and 400 nm. For the first case $n_3 - N_f = 26 \cdot 10^{-8}$ and $\phi \sim 1.5$ over a wide range of L. Deviation from $\phi = 1.5$ is attributed to the fact that $|H|^2$ is not a constant for $k_z \sim \beta_s$. The second case corresponds to "classical"

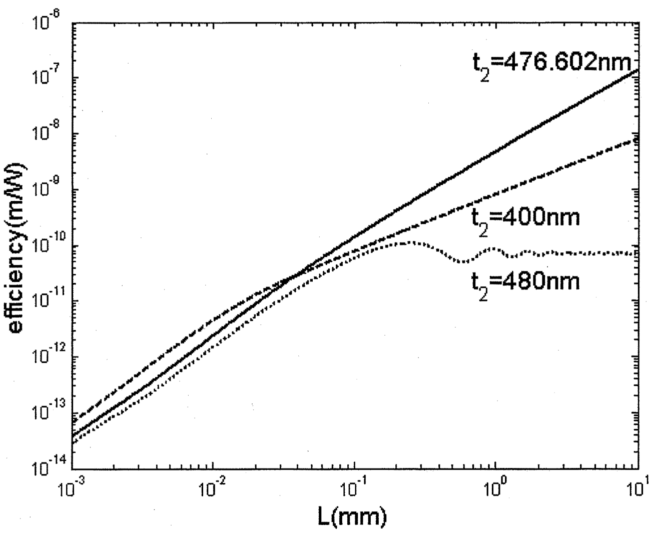

Fig. 6. Calculated conversion efficiencies, $\eta(L)$, according to (17) for the structure given in Table I, for various values of the thickness of the guiding layer.

CSHG, $\beta_s << K$ and a conversion rate $\eta \propto L$ is found for not too small values of L.

In order to show some typical examples Fig. 6 depicts the calculated conversion efficiency for thicknesses of the guiding layer of $t_2 = 400$ nm (dashed line), 476.602 nm (solid line), and 480 nm (dashed line). The first two values of t_2 correspond to "classical" CSHG and CSHG for parameters close to the highest possible efficiency. The difference in conversion rate becomes, as may be anticipated, quite pronounced for larger propagation lengths, $L > 100\ \mu$m. The value $t_2 = 400$ nm corresponds to a situation for which there is no phase matching to the radiation modes, i.e., $N_f = 1.8880 >\sim n_3$, but as argued below (36), a length dependence approximately proportional to $L^{1.5}$ is expected for not too large lengths. For large L a conversion rate according to (30) is observed.

VI. Conclusion

A simple theory leading to an analytical expression for the conversion rate of CSHG in a three-layer planar structure has been given. The theory is based on a Fourier transformation of the source term due to a uniform nonlinearity in the substrate and assumes TE-TE conversion and no depletion of the mode at the fundamental wavelength. Results of the presented method show nice agreement with that of BPM. Various types of CSHG have been classified.

The theory can as well be used for the case of a longitudinally varying nonlinearity as used for quasi phase matching and, with small adaptations, for other polarizations and for multilayer structures.

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