THE EFFECTIVE COORDINATION NUMBER OF SELF-AVOIDING TWO-DIMENSIONAL RANDOM WALKS

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It is demonstrated that the effective coordination number of self-avoiding random walks in a plane is given by $\exp(-\pi^2/24)=0.66283$.

The total number of self-avoiding random flights of N steps of equal lengths in a plane, which start in the origin of coordinates, can be written in the form $Q_N=A\mu^N N^{-a}$ provided $N\gg 1$. Most research has been directed towards the calculation of the exponent a, which is believed to be universal, i.e. independent of the details of the model (apart from the dimension of space). In contrast with a the value of the effective coordination number μ *does* depend on details of the model. As μ is nevertheless the basic measure for the attrition of a random walk due to the constraint of self-avoidingness an analytical calculation of μ for *some* model is still of interest. In this note we present arguments which indicate that such a calculation is possible for continuous, two-dimensional, self-avoiding random flights, with the result $\mu=\exp(-\pi^2/24)$.

For this model it was shown in ref. [1] that the constraint that a configuration should be self-avoiding is automatically taking into account it one counts all random flights (not necessarily self-avoiding ones) provided they are given complex weights

$$W(C)=\exp[\tfrac{1}{2}i\phi(C)+i\pi n(C)]. \quad (1)$$

Here $\phi(C)$ denotes the total accumulated increase in the angle which the tangent to the random flight C makes with the positive x-axis, and $n(C)$ denotes the total number of times that C crosses a certain branchline T which is specified in detail in ref. [1]. We therefore have to take the following two steps:

(a) One needs to know the probability density $p_N(x, y, \phi)$ that a random flight of N steps, which started at the origin, will reach x, y with a total angle of rotation equal to ϕ. This problem has now been studied in considerable detail by various authors [2–4]; all one needs to know is the asymptotic form of p_N for $N\gg 1$

$$p_N(x, y, \phi)=(N\pi l^2)^{-1}\,(\tfrac{2}{3}N\pi^3)^{-1/2} \times\exp\left(-\frac{x^2+y^2}{Nl^2}-\frac{3\phi^2}{2N\pi^2}\right),\quad N\gg 1. \quad (2)$$

(b) The second term $i\pi n(C)$ in the exponential of (1) simply means that the complex weight W acquires a factor -1 each time the random flight crosses the branchline T. The effect of these factors -1 on the total weight of all plane random flights has been studied in detail in refs. [5,6] for a somewhat simplified form of the branchline. It is shown there that these factors lead to a depletion of the total weight which has an N-dependence $\sim N^{-1/4}$, hence they do not change the effective coordination number.

Combination of the results quoted under (a) and (b) shows that the total number of self-avoiding plane random flights of $N\gg 1$ steps has the asymptotic form

$$Q_N=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\int_{-\infty}^{\infty}d\phi\, p_N(x, y, \phi)\exp(\tfrac{1}{2}i\phi) =\exp(-N\pi^2/24). \quad (3)$$

As the total number of free plane random walks was normalized to unity this implies an effective coordination number equal to $\exp(-\pi^2/24)=0.66283$.

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