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Coupled mode theory for resonant excitation of waveguiding structures

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Abstract. Resonant coupling of light beams via high-index media or gratings to planar waveguiding structures are of interest for both applications and from a theoretical point of view. Coupled Mode theory (CMT) can give an accurate description of the coupling process in terms of relatively simple expressions involving often a large number of coupling parameters. In this paper it is shown, using time reversal and energy conservation how these parameters are interrelated. The evaluation of the remaining independent parameters is shown to be possible using a few reflection and transmission coefficients for incoming plane waves, including in the calculations, if present, the effect of the grating. Further, it is proved that under certain condition a grating coupler may show exactly 100% reflection. Analytical expressions for the reflected and transmitted beams and the amplitude distribution of the excited mode are given for the case of incoming Gaussian beams. A few applications of the theory and considerations on its applicability are presented.

Key words: coupled mode theory, coupler, filter, grating, prism coupler, resonance, waveguide

1. Introduction

Resonant coupling of light into waveguiding structures has potentially many applications and has attracted the attention of many researchers (Wood 1902; Hessel and Oliner 1965; Ulrich 1970; Peng *et al.* 1975; Avrutsky *et al.* 1989; Bertoni *et al.* 1989; Rosenblatt *et al.* 1997). Due to the usually high reflection coefficient of light propagating in a waveguiding layer such structures can be used as high finesse filters, polarizers and optical switches. Besides this, coupling of light by gratings or high-index media offer the possibility to excite waveguides, leading to large increase of intensity which could be used for nonlinear effects.

In most cases such structures can be described quite accurately by coupled mode theory (CMT). The advantage of such a parametric description is that the effect of such a resonant structure as part of a device can be evaluated easily. This paper concentrates on CMT for three different types of 2D structures near resonance. Here the coupling occurs either via high-index media, being the two outermost layers or by symmetric gratings, which are resonant at off-normal incidence, or at normal incidence. These three different cases have been chosen due to the large similarity in the theoretical treatment on solving the coupled mode equations (CMEs). The advantage of the presented method is that it is not based on perturbation theory (Rosenblatt *et al.* 1997), which holds only for weak effects, but uses a few exact field solutions of the corresponding wave equation to calculate the CMT parameters.

In Section II it is discussed how the large number of coupling parameters occurring in the CMEs are interrelated, leading to a considerable reduction in the number of independent parameters. Here use is made of time reversal, energy conservation and the properties of the scattering matrix (Haus 1982). It is also discussed how these parameters can be evaluated for a given structure by using the outcome of a few field calculations on the considered structure. The CMT used here leads to simple equations for reflected and transmitted beams and the modal power distribution belonging to arbitrarily shaped incoming beams. For incoming Gaussian beams it is shown that these quantities can be expressed analytically. Examples of applications of the theory are given in Section III. The paper ends with concluding remarks, Section IV. In Appendix A, a ray picture is used to discuss the validity of the CMT for resonant coupling via high index media. In Appendix B, a ray picture is used to describe a grating coupler. Using general arguments it is shown that for some situations grating couplers show 100% reflection, thus demonstrating the applicability of such a device as a frequency and polarization selective end-face for a laser.

2. Theory

We will illustrate the theory by considering three typical situations for resonant coupling to planar waveguiding structures. A time dependence $\exp(i\omega t)$, corresponding to the considered wavelength, is assumed throughout this paper. In the examples we assume TE polarization, for which one has to solve the wave-equation (see Fig. 1):

$$\{\partial_{xx} + \partial_{zz} + k_0^2 n^2(x, z)\} E_y(x, z) = 0. \tag{1}$$

Here k_0 is the wavenumber in vacuum, and the index n depends on z if gratings are involved. The scheme outlined below is similar for the case of TM.

2.1. COUPLING VIA HIGH-INDEX MEDIA

We will first consider a waveguiding structure which is leaky due to the presence of high-index top and bottom layers (see Fig. 1). Such a structure may be considered as an extension of the prism coupler, where coupling can

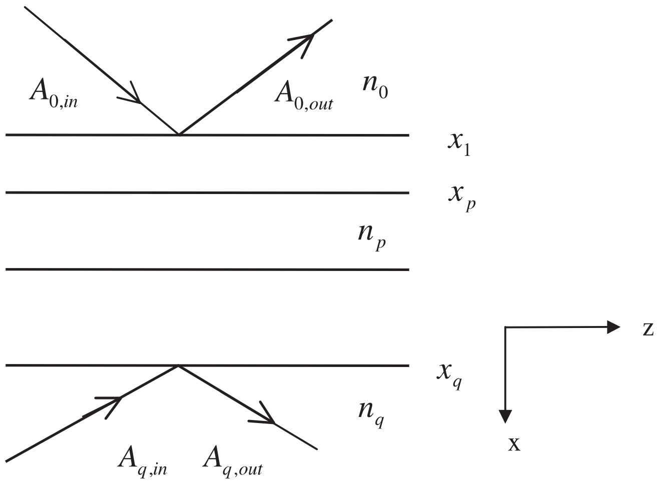

Fig. 1. Geometry of a structure for resonant excitation of leaky modes via high-index media.

occur from one side (Ulrich 1970). It is assumed that the coupling between the external fields (in layers 0 and q) and the internal field, having a shape $\phi(x)$, with $x_1 < x < x_q$ is small. In Appendix A, it is made plausible that then the shape of $\phi(x)$ depends only weakly on the propagation constant of incoming waves, if these are close to that for resonant excitation, and also that this shape depends weakly on the way of excitation (from top or bottom layers, layers 0 and q, respectively). It is also made plausible that for weak coupling the shape $\phi(x)$ may be chosen real. These assumptions, which are typical for CMT, are in general not valid. They are approximately valid in the case of weak coupling, where the high index layers introduce only a small perturbation on the guiding structure (layers 1 to $q-1$). So, CMT does in general not predict the excited field exactly, but, as the reflection and transmission coefficients, as calculated with CMT, are quite accurate for incoming fields close to resonance and for not too strong coupling, it can be argued that the power transport in the inner section may be predicted quite well by CMT.

With the above assumptions the y-component of the electric field is defined by:

$$E_r(x,z) = A_r(z)\phi(x)\exp(-\mathrm{i}\,\beta_{\mathrm{res}}z) \quad x_1 < x < x_q, \tag{2}$$

$$E_0(x_1,z) = \{A_{0,\mathrm{in}}(z) + A_{0,\mathrm{out}}(z)\}C_0 \exp(-\mathrm{i}\beta_{\mathrm{res}}z), \tag{3}$$

$$E_q(x_q,z) = \{A_{q,\mathrm{in}}(z) + A_{q,\mathrm{out}}(z)\}C_q \exp(-\mathrm{i}\beta_{\mathrm{res}}z). \tag{4}$$

In the above $\phi(x)$ is defined in such a way that $|A_r|^2$ is the power (in W/m) in the inner section, with A_r the slowly varying envelope (SVE) belonging to the field propagating in the inner section. The normalization constants $C_{0/q}(\equiv [2\omega\mu_0/(k_0^2 n_{0/q}^2 - \beta_{\rm res}^2)^{1/2}]^{1/2})$ are such that $|A_{0/q,{\rm in/out}}|^2$ represents the incoming/outgoing intensity (in W/m^2) in the 0th/qth layer. The value of $\beta_{\rm res}$is chosen to be the propagation constant of the plane wave that leads to maximum excitation of the inner section.

Then, using e.g. a ray picture, similar to (Ulroch 1970), the CMEs can be formulated:

$$\partial_z A_r = -\kappa A_r + c_1 A_{0,{\rm in}} + c_2 A_{q,{\rm in}}, \tag{5}$$

$$A_{0,{\rm out}} = c_3 A_r + c_4 A_{0,{\rm in}} + c_5 A_{q,{\rm in}}, \tag{6}$$

$$A_{q,{\rm out}} = c_6 A_r + c_7 A_{0,{\rm in}} + c_8 A_{q,{\rm in}}. \tag{7}$$

So, 9 complex unknown parameters ($\beta_{\rm res}, \kappa, c_{1\text{–}8}$) are involved in the CMEs and our task will consist in both finding out how these unknowns are interrelated, and next how these can be calculated for a given structure.

We consider first the case that there are no incoming fields in the interval $z \in [0, \infty)$. Then, using Equations (5–7) it follows:

$$A_r(z) = A_r(0)\exp(-\kappa z), \tag{8}$$

$$A_{0,{\rm out}}(z) = c_3 A_r(z) \text{ and } A_{q,{\rm out}}(z) = c_6 A_r(z). \tag{9}$$

The above corresponds to a leaky mode with only outgoing radiation and with a complex propagation constant $\beta_{\rm leaky} = \beta_{\rm res} - {\rm i}\kappa$. So, using a (standard) solver for leaky modes (Smith *et al.* 1992) the two latter parameters can be found. Note that also a leaky mode can be found with $\beta_{\rm leaky} = \beta_{\rm res} + {\rm i}\kappa$, corresponding to a situation with no outgoing fields.

Using Equations (8) and (9) and energy conservation by comparing the outgoing and incoming powers, i.e.,

$$|A_r(0)|^2 = \int_0^\infty |A_{0,{\rm out}}(z)|^2 + |A_{q,{\rm out}}(z)|^2 \, {\rm d}z$$

it follows with (6) and (7) that

$$|c_3|^2 + |c_6|^2 = 2\kappa. \tag{10}$$

Before considering time reversal we remark that due to symmetry of the structure the CMEs for fields moving to the left can be found from Equations (5)–(7) by replacing the set $\{z, A_r, A_{0/q,{\rm in/out}}\}$ by $\{-z, A_l, A^l_{0/q,{\rm in/out}}\}$, here the subscript and superscript, l, denotes that the amplitudes correspond to a modal field moving to the left. So, the CMEs for this case are:

$$-\partial_z A_l = -\kappa A_l + c_1 A^l_{0,\text{in}} + c_2 A^l_{q,\text{in}}, \tag{11}$$

$$A^l_{0,\text{out}} = c_3 A_l + c_4 A^l_{0,\text{in}} + c_5 A^l_{q,\text{in}}, \tag{12}$$

$$A^l_{q,\text{out}} = c_6 A_l + c_7 A^l_{0,\text{in}} + c_8 A^l_{q,\text{in}}, \tag{13}$$

with

$$E_l(x,z) = A_l(z)\phi(x)\exp(\mathrm{i}\,\beta_{\text{res}} z) \quad x_1 < x < x_q, \tag{14}$$

$$E^l_0(x_1,z) = \{A^l_{0,\text{in}}(z) + A^l_{0,\text{out}}(z)\}C_0 \exp(\mathrm{i}\,\beta_{\text{res}} z), \tag{15}$$

$$E^l_q(x_q,z) = \{A^l_{q,\text{in}}(z) + A^l_{q,\text{out}}(z)\}C_q \exp(\mathrm{i}\,\beta_{\text{res}} z). \tag{16}$$

As we have to do with real indices, the complex conjugate of the field solutions (14–16) correspond to time reversal of the solutions (2–4), and vice versa (Haus 1982). So, using $E_l = E_r^*$ and also that $\phi(x)$ has been chosen real, for the solutions (8) and (9) it follows that

$$A_l(z) = A_r^*(0)\exp(-\kappa z), \tag{17}$$

$$A^l_{0/q,\text{in}}(z) = c^*_{3/6} A^*_r(z), \tag{18}$$

and

$$A^l_{0/q,\text{out}} = 0 \tag{19}$$

should be solutions of Equations (11)–(13). After substitution of Equations (17)–(19) into Equations (11)–(13) the following helpful equalities can be obtained:

$$c_1 c_3^* + c_2 c_6^* = 2\kappa, \tag{20}$$

$$c_3 + c_4 c_3^* + c_5 c_6^* = 0, \tag{21a}$$

$$c_6 + c_7 c_3^* + c_8 c_6^* = 0. \tag{21b}$$

Next we consider the situation of incoming plane waves having a propagation constant β, with $\Delta\beta \equiv \beta - \beta_{\text{res}}$, and a corresponding solution to Equation (5) of the form:

$$A_r(z) = A_r(0)\exp(-\mathrm{i}\Delta\beta z). \tag{22}$$

Then, using Equation (5) it follows:

$$A_r(z) = \{c_1 A_{0,\text{in}} + c_2 A_{q,\text{in}}\}/B, \quad B \equiv \kappa - \text{i}\Delta\beta. \tag{23}$$

Substituting Equation (23) into Equations (6) and (7) leads to

$$\mathbf{A}_{\text{out}} = \mathbf{S}\mathbf{A}_{\text{in}}, \tag{24}$$

where

$$\mathbf{A}_{\text{in/out}} \equiv \begin{pmatrix} A_{0,\text{in/out}} \\ A_{q,\text{in/out}} \end{pmatrix}, \tag{25}$$

and $\mathbf{S}$ is the scattering matrix, with

$$\begin{aligned} S_{11} &\equiv c_1 c_3/B + c_4; \quad S_{12} \equiv c_2 c_3/B + c_5; \\ S_{21} &\equiv c_1 c_6/B + c_7; \quad S_{22} \equiv c_2 c_6/B + c_8. \end{aligned} \tag{26}$$

Assuming small values of $\Delta\beta$ the following should hold for the scattering matrix (Haus 1982):

$$\mathbf{S} = \mathbf{S}^{\text{t}} = \mathbf{S}^{*-1}. \tag{27}$$

Here the superscripts t and $*$ indicate the transpose and the complex conjugate, respectively.

Based on Equation (27) with Equation (26), also using Equations (10), (20) and (21) the following new relations can be obtained:

$$c_5 = c_7, \tag{28}$$

$$c_1 = c_3, \quad c_2 = c_6, \tag{29}$$

and

$$|c_4|^2 + |c_5|^2 = |c_5|^2 + |c_8|^2 = 1, \tag{30}$$

Here, Equation (28) follows directly from symmetry of the scattering matrix. The first equality (29) can be proved by substituting c_4, expressed using Equation (21a), into $|c_1 c_3|^2 + |c_1 c_6|^2 + 2\kappa(c_1 c_3 c_4^* + c_1 c_6 c_5^*) = 0$, which follows from $|S_{11}|^2 + |S_{21}|^2 = 1$ for all $\Delta\beta$. Next, substituting 2κ, from Equation (20), and $c_6 (= c_2 c_3/c_1)$, following from Equation (27), the first equality (29) follows. The second equality can be found in a similar way. Equation (30) can be derived directly from Equation (27).

So, for a given structure of the kind considered the parameters of the CMEs are all defined by the values of $\beta_{\mathrm{res}}, \kappa, c_1, c_2$ and c_4. The others follow from these, on using Equations (21) and (28)–(30).

The parameter β_{leaky} for such a structure can be calculated by solving the wave equation for the leaky mode, or by calculating the reflection coefficient, say S_{11}, for three different values of β close to the dip in $|S_{11}|$ and to solve for κ and β_{res}. In particular for grating couplers (see below) this strategy is found to be more straightforward.

Next, considering reflection and transmission of plane waves, for two values of $\Delta\beta$, coming in from, say, layer 0, leads to the values of c_1^2, c_4 and $c_1 c_2$ by solving two equations with two unknowns. Note that there is an ambiguity in the choice of the sign of c_1 (and so of c_2). This is related to the choice of the sign for the modal field shape, $\phi(x)$. If one is only interested in the modal power distribution this sign can be chosen freely. If one is also interested in the field distribution, as calculated with CMT, the shape $\phi(x)$ should be defined. With a mode solver the inner field, E_r can be found corresponding to, e.g., an incoming field from layer 0 at resonance. Then, using Equations (2) and (5) it follows

$$\phi \approx \mathrm{Re}(\psi), \tag{31}$$

with $\psi \equiv E_r(x, z = 0)\kappa/(c_1 A_{0,\mathrm{in}})$ and the equality $\mathrm{Im}(\psi) \approx 0$ should hold according to the assumptions made above.

We remark that from the above the corresponding relations for the prism coupler (i.e., in the presence of only one high-index layer for resonant excitation) can be found by assuming an infinite thickness for one of the low index layers between the guiding layer, p, and layer q. Then: $c_2, c_5, c_6 \to 0$ and $|c_8| \to 1$ which leads to $|c_1|^2 = 2\kappa$ and $c_4 = -c_1^2/(2\kappa)$.

2.2. GRATING COUPLER, SINGLE MODE EXCITATION

We consider a symmetric grating, and choose the origin such that the structure is invariant for inversion of the z-axis (see Fig. 2). With this assumption we can use the symmetry considerations given above leading to relatively simple CMEs. We further assume that only a single mode is excited with considerable amplitude, or that excitation of other modes can be neglected in the CMT picture.

The relations between the various amplitudes and the y-component of the electric fields are:

$$E_r(x, z) = A_r(z)\phi(x)\exp(-\mathrm{i}\,\beta_{\mathrm{res}} z), \tag{32}$$

$$E_{\mathrm{rad}}(x_1, z) = \{A_{0,\mathrm{in}}(z) + A_{0,\mathrm{out}}(z)\}C_0 \exp\{-\mathrm{i}(\beta_{\mathrm{res}} - K)z\}, \tag{33}$$

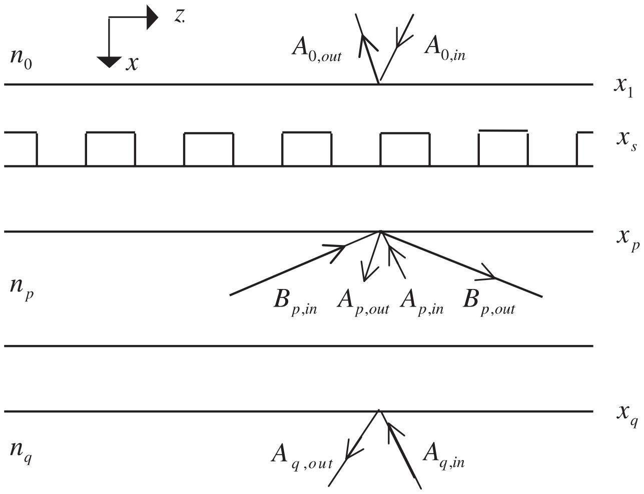

Fig. 2. Geometry of a structure for resonant excitation of leaky modes by a grating.

$$E_{\rm rad}(x_q, z) = \{A_{q,{\rm in}}(z) + A_{q,{\rm out}}(z)\}C_q \exp\{-{\rm i}\,(\beta_{\rm res} - K)z\}. \tag{34}$$

Here $C_{0/q} \equiv \sqrt{2\omega\mu_0/|\alpha_{0/q,{\rm rad}}|}$ and K is the wavenumber of the grating. The CMEs can be shown to be of the same form as Equations (5)–(7), by using, e.g., a ray-picture or Greens function theory (Rosenblatt *et al.* 1997). Next using the same procedure as in Section 2.1 the relations (10), (21) and (28)–(30) can be derived. The coupling constants can be determined for a given structure by comparing these relations with the outcome of calculations of the reflection coefficients for a few angles of incidence near resonance, similarly as discussed at the end of Section 2.1. As is shown in the Appendix B, the considered structure may have the property of full reflection of an incoming plane wave for certain angle-wavelength combinations. For the propagation constant corresponding to full reflection, say at an offset $\Delta\beta_1$, the extra relation $\kappa - {\rm i}\,\Delta\beta_1 = -c_1c_2/c_5$ can be obtained with Equation (26).

For practical applications of such grating couplers it is of interest to be able to calculate the amplitude distributions for arbitrarily shaped incoming beams. In most cases this has to be done numerically, but in the case of incoming Gaussian beams analytical expressions can be found as described below. We consider an incoming Gaussian beam with a central propagation constant differing $\Delta\beta_c$ from $\beta_{\rm res}$. The amplitude of the field is given by:

$$A_{0,{\rm in}}(z) = \sqrt{\sqrt{(2/\pi)}/w}\,\exp(-{\rm i}\,\Delta\beta_c z - z^2/w^2) \tag{35}$$

$$= a_1 \exp(-{\rm i}\,\Delta\beta_c z)\int {\rm d}\,k_z \exp(-{\rm i}\,k_z z - k_z^2 w^2/4), \tag{36}$$

where Equation (36) shows the distribution among the Fourier components, with $a_1 \equiv [w/\sqrt{2\pi}]^{1/2}$. The reflected amplitude distribution can now be calculated using Equations (26) and (29):

$$A_{0,\text{out}} = c_4 A_{0,\text{in}} + c_1^2 a_1 \int \mathrm{d}k_z \exp(-\mathrm{i}\,\Delta\beta_c z - \mathrm{i}\,k_z z - k_z^2 w^2/4)/\{\kappa - \mathrm{i}(k_z + \Delta\beta_c)\}$$
$$= c_4 A_{0,\text{in}} + c_1^2 \sqrt{w\sqrt{\pi/8}} \exp(-\kappa z + a_2^2)\{\text{erf}\,(z/w + a_2) + 1\}, \qquad (37)$$

where erf is the complex error function and $a_2 \equiv (-\kappa + \mathrm{i}\,\Delta\beta)w/2$. For numerical accuracy the error function should be evaluated by considering $\exp(a^2)\text{erf}\,(a + ib) = \exp(a^2)[\{1 - \text{erf}\,(-a)\} + p(a,b)]$, with a and b real, using the function erfcx of Matlab (The Math Works) for the first part, and Equation [7.1.29] of (Abramowitz and Stegun) for the part containing the polynomial, $p(a,b)$.

Similarly as above expressions for the transmitted amplitude and modal amplitude distributions can be obtained.

2.3. GRATING COUPLER AT NEAR-NORMAL INCIDENCE

If resonance occurs near normal incidence modes running to the right and the left are excited. The CMEs can be formulated quite generally as:

$$\partial_z A_r = -\kappa A_r - \mathrm{i}\,(\Delta\beta_\lambda + \gamma_1)A_r + (\gamma_0 + \mathrm{i}\,\gamma_1')A_l + c_1 A_{0,\text{in}} + c_2 A_{q,\text{in}}, \qquad (38)$$

$$\partial_z A_l = \kappa A_l + \mathrm{i}\,(\Delta\beta_\lambda + \gamma_1)A_l - (\gamma_0 + \mathrm{i}\,\gamma_1')A_r - c_1 A_{0,\text{in}} - c_2 A_{q,\text{in}}, \qquad (39)$$

$$A_{0,\text{out}} = c_3(A_r + A_l) + c_4 A_{0,\text{in}} + c_5 A_{q,\text{in}}, \qquad (40)$$

$$A_{q,\text{out}} = c_6(A_r + A_l) + c_7 A_{0,\text{in}} + c_8 A_{q,\text{in}}. \qquad (41)$$

Here we have used that the grating is symmetric with respect to inversion of the z-axis and, as a consequence, that the equations remain unchanged on simultaneously replacing $\{z, r, l\}$ by $\{-z, l, r\}$. The parameters γ_0, γ_1 and γ_1' have been introduced for generality. The amplitudes in Equations (38)–(41) are related to the electric field as follows:

$$E_r(x,z) = A_r(z)\phi(x)\exp(-\mathrm{i}\,Kz), \qquad (42)$$

$$E_l(x,z) = A_l(z)\phi(x)\exp(\mathrm{i}\,Kz), \qquad (43)$$

$$E_{\text{rad}}(x_1,z) = \{A_{0,\text{in}}(z) + A_{0,\text{out}}(z)\}C_0, \qquad (44)$$

$$E_{\text{rad}}(x_q,z) = \{A_{q,\text{in}}(z) + A_{q,\text{out}}(z)\}C_q, \qquad (45)$$

with $C_{0/q} \equiv \sqrt{2\omega\mu_0/(k_0 n_{0/q})}$. Here K is the wavenumber of the grating. In the CMEs $\Delta\beta_\lambda$ is the shift in the modal propagation constant due to changes in the wavelength with respect to that for resonant coupling, say λ_{res}. We will assume $\Delta\beta_\lambda = \partial\beta/\partial k_0 \Delta k_0$, where Δk_0 is the shift in wavenumber.

We assume for the moment plane waves at normal incidence in order to take full advantage of the symmetry. For this situation it follows that $A_r = A_l$ and that $\partial_z A_{r,l} = 0$, due to Equations (42) and (43) and Floquet's theorem. Using this for Equations (38) and (39) it follows that with the choice

$$\gamma_1' = \gamma_1 \tag{46}$$

$$A_{r,l} = (c_1 A_{0,\mathrm{in}} + c_2 A_{q,\mathrm{in}})/(\kappa - \gamma_0 + \mathrm{i}\,\Delta\beta_\lambda) \tag{47}$$

resonance (i.e., maximum modal amplitudes) occurs, as desired, for $\Delta\beta_\lambda = 0$, corresponding to a wavelength, say, λ_{res}.

In order to derive relations between the coupling parameters we consider a situation with no incoming fields. Then:

$$\partial_z A_{r/l} = -/+ (\alpha A_{r/l} - \gamma A_{l/r}) \tag{48}$$

$$A_{0/q,\mathrm{out}} = c_{3/6}(A_r + A_l), \tag{49}$$

with

$$\alpha \equiv \kappa + \mathrm{i}\,(\Delta\beta_\lambda + \gamma_1) \text{ and } \gamma \equiv \gamma_0 + \mathrm{i}\,\gamma_1. \tag{50}$$

The amplitudes corresponding to the reciprocal situation (no outgoing fields) are:

$$A_l^{\mathrm{rec}} = A_r^*;\ A_r^{\mathrm{rec}} = A_l^*;\ A_{0,\mathrm{in}}^{\mathrm{rec}} = c_3^*(A_r^* + A_l^*);\ A_{q,\mathrm{in}}^{\mathrm{rec}} = c_6^*(A_r^* + A_l^*). \tag{51}$$

Substitution into Equations (38)–(41) leads to:

$$\partial_z A_{r/l}^{\mathrm{rec}} = -/+ \{\alpha A_{r/l}^{\mathrm{rec}} - \gamma A_{l/r}^{\mathrm{rec}} - (c_1 c_3^* + c_2 c_6^*)(A_r^{\mathrm{rec}} + A_l^{\mathrm{rec}})\} \tag{52}$$

$$0 = (c_3 + c_4 c_3^* + c_5 c_6^*)(A_r^{\mathrm{rec}} + A_l^{\mathrm{rec}}) \tag{53}$$

$$0 = (c_6 + c_7 c_3^* + c_8 c_6^*)(A_r^{\mathrm{rec}} + A_l^{\mathrm{rec}}). \tag{54}$$

Now, taking the conjugate of the amplitudes, also using Equation (51), it follows:

$$\partial_z A_{l/r} = -/+ \{\alpha^* A_{l/r} - \gamma^* A_{r/l} - (c_1^* c_3 + c_2^* c_6)(A_r + A_l)\}. \tag{55}$$

Comparing Equation (55) to Equation (38) and (39) leads to:

$$\gamma_0 = -\kappa, \tag{56}$$

and

$$c_1^* c_3 + c_2^* c_6 = 2\kappa. \tag{57}$$

From Equations (53) and (54) it follows:

$$c_3 + c_4 c_3^* + c_5 c_6^* = c_6 + c_7 c_3^* + c_8 c_6^* = 0. \tag{58}$$

Another helpful equation follows by using the requirement of energy conservation for one of the solutions of Equation (38)–(41), assuming no incoming fields, which can be calculated to be:

$$A_r = \exp(-wz); \; A_l = v \exp(-wz) \tag{59}$$

with

$$w \equiv \sqrt{\alpha^2 - \gamma^2}; \quad \mathrm{Re}(w) > 0 \tag{60}$$

and

$$v \equiv (\alpha + w)/\gamma = \gamma/(\alpha - w). \tag{61}$$

Now considering the energy balance by comparing the input and output power in the interval, say, $[0, \infty)$ it follows, also using Equations (40) and (41):

$$1 - |v|^2 = (|c_3|^2 + |c_6|^2)|1 + v|^2/\{2 \text{ Re } (w)\}. \tag{62}$$

From Equation (62) one may prove:

$$|c_3|^2 + |c_6|^2 = 2\kappa, \tag{63}$$

by performing (e.g.) the following steps: substitute v using the first equality (61) into Equation (62), substitute v using the second equality, remove the nominators in both equations and subtract the two equations after expressing v, w, α and γ in terms of κ, γ_1 and $\Delta\beta_\lambda$.

The above can be used to simplify the expressions for the elements of the scattering matrix, **S** , assuming normal incidence. Using Equations (38)–(41), (46), (47) and (56) it follows:

$$S_{11} = 2c_1c_3/d + c_4; \quad S_{12} = 2c_2c_3/d + c_5;$$
$$S_{21} = 2c_1c_6/d + c_7; \quad S_{22} = 2c_2c_6/d + c_8. \tag{64}$$

Here $d \equiv 2\kappa + \mathrm{i}\,\Delta\beta_\lambda$. Using the properties of the scattering matrix, and the Equations (57), (58) and (63) the relations (27)–(29) can also be derived for the grating coupling near normal incidence.

Below we will discuss how to evaluate the CMT parameters for a given structure. First we consider the reflection and transmission of a plane wave coming in from layer 0, with a propagation constant $\Delta\beta$, i.e., $A_{0,\mathrm{in}} = \exp(-\mathrm{i}\,\Delta\beta z)$. Assuming the same z-dependence for the modal amplitudes it follows from Equations (38) and (39):

$$A_{r/l} = c_1 \exp(-\mathrm{i}\,\Delta\beta z)(a +/- \mathrm{i}\,\Delta\beta)/(b^2 + \Delta\beta^2), \tag{65}$$

with $a \equiv \mathrm{i}\,(\Delta\beta_\lambda + 2\gamma_1)$ and $b^2 \equiv a(2\kappa + \mathrm{i}\,\Delta\beta_\lambda)$.

With Equations (65), (40) and (41) CMT expressions for the reflection and transmission coefficients can be obtained. Using these and also Equations (64), (47) and (56) for incoming plane waves at angles and wavelengths near resonance sufficient relations can be obtained to evaluate the CMT parameters. Hereby, one should as mentioned before, compare the CMT expressions with the results of field calculations.

Next, we will derive expressions for the amplitude distributions corresponding to the case of incoming Gaussian beams. Using Equation (65) for such an incoming Gaussian beam, with

$$A_{0,\mathrm{in}} = \sqrt{\sqrt{2/\pi}/w}\exp(-\mathrm{i}\,\Delta\beta_c z - z^2/w^2), \tag{66}$$

where $\Delta\beta_c$ is the central propagation constant, leads to

$$A_{r/l} = c_1\sqrt{w\sqrt{\pi/8}}\left\{-\left(\frac{a}{2b} -/+ \frac{1}{2}\right)\exp\left(bz + b_+^2\right)\left\{\mathrm{erf}\left(\frac{z}{w} + b_+\right) - 1\right\} + \left(\frac{a}{2b} +/- \frac{1}{2}\right)\exp\left(-bz + b_-^2\right)\left\{\mathrm{erf}\left(\frac{z}{w} - b_-\right) + 1\right\}\right\}. \tag{67}$$

Here $\mathrm{Re}(b) > 0$ and $b_\pm \equiv (b \pm \mathrm{i}\,\Delta\beta_c)w/2$. The error functions can be evaluated as described above, also using that $\mathrm{erf}\,(x) = -\mathrm{erf}\,(-x)$ (Abramowitz and Stegun). The reflected and transmitted amplitude distributions can now be found from:

$$A_{0/q,\mathrm{out}} = c_{1/2}(A_r + A_l) + c_{4/5}A_{0,\mathrm{in}}. \tag{68}$$

3. Numerical results

We will illustrate the presented method with a few examples for each type of the structures considered above (see Table 1). Once it is known how the different coupling parameters are interrelated the evaluation of such a structure is simple. For the field calculations on the structures with a grating we have used a Fourier expansion of the index in the grating layer, like in (Rosenblatt *et al.* 1997). A sufficiently large number of Floquet modes have been taken into account to calculate the reflection and transmission coefficients of incoming plane waves. The higher order Floquet modes, which are evanescent along x, could quite conveniently be included by using the immitance matrix function of x, according to (Ctyroky *et al.* 1998). For other methods to deal with grating structures the reader is referred to (Tamir and Zhang 1995) for (more or less) rectangularly shaped gratings, or to the transformation given in (Chandezon *et al.* 1982) for multicoated, smoothly varying structures.

For structures 1 and 2, S_{11} has been calculated for three values of β close to the extreme value of $|S_{11}|$, to solve for κ, and β_{res}. Next, the parameters c_1^2, c_4, c_1c_2 and c_5 have been calculated from S_{11} and S_{12} for two values of the relevant variable. The other parameters follow from Equations (28), (29) and (21b). For structure 3 first λ_{res}, $(\partial\beta/\partial k_0)/\kappa$ and γ_1 have been evaluated using Equations (47) and (65). Next a similar procedure as for structures 1 and 2 has been followed, but with $\lambda(\approx\lambda_{\text{res}})$ as a variable instead of β, and considering plane waves at normal incidence (see Table 2). As a measure of the validity of the applied CMT we introduce a parameter, T, which should be equal to 0 in the limit of weak coupling according to Equation (21a)

$$T \equiv |(c_1 + c_4c_1^* + c_5c_6^*)/c_1|. \tag{69}$$

Table 1. Parameters for the considered structures. As unit of length μm has been used. Structures 2 and 3 correspond to Si_3N_4 on KTP

	Structure 1		Structure 2		Structure 3	
	Thickness	Index	Thickness	Index	Thickness	Index
Layer number						
0	∞/2	1.58	∞/2	1	∞/2	1
1	1; var	1.5	0.5; var	grating	0.5	grating
2	1	1.6	0.47571	1.98595	0.47571	1.98595
3	1; var	1.5	∞/2	1.82951	∞/2	1.82951
4	∞/2	1.58				
Grating period	–		0.5		0.5623	
Wavelength	1		1.063		–	

The annotation 'var' indicates that in some of the presented examples the corresponding thickness has been varied. The gratings are rectangular, with a duty cycle of 0.5, and consisting of the materials of the two adjacent layers.

Table 2. CM parameters for the considered structures, obtained as described in the text

	Structure 1	Structure 2	Structure 3
c_1	0.021086 exp(−i0.3694)	0.0969 exp(i0.7738)	0.0872 exp(i0.6027)
c_2	0.021086 exp(−i0.3694)	0.0962 exp(−i2.1258)	0.0801 exp(−i2.1889)
c_4	0.9985 exp(i2.3967)	0.2488 exp(−i2.8852)	0.2962 exp(−i2.9389)
κ	4.453×10^{-4}	9.319×10^{-3}	7.008×10^{-3}
β_{res}	9.86158	11.1619	–
λ_{res}	–	–	1.0605
γ_1	–	–	0.0180
T	3.0×10^{-4}	6.1×10^{-5}	2.1×10^{-5}
$\partial\beta/\partial k$	–	–	1.8788

As unit of length μm has been used. The relatively small values of the test parameter, T, is an indication that the condition of weak coupling holds for the considered structures.

In Fig. 3 results for structure 1 are given. The values for κ agree with that obtained with a solver for leaky modes with relative errors of 0.5% for $t(\equiv t_1 = t_3) = 0.3\,\mu\text{m}$ to $<10^{-5}$ for $t > 0.8\,\mu\text{m}$. It can be seen from Fig. 3 that the requirement $\kappa W \ll 1$, where W is the distance along z for one zigzag of the light in the guiding layer, including the Goos-Hänchen shift (Hoekstra *et al.* 1993), no longer holds for $t(\equiv t_1 = t_3) < 0.5\,\mu\text{m}$. So, as argued in Appendix A in this region the presented CMT is less accurate, as also can be seen from the quantity T. A similar conclusion follows from Fig. 4. Here, both the real ($\equiv \phi(x)$) and imaginary parts of the field ψ, defined in Section 2.1, are given. Im(ψ), which should be equal to 0, starts to deviate from this

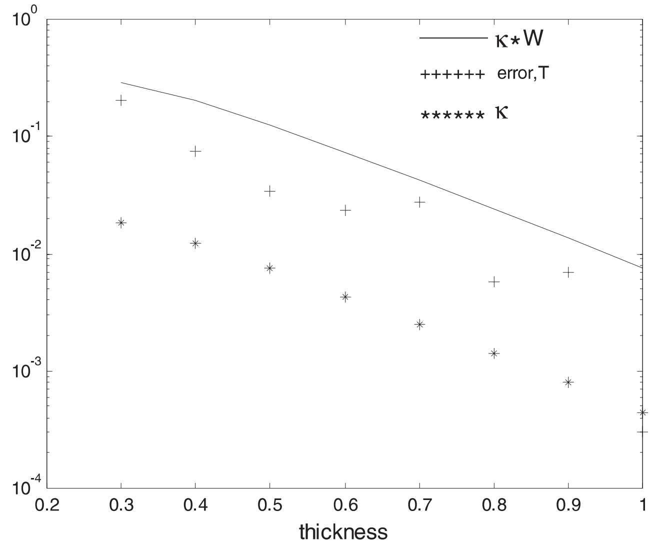

Fig. 3. Some relevant parameters for structure 1 as a function of $t \equiv t_1 = t_3$, where the subscripts indicate the layer number. κ is given in μm^{-1}.

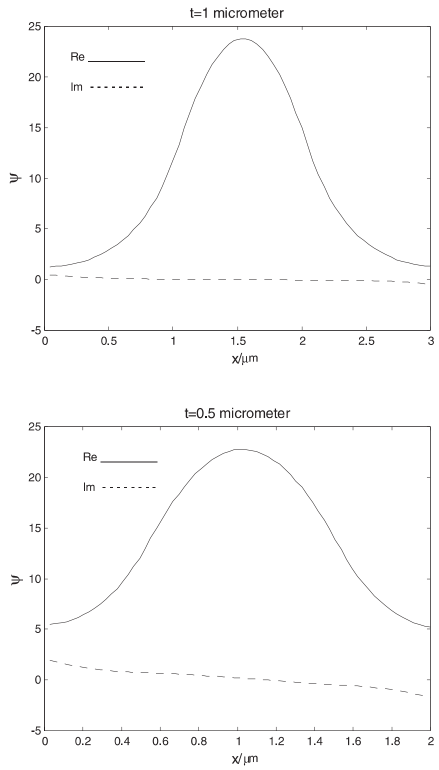

Fig. 4. Real and imaginary parts of the field ψ (in $V/\sqrt{Wm}$), defined in Section 2.1, for structure 1, for different values of $t \equiv t_1 = t_3$.

value for $t < 0.5\,\mu\text{m}$. The error in the CMT results can be estimated by considering the quantity

$$\int_{x_p}^{x_{p+1}} \phi^2 \mathrm{d}x \beta_{\mathrm{res}}/(2\omega\,\mu_0) \tag{70}$$

which should be equal to unity as can be deduced from Section 2.1. For the parameters considered in Fig. 4 we find a deviation in this quantity of 0.25% and 3.9% for $t(\equiv t_1 = t_3) = 1$ and $0.5\,\mu\text{m}$, respectively. This, and also the results in Fig. 5, confirms that the CMT is only approximately valid for strong coupling.

Similar conclusions may be shown to hold for the considered grating couplers. Here, we will show only a few computational results to illustrate the presented method. In Fig. 6 the reflection curves, $|r|$, as a function of the effective index of a plane wave, coming in form layer 0 are given for different thicknesses of the grating layer of structure 2. The agreement between the results of CMT and that of modal field calculations is fairly well. Fig. 7

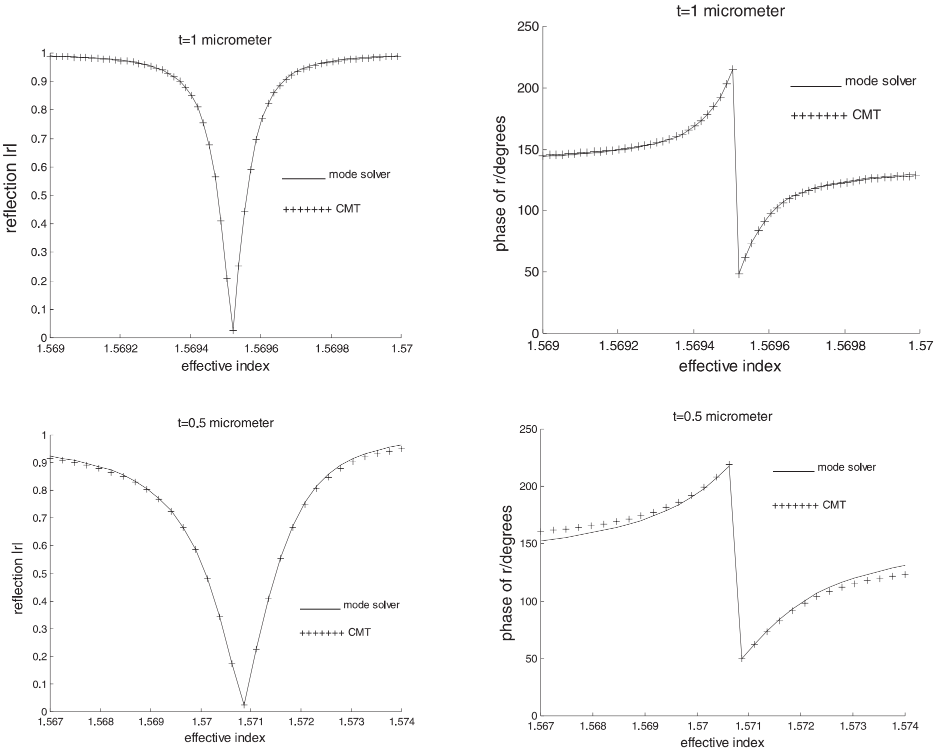

Fig. 5. A comparison between exact calculations and CMT results for the reflection coefficient of structure 1.

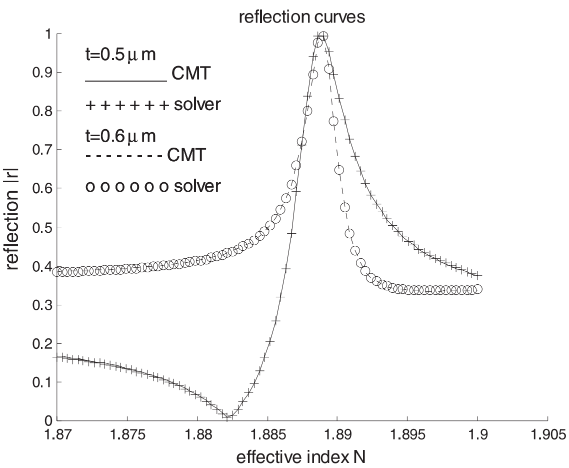

Fig. 6. Reflection curves, $|r|(\equiv |S_{11}|)$, for structure 2 (see Table 1), except that here different values of the grating layer thickness, t_1. The effective index, N, indicates index of the excited field.

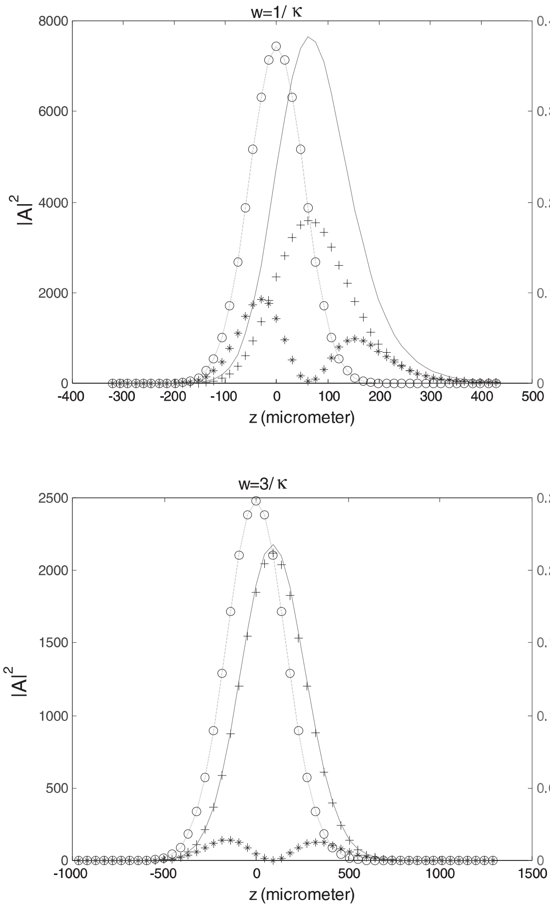

Fig. 7. Amplitude distribution, $|A_{0,\mathrm{in}}|^2$ (○○○), $|A_{0,\mathrm{out}}|^2$ (+++), $|A_{q,\mathrm{out}}|^2$ (∗∗∗), left scale (in W/m^2) and $|A_r|^2$ (solid line), right scale (in W/m) for structure 2, corresponding to incoming Gaussian beams (see text, Equation (37) with a power of 1 W/m. The reflected intensities are: 66% (top) and 93% (bottom).

shows, for structure 2, results for an incoming Gaussian beam, with a power of 1 W/m, at an angle of incidence corresponding to maximum reflection, i.e., (see Equation (26)):

$$\Delta\beta_c = \mathrm{Im}(c_1 c_2 / c_5), \tag{71}$$

corresponding to zero transmission.

Finally, in Fig. 8 the various relevant amplitude distributions are given, for structure 3, for incoming Gaussian beams, each with a power of 1 W/m, at normal incidence for a wavelength corresponding to maximum reflection, i.e., (see Equation (64)):

$$\Delta\beta_\lambda = -\mathrm{Im}(2c_1 c_2 / c_5). \tag{72}$$

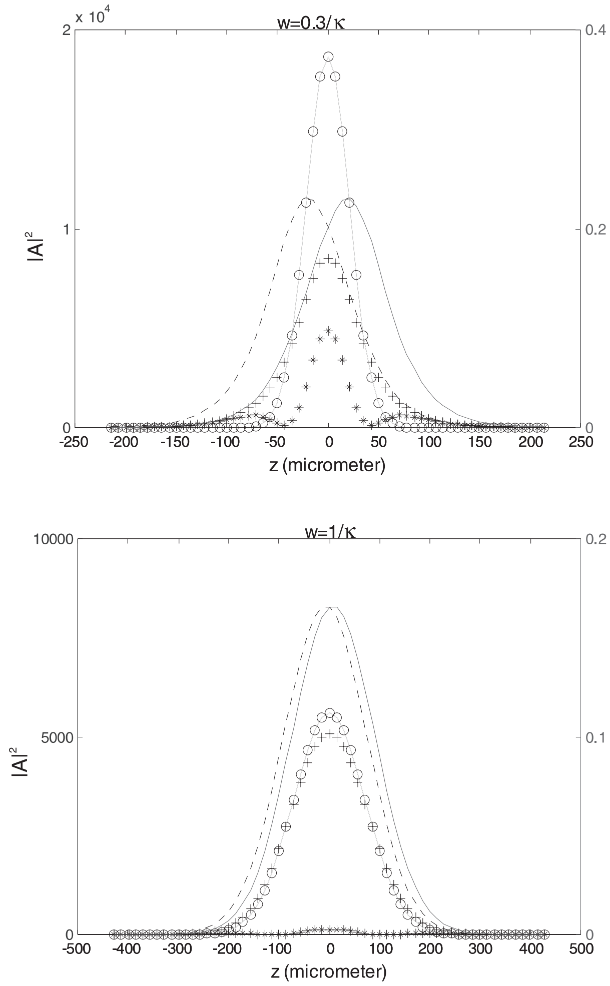

Fig. 8. Amplitude distribution $|A_{0,\mathrm{in}}|^2$ (○○○), $|A_{0,\mathrm{out}}|^2$ (+++), $|A_{q,\mathrm{out}}|^2$ (∗∗∗), left scale (in W/m^2) and $|A_r|^2$ (solid line), $|A_l|^2$ (dashed line), right scale (in W/m) for structure 3, corresponding to incoming Gaussian beams at normal incidence, and a wavelength according to Equation 72 with a power of 1 W/m. The reflected intensities are: 67% (top) and 96.7% (bottom).

4. Concluding remarks

We have presented an accurate method to find out how the coupling coefficients, occurring in the CMEs for a few resonant structures, are interrelated, and to evaluate these coefficients from a few field calculations. The CMT, as presented here, is useful for the following reasons:

- the evaluation of a given structure is quite simple as only a few reflection and transmission coefficients are required to find all the relevant parameters,

- in the case of incoming Gaussian beams the various amplitude distributions can be expressed analytically, in terms of the error function,
- with the CMT it is quite easy to take into account all kind of effects, like 2nd and 3rd order nonlinearities, scattering, gratings of finite length, and
- the structure may be fully characterized by a few parameters, which is quite convenient for design purposes in order to be able to predict the effect of such a resonant waveguiding structure as a part of a device.

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Appendix A

In order to discuss the validity of the CMT, as presented in Section 2.1, to a certain extent, we consider a structure of the type treated there, with a high-index layer, p, in which the field becomes resonantly enhanced on excitation. Using Fresnel reflection and transmission laws the field in this layer, assuming incoming plane waves with the same propagation constant ($\beta = \beta_{\rm res} + \Delta\beta$), is given by:

$$E_p = \{C_0 A_{0,\rm in} t_{0p} g_{p0}(x) + C_q A_{q,\rm in} t_{qp} g_{pq}(x)\} \exp(-\mathrm{i}\beta_{\rm res} z - \mathrm{i}\varphi/2)/N, \qquad \text{(A1)}$$

with

$$g_{p0} \equiv \exp(-\mathrm{i}k_x x) + r_{pq}\exp(\mathrm{i}k_x x - \mathrm{i}\varphi),$$
$$g_{pq} \equiv \exp(\mathrm{i}k_x x) + r_{p0}\exp(-\mathrm{i}k_x x - \mathrm{i}\varphi),$$
$$N \equiv 1 - r_{pq} r_{p0}\exp(-2\mathrm{i}\varphi),$$

and

$$\varphi \equiv k_0 n_p \cos(\theta_p) t_p \equiv k_x t_p.$$

Here $x = 0$ in the middle of layer p, and r and t are Fresnel reflection and transmission coefficients, respectively.

We will limit the discussion to layer p, for neighbouring layers similar arguments as below could be used to make plausible that

$$E_p \cong \text{const. } A_r \phi \exp(-\mathrm{i}\beta_{\text{res}} z), \tag{A2}$$

where $\phi(x)$ can be chosen real, and that in agreement with Equation (5)

$$A_r \cong (c_1 A_{0,\text{in}} + c_2 A_{\text{q,in}})/(\kappa - \mathrm{i}\Delta\beta). \tag{A3}$$

Using arguments similar as given in (Hoekstra *et al.* 1993) it follows that:

$$N \equiv 1 - \exp(-\kappa W + \mathrm{i}\Delta\beta W) \cong \kappa W - \mathrm{i}\Delta\beta W, \tag{A4}$$

for weak coupling and close to resonance, i.e., if

$$\kappa W \ll 1 \tag{A5}$$

and

$$|\Delta\beta W| \ll 1. \tag{A6}$$

In the above W is the distance along z needed for one zigzag of the light in layer p, including the Goos-Hänchen shift (Hoekstra *et al.* 1993).

As a consequence of Equations (A4) and (A5) $|r_{p0}| \cong 1$ and so, the phase of g_{p0} depends only weakly on x. Using similar arguments it can be shown that this also is true for g_{pq}. The functions g_{p0} and g_{pq} have approximately the same shape. This can be seen by writing $r_{p0} \cong \exp(2\mathrm{i}\phi_{p0})$ and $r_{pq} \cong \exp(2\mathrm{i}\phi_{pq})$ and using the transverse resonance condition:

$$\varphi - \phi_{p0} - \phi_{pq} = m\pi, \quad m = 0, 1, \ldots, \tag{A7}$$

leading to:

$$g_{p0}, g_{pq} \cong \text{const.} \cos(k_x x - \varphi/2 + \phi_{pq}). \tag{A8}$$

So, using Equations (A5) and (A6) and also that all quantities in Equation (A1), except N, depend only weakly on β close to resonance, it follows that both Equations (A2) and (A3) hold approximately.

Appendix B

Here we will show that the considered resonant grating couplers can show exactly 100% reflection for a specific incoming plane wave close to resonance. The discussion is based on the ray picture, and will also show the similarity between this picture and the CMT. The considered structure is given in Fig. 2, and it is assumed that there is a high-index guiding layer (layer p)

below the grating structure. First we will discuss oblique incidence at such an angle that there are only two running waves in layer p (see Fig. 2). So, the other Floquet modes are assumed to be evanescent there.

Splitting the total structure into two parts, part I with layers $(1-p)$ and part II with layers $(p-q)$ the scattering matrix for both systems can be defined. For system I we write:

$$\mathbf{A}_{I,\mathrm{out}} = \mathbf{S}_I \mathbf{A}_{I,\mathrm{in}} \tag{B1}$$

with

$$\mathbf{A}_{I,\mathrm{in/out}} \equiv \begin{pmatrix} A_{0,\mathrm{in/out}} \\ A_{p,\mathrm{in/out}} \\ B_{p,\mathrm{in/out}} \end{pmatrix} \text{ and } \mathbf{S}_I \equiv \begin{pmatrix} r_{0p} & \tau_{p0} & T_{p0} \\ \tau_{p0} & r_{p0} & R_{p0} \\ T_{p0} & R_{p0} & r_{g,p0} \end{pmatrix}, \tag{B2}$$

and for system II

$$\mathbf{A}_{II,\mathrm{out}} = \mathbf{S}_{II} \mathbf{A}_{II,\mathrm{in}},$$

with

$$A_{II,\mathrm{in/out}} \equiv \begin{pmatrix} A'_{p,\mathrm{in/out}} \\ A_{q,\mathrm{in/out}} \\ B_{p,\mathrm{in/out}} \end{pmatrix} \text{ and } \mathbf{S}_{II} \equiv \begin{pmatrix} r_{pq} & \tau_{qp} & 0 \\ \tau_{qp} & r_{qp} & 0 \\ 0 & 0 & r_{g,qp} \end{pmatrix}. \tag{B3}$$

Here all amplitudes are defined such that the squared absolute value is the intensity in, say, W/m^2. The elements r denote the reflection coefficients for the electric field and the elements τ are related to the transmission coefficients for the electric field, t, as follows, e.g.,

$$\tau_{p0} = t_{p0}\sqrt{\alpha_0/\alpha_p}.$$

The elements T and R denote the interaction between free and (nearly) guided wave, i.e., waves with propagation constant $(\beta - K)$ and β, respectively. The amplitudes A and B belong to free and guided waves, respectively. The prime in Equation (B3) indicates that the corresponding amplitudes are evaluated in layer p, next to layer $p+1$.

The two systems considered above can be combined to the system discussed in Section 2.2. On joining these two systems it is assumed that the Floquet modes with an evanescent tail into layer p (and so the elements of the matrix in Equation (B2)) are hardly affected. Assuming a thickness t_p for layer p, the following relations hold:

$$A'_{p,\mathrm{in/out}} = \exp(-/+\mathrm{i}\varphi)A_{p,\mathrm{out/in}} \text{ and } B'_{p,\mathrm{in/out}} = \exp(-/+\mathrm{i}\varphi_g)B_{p,\mathrm{out/in}}, \quad \text{(B4)}$$

with $\varphi_{(g)} \equiv k_0 n_p t_p \cos\theta_{(g),p}$, where $\theta_{(g),p}$ is the angle with the x-axis of the rays of the free (guided) wave in layer p.

Before deriving expressions for the combined system we first note that

$$\mathbf{S}_I\mathbf{S}_I^* = \mathbf{I}. \quad \text{(B5)}$$

This general property for the scattering matrix leads, in this case, to 6 equations involving inner products of the rows. Rather straightforward manipulations of these equations lead to:

$$|(\tau_{p0}r_{g,p0} - T_{p0}R_{p0})/\tau_{p0}(\equiv \exp\ \mathrm{i}\theta_1)| = 1. \quad \text{(B6)}$$

We will now consider the combined system (system I and II) for the situation that there is a single incoming wave, in layer 0, so

$$A_{0,\mathrm{in}} \neq 0,\ A_{q,\mathrm{in}} = 0. \quad \text{(B7)}$$

In order to prove that there may be full reflection for a certain angle of incidence it is sufficient to prove that $A_{q,\mathrm{out}} = 0$, or equivalently, see Equations (B3) and (B7) that $A_{p,\mathrm{out}} = 0$, as

$$A_{q,\mathrm{out}} = \tau_{qp}\exp(-\mathrm{i}\varphi)A_{p,\mathrm{out}}. \quad \text{(B8)}$$

By substitution of Equations (B3) and (B4) into (B2) three equations are obtained containing $A_{0,\mathrm{in}}$ and the three unknowns $A_{0,\mathrm{out}}, A_{p,\mathrm{out}}$ and $B_{p,\mathrm{out}}$. Solving for the two latter amplitudes leads to:

$$A_{p,\mathrm{out}} = (D/N)A_{0,\mathrm{in}}, \quad \text{(B9)}$$

with

$$D \equiv \tau_{p0}F_g + T_{p0}r_{g,pg}R_{p0}\exp(-2\mathrm{i}\varphi_g), \quad \text{(B10)}$$

$$N \equiv F_gF - r_{g,pq}r_{pq}R_{p0}^2\exp\{-2\mathrm{i}(\varphi + \varphi_g)\}, \quad \text{(B11)}$$

and

$$F_{(g)} \equiv \{1 - r_{(g),pq}r_{(g),p0}\exp(-2\mathrm{i}\varphi_{(g)})\}, \quad \text{(B12)}$$

and

$$B_{p,\mathrm{out}} = (D_g/N)A_{0,\mathrm{in}}, \quad \text{(B13)}$$

with

$$D_g \equiv T_{p0}F + \tau_{p0}R_{p0}r_{pq}\exp(-2i\varphi). \tag{B14}$$

Using Equations (B6), (B10) and (B12) it follows that

$$D = \tau_{p0}\{1 - r_{g,pq}\exp(i\theta_1 - 2i\varphi_g)\}, \tag{B15}$$

from which it follows, also using $|r_{g,pq}| = 1$, that on varying the thickness t_p and so φ_g a value $D = 0$ can be found. From Equation (B10) it can be seen that in the case of weak grating coupling (small values of R_{p0} and T_{p0}) this happens close to the thickness corresponding to the minimum value of $|F_g|$. So, using general arguments we have shown that conditions can be found for zero transmission and 100% reflection. Similar proofs, using however simplifying assumptions, can be found in (Rosenblatt *et al.* 1997; Sychugov and Tishchenko 1993).

Similar arguments as above can be used to explain the total reflection at normal incidence. The same system I as above is considered. A plane wave at normal incidence, and a given wavelength, will excite two running waves in layer p with equal amplitudes $B^r_{p,\text{out}}$ and $B^l_{p,\text{out}}$, and propagation constants $\pm K$. Also due to the symmetry of the grating is the following: if $B^r_{p,\text{in}} = -B^l_{p,\text{in}}$ there will be no outgoing waves along the normal. This suggests the transformation:

$$\begin{pmatrix} B_+ \\ B_- \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} B^r_p \\ B^l_p \end{pmatrix}, \tag{B16}$$

where B_- is completely decoupled from the waves A_0 and A_p.

So, replacing in Equation (B2) etc. B_p by B_+ the same arguments as above can be used to demonstrate the 100% reflection for a certain thickness, t_p.

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