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History The golden age of mathematics

Stevin, Huygens and the Dutch republic

Mathematics played its role in the rise of the Dutch Republic. Simon Stevin and Christiaan Huygens were key figures in this. Though from very different backgrounds, both gave shape to Dutch culture. Fokko Jan Dijksterhuis, who recently received an NWO-vidi grant for his research on the history of mathematics, discusses their influence on mathematics and its historical significance.

The 17th century was the golden age of the Dutch republic. A small state lacking substantial natural resources, from the 1580s onwards it quickly became a wealthy trade centre that played a leading part in international politics. It produced cultural tour de forces in the arts, sciences and technology. Mathematics is represented by Simon Stevin (1548–1620) and Christiaan Huygens (1629–1695), spanning the main part of the golden age.

Figure 1 Christiaan Huygens, pastel drawing Bernard Vaillant, 1686. Collectie Huygensmuseum Hofwijck, Voorburg

Recent developments in cultural history have opened new perspectives on the mathematical pursuits of the Dutch republic.[1] Understanding the brilliant ideas of Stevin and Huygens in their historical context promises a new and deeper insight into their achievements. Stevin and Huygens lived at different stages of the golden age and their mathematics likewise differed. Stevin witnessed the birth of the Dutch republic and played an active role in its formation. Half a century later, the wealth, the power and the splendour of the republic had been established (and had even begun to wane) with Huygens as its main mathematical representative. The hands-on mathematics of Stevin differed greatly from Huygens' aristocratic geometry and yet, as will be argued here, they had essential features in common. In this article, the work of Stevin and Huygens will be sketched as part of, and giving shape to, the golden age.

Figure 2 Simon Stevin, Universiteitsbibliotheek Leiden

Mixed mathematics

A history of mathematics in the golden age begins with the realisation that early modern mathematics was an endeavour quite different from its present form. Viewed with our modern conception of mathematics, our main heroes Stevin and Huygens instantly disappear from the history of mathematics. Their contributions to mathematics per se were minor (Huygens) or virtually zero (Stevin). More importantly, by trying to single out their mathematical achievements we lose sight of the structure of their work and consequently their ingenuity. The first thing we need to realise is that the idea of pure mathematics dates from the early nineteenth century. Prior to Lagrange's and Cauchy's rationalist purification of mathematics, the whole range of mathematical sciences (geometry, arithmetic, astronomy, music, optics, etc.) were seen as *parts* of mathematics. The idea of an a priori given pure mathematics applied to physical domains simply did not exist. In early modern conceptions 'mathematica pura' was distinguished from 'mathematica mixta', indicating the level of abstraction of the object of study.[2] Geometry and arithmetic studied quantity as subsisting in itself; astronomy, music and optics studied quantity as subsisting in matter. The mixed parts were not separated, let alone subordinate to the pure parts of geometry and arithmetic: mathematics consisted of the study of reality in its quantitative aspect. From this historical point of view, Stevin and Huygens were full-blown mathematicians, and brilliant at that.

The beginning of the Dutch Golden age

When the northern Netherlands in 1579 revolted against Spanish rule and liberated themselves, a 50-year exodus of capital, intellect and skills from the southern Netherlands began. Although his motives are unclear, Stevin moved too and he turns up in Leiden in 1581, where he enrolled at the university in 1583. He had been born in Brugge in 1548, illegitimate son from a well-to-do family, and had worked as a bookkeeper in Antwerp and at the Brugge tax office.[3] Although little is known of his scholarly and practical education, the documented activities in his early years in Holland show that he was both literate and skilled, and that he applied himself to the whole spectrum of mathematics. During the 1580s, he published a range of books in Dutch, French and Latin, applied for patents for the construction of mills and performed experiments. These activities on the one hand reflected his previous experiences and on the other hand opened up new areas of interest, areas that fitted particularly well with his new environment. During these early Leiden years Stevin also developed relationships with notable Dutchmen, for example the future Delft burgomaster Johan Cornets de Groot (1554–1640) with whom he collaborated on mill projects, and the future stadholder Maurits van Nassau (1567–1625) who studied at Leiden university at the same time. In short, we see a Flemish exile in his thirties who energetically and quite successfully pursued his interests in matters mathematical.

Simon Stevin

The publications that reflected Stevin's previous occupations were *Nieuwe Inventie van rekeninghe van compaignie* (New invention of business calculation, 1581) and *Tafelen van Interest* (Tables of Interest, 1582).[4] *De Thiende* (The Dime, 1585) also built on his bookkeeping experience but had a much wider scope. It was an inspired plea for the use of decimal fractions in all calculating professions: "stargazers, surveyors, measurers of carpet, of wine, of bodies in general, cashiers, and all merchants". With works like these Stevin contributed to the growing amount of calculating in all kinds of professions. He did so by proposing specific methods and elucidating their character and application — to the point of disclosing professional secrets. Twenty years later Stevin got the opportunity to put his ideas about sophisticated bookkeeping into practice when he became administrator of Maurits' domains. He was given permission to introduce Italian double entry bookkeeping. In *Vorstelicke bouckhouding* (Royal bookkeeping, 1608) he described and explained his ideas and methods.

Stevin has achieved lasting fame with *De Beghinselen der Weeghconst* (Elements of the Art of Weighing, 1586), appended with *De Weeghdaet* (The Act of Weighing) and *De Beghinselen des Waterwichts* (The Elements of Waterweight). These publications reflect the interests of his new country, of machines and of water management. Stevin employed his findings in new mill designs, for which he got patents and which he realised in a couple of places — albeit not fully satisfactorily.[5] At the same time, however, these works reflected an international — most notably Italian — interest in the mathematics of mechanisms and the Renaissance of Greek mathematics. In particular the rediscovery of Archimedes had sparked a lively interest in this particularly 'tactile' mathematics and methods of analysis.

Stevin contributed to this Archimedean renaissance with an inventive inquiry into statics and hydrostatics that extended well beyond the original of Archimedes. The 'clootcrans' (wreath of spheres) is the most famous example. He proved by reductio ad absurdum that two bodies on an inclined plane are in balance when their weights are proportional to the length of the planes. Imagine a wreath of equal spheres at equal distances that can move freely. The number of spheres is proportional to the length of the sides of the triangle. If the spheres on the one side would *not* keep those at the other side in balance, the wreath would begin to move. However, it would soon reach the same state and thus a perpetual motion would occur. Because a perpetuum mobile is impossible, the supposition is false, and thus the spheres must be in balance. Stevin was so proud of this proof that he used it as the frontispiece of the *Weeghconst*.

With the *Weeghconst* Stevin expressly presented himself as a man of learning, displaying his knowledge of mathematical literature and elaborating his account in geometric fashion by formulating principles, drawing up thought-experiments and deducing propositions. He did so, however, in an idiosyncratic manner. He explicitly combined his Archimedean analyses with practical pursuits. The titles of his books reflect his views: the *elements* of weighing was appended with the *act* of weighing, and the elements of waterweighing contained a commencement of the act of waterweighing. He called these two aspects 'spiegheling en daet', reflection and action.[6] Both were requisite: just like a foundation without a building is futile "so the reflection on the principles of the arts is lost labour where the end does not tend to the action".[7] In all his work Stevin consequently combined practical work and literary pursuits. In bookkeeping or mill building he did not content himself with getting the paperwork right and putting machines to work. He also committed his opinions and inquiries to paper, learnedly elaborating them.

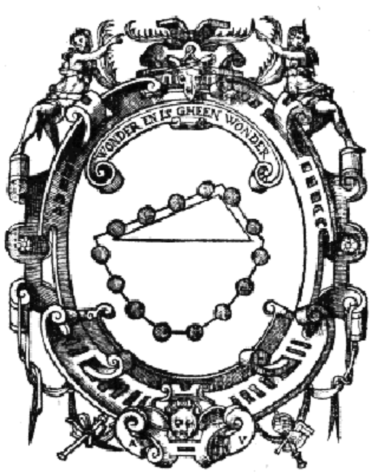


Figure 3 The 'clootcrans'

'Spiegheling' and 'daet' are usually interpreted as 'theory' and 'practice' but we should be careful interpreting these words with our modern understanding. By 'spiegheling' Stevin understood the intellectual analysis aimed at disclosing principles and causes that should precede the 'daet' of understanding and employing phenomena in the real world. The *Weeghconst* and the *Weeghdaet* both contain theories of weighing, with principles, deductions and so on. The former considers weights per se whereas the latter considers them in their physical appearance, Stevin said.[8] For us (early 21st-century mathematicians) such a distinction is not self-evident but Stevin's explanation corresponds quite well with the distinction 'pure' and 'mixed'. This is remarkable, because 'spiegheling en daet' may best be translated

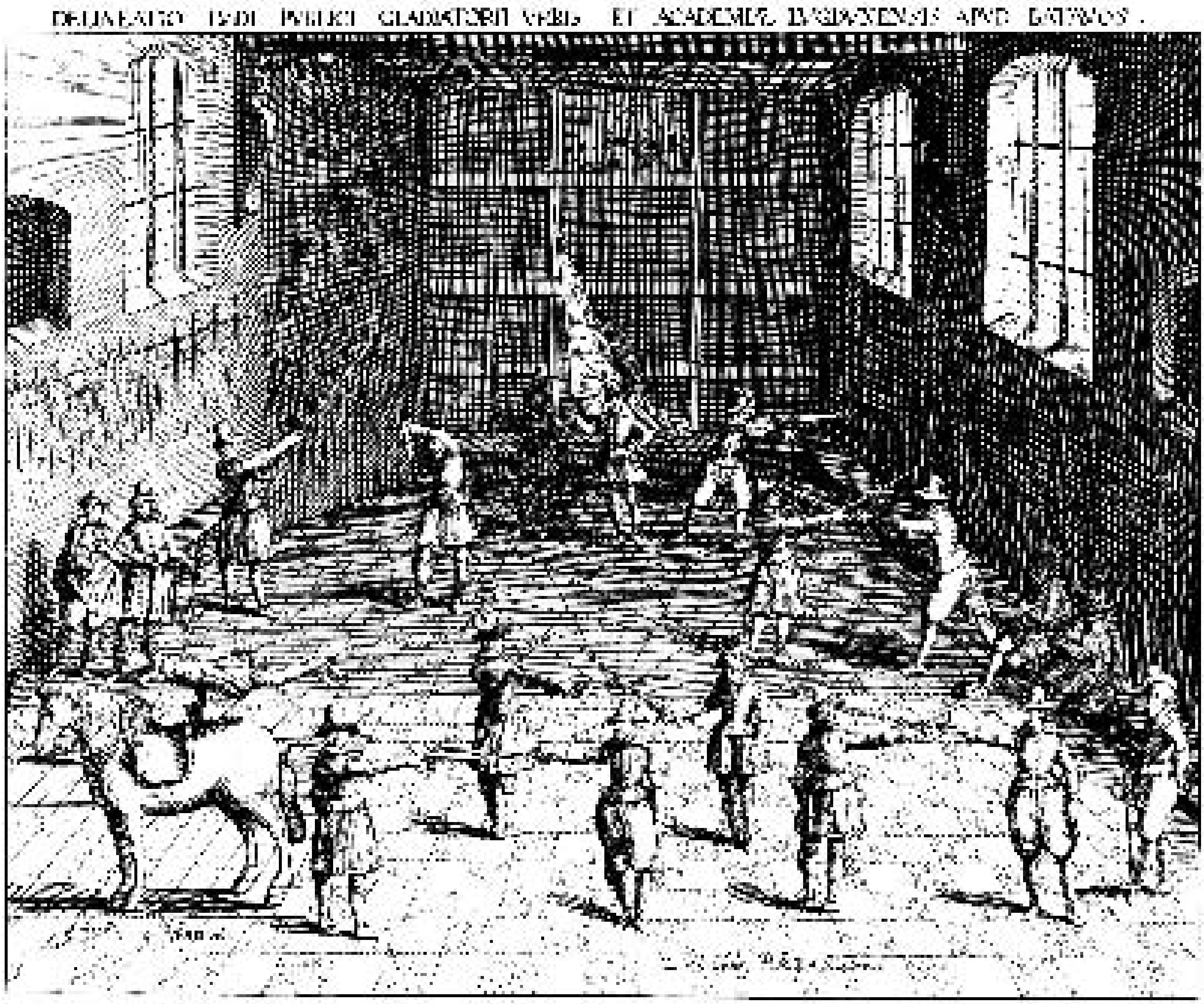


Figure 4 The 'Duytsche Mathematique' found its first home in the fencing school of its first professor, Ludolf van Ceulen, on the ground floor of the University Library. This picture nicely illustrates the close links that existed between mathematics and the military at that time. Copper engraving: Jan Cornelisz. van't Woudt (1610)

as 'reflection' and 'action'. That was a second distinction drawn in early modern mathematics, signifying the goal of study rather than the object.[9] Stevin's oblique use of reflection and action may be explained by realising that he was primarily interested in the mathematics of an artisan — bookkeeping, machines and, later on, fortifications and navigation — to which he applied his considerable speculative ingenuity in finding principles and causes. The mathematics of natural phenomena considered outside a context of artefacts and 'practices' lay beyond his interest.

The use of the vernacular was important for Stevin. Not only did he consider Dutch the best way to reach his intended audience (men of action), he even went so far as to regard Dutch the language best suited for learning. The *Weeghconst* opened with a long digression on the virtues of Dutch, being more clear and more efficient than any other language. He rejected the tendency of other writers in the vernacular to Dutchify Latin words and proposed a whole list of truly Dutch words. Irrespective of the virtues of Stevin's linguistic theories, he did give Dutch its own word for mathematics: wisconst, nowadays wiskunde, literally 'art of knowing'.

The establishing of the Leiden engineering school in 1600

Stevin's greatest success in realising his ideas was the establishment in 1600 of the 'Duytsche Mathematique', the engineering school at Leiden University. It would train fortificationists with precisely the combination of 'spiegheling en daet' Stevin envisioned in his writings. The teaching was in the vernacular, hence *Duytsche* mathematics. Although Stevin never taught at the engineering school, he drew up a detailed curriculum in which a careful selection of mathematical theory was combined with field practice. For example:

"The measuring of circles with segments of that sort, further the area of spheres. The shapes named ellipsis, parabola, hyperbola and the like, that is not necessary here, because engineers are very seldom made to perform such measurements; but only they shall learn with straight planes, after that curvilinear in surveyor's manner, measuring thus a plane by various division, like in triangles or other planes to see how this matches with that." [10]

as a victory of Stevin over the practically inclined engineers in the Dutch army. The idea that engineers should also learn from books was not obvious, and would not be for at least a century.

Stevin's success can be explained by his close ties to Count Maurits, the Dutch stadholder and army commander.[11] They probably knew each other from their student year in Leiden in the 1580s, and in the 1590s they established a close relationship. In 1593 Stevin entered the service of Maurits as quartermaster of the army.[12] From this time his publications dealt with issues of state building and reinforcement, for example *Sterctenbouwing* (Fortification, 1594) and *De Havenvinding* (The finding of harbours, 1599). These had been preceded by a treatise on citizenship *Vita Politica. Het Burgerlick Leven* (Political Life. The Civil Life, 1590). Stevin's ideas and pursuits fitted particularly well with his employer's interests. Maurits was greatly interested in mathematics and shared Stevin's idea of combining practical improvements with theoretical reflection. The two had lengthy conversations on these matters, which Stevin eventually put down in his *Wisconstighe Ghedachtenissen* (Mathematical Memoirs, 1605–1608). These reveal several original contributions of Maurits.[13] Maurits played a central role in the military reforms that turned the Dutch army into a highly skilled, professional and quite successful force. Part of these reforms was the 'Duytsche Mathematique' that Maurits installed high-handedly at Leiden university.

The promotion of mathematics in warfare, state building and land development was not just a Dutch affair. The second stadholder of the republic Willem Lodewijk of Friesland had similar ideas about the intellectual and material reinforcement of the new state. His scholarly interests were mainly aimed at classical military theories but he provided for the cultivation of mathematics by appointing Adriaan Metius in the chair of mathematics at Franeker University. Like Stevin, Metius disclosed new developments in the mathematical sciences to a broad audience by publishing a range of books in both Dutch and Latin.[14] In Franeker as well as Leiden, mathematics figured next to (Calvinist) theology as the means of intellectually reinforcing the new republic. Stevin's religious inclinations are somewhat diffuse but we have acquired a good understanding of his mathematical persona. His specific and explicit mixing of reflection and action resulted in a series of textual and material works in which he ingeniously extended the renaissance mathematics inspired by Archimedes. The beautiful Dutch word 'vernufteling' is most appropriate for Stevin, indicating cleverness of both hand and mind.

French mathematics brought to Leiden

The historical link between Stevin and Huygens is established by Frans van Schooten Jr. (1615–1660). Van Schooten's career reflects the transition from the Dutch-spoken, utilitarian mathematics of Stevin to the Latin-written, academic mathematics of Huygens. After his studies at Leiden university, he began replacing his father, Frans van Schooten Sr., the then professor of 'Duytsche Mathematique' in the 1630s, before succeeding him in 1645. In the meantime, he had established himself as an advocate of the new mathematics coming from France. He gathered and published papers by Fermat and Viète and collaborated with René Descartes. He commented on the manuscript of *La Géométrie* and made the illustrations. His claim to fame is *Geometria a Renato Des Cartes*, a Latin translation with commentaries and supplements disclosing Descartes' new geometry to the international mathematical community. A first edition appeared in 1649, followed by an expanded edition in 1659–1661. This edition contained contributions from elite students Van Schooten had attracted to Leiden: patrician sons like Johan de Witt (future governor of the republic), Johannes Hudde (future burgomaster of Amsterdam) and Christiaan Huygens.

Christiaan Huygens

It remains one of the miracles of the golden age that patricians like De Witt and Hudde developed and cultivated an interest in mathematics, a field of study that apparently bore no relevance to their administrative ambitions. Their amateur interest in the mathematical sciences may be a major key to understanding the unprecedented rise of the epistemic and cultural status of mathematics in the 17th century. In the course of the scientific revolution, mathematics rose from being an art pursued by ingenious professionals to the language of philosophy — and even nature — cultivated by savants of all ranks. The growing interest in natural inquiry among Dutch patricians has been explained by the 'aristocratisation' of this group. They were no nobles but they increasingly took up noble activities like investing in landed property, building dynasties of power, and taking up arts and letters.

Huygens too was an amateur in mathematics; until the 19th century any true scientist was. His case is different, however, from his fellow students. He managed to devote his entire life to his main pleasure: mathematics. Huygens was the second son of Constantijn Huygens, secretary of the stadholder and a leading man of learning and art in the Dutch republic.[15] His predilection for mathematics showed at an early age, when he and his older brother Constantijn were educated by private tutors. He helped his brother with mathematical exercises and was soon bored with writing poems. After the brothers had gone through university, their father could set out to plot a diplomatic career for his sons. However, in 1650 the first stadholderless period began and with the Oranges out of power no posts were available to the Huygenses. Huygens was free to pursue his interests (and happy too, we may presume). He would do so for the rest of his life, even when his brother made a diplomatic career some years later. For a man of learning of Huygens' rank few public positions were available and in the republic next to none. A university professorship was far below his standing and the republic did not have a lustrous court. The only options were the great courts of Italy, France and England. His bid for Florence was unsuccessful but in 1666 Colbert brought him to Paris to chair Louis XIV's Académie des Sciences. He would stay there until the early 1680s, spending his last years at his family residences in The Hague and Voorburg: a life in mathematics, free from public obligations — a savant's paradise.

What did this pastime look like? In the course of his life, Huygens worked on a wide range of topics: the catenary and floating bodies; Cartesian ovals, quadrature and rectification; lenses, telescopes, heavenly observations and light waves; impact, fall, circular and pendulum motion, clocks and longitude; probability; tuning; and so on. Historians have lamented that Huygens' oeuvre lacks a unifying idea, that it is an eclectic collection of puzzles that came his way through coincidental encounters and the like. From the viewpoint of early modern natural philosophy and in comparison with men like Galileo, Descartes and Newton, Huygens' oeuvre indeed looks fragmentary. Yet, Huygens had no taste for philosophy, as his pupil Leibniz would aptly observe. He was a mathematician. From the perspective of early modern mathematics, Huygens' oeuvre does not look incoherent at all. All his activities were mathematical and he covered almost the entire range of the mathematical sciences: geometry, (hydro)statics, optics, astronomy, mechanics, arithmetics and music. Furthermore, and this is important to realise, he barely went beyond the domain of mathematics, except for one important case that will be discussed below. In other words Huygens was a full-blown, 17th-century mathematician.

Huygens' first steps in the Republic of Letters earned him the epithet 'little Archimedes'. A discussion of projectile motion greatly impressed Mersenne, who wrote to Huygens' father saying that his boy would soon become a new Archimedes.[16] From then on, Constantijn would call Christiaan his little Archimedes.

Huygens' style was indeed Archimedean. He had a visual and tangible approach, in which he *saw* the ratios fundamental to classical geometry and favoured the kinematic understanding of curves Van Schooten had taught him in his introduction to Descartes' new geometry. His theory of evolutes is based on the actual motion of a thread unwinding from a curve. Archimedes was also prominent in Huygens' first publication in 1651, a treatise on quadrature in which he revealed mistakes in a recent treatise on the quadrature of the circle. He proposed an improved method and applied this to conic sections as well. He showed how to limit the amount of calculations for the method of exhaustion by using the centre of gravity of line segments. The visual and tangible style of Huygens' earliest work would be a distinctive feature of his mathematics.

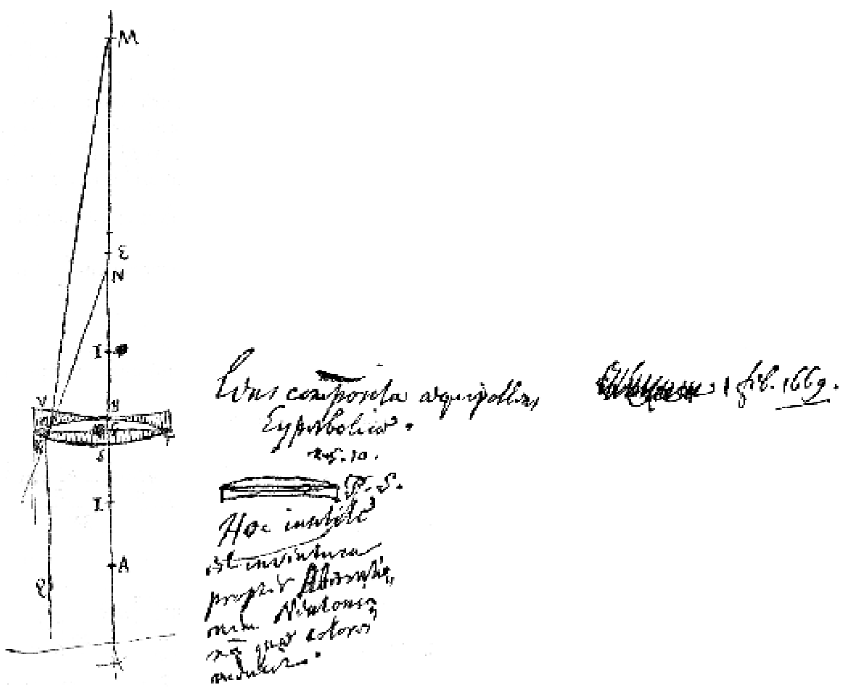
Lenses and telescopes

In 1653 Huygens became 'totally absorbed' by dioptrics, the mathematical analysis of lenses and their configurations.[17] Although the sine law had been published by Descartes in 1637, Huygens was the first to apply it to lenses actually used in telescopes. Descartes had only been interested in determining the shape of lenses with a perfect focus, finding them elliptic or hyperbolic. Those lenses, however, turned out to be very difficult to manufacture and telescopes were fitted with ordinary spherical lenses. In a study that would be published only posthumously, Huygens employed the sine law to derive the properties of lenses in a rigorous fashion. Due to spherical aberration the focus is not perfect and Huygens defined it as the limit of intersections of the axis. He then derived exact expressions for the focal distance of any kind of lens: plano-convex, plano-concave, bi-convex, etc. Only then did he show how these cumbersome expressions reduced to the simple lens formulas when the thickness of the lens is neglected.

Huygens' theory of the telescope reveals a second feature of his mathematics: the rigorous elaboration of mathematical problem focused on concrete topics. His oeuvre consists of a couple of clusters of topics that are bound together by a marked interest in instruments. This is not to say that material problems guided his mathematical studies. He valued the mathematics of things greatly, rigorously elaborating the dioptrics of lenses. Still, he selected mathematical topics for their relevance to understanding the telescope, leaving out sophisticated problems like obliquely incident rays.

After he finished his theory of the telescope, Huygens turned to the practice of lenses. He inquired about the grinding and polishing of lenses and began manufacturing telescopes with his brother Constantijn. In 1655 he discovered a satellite of Saturn with one of his telescopes, the first new body in the solar system since Galileo's famous discoveries in 1610. Soon after he was able to figure out the cause of the strange shape of Saturn: the planet was surrounded by a flat, circular disc. These discoveries gained him fame, but there is something odd about Huygens' use of the telescope. Apart from these and a couple of other observations, Huygens never became a systematic astronomical observer. His interest in the telescope on the other hand was virtually unlimited. He endlessly manufactured lenses and telescopes, developing configurations with excellent optical properties and inventing improvements like the diaphragm and a rudimentary micrometer. With instruments so central to Huygens' mathematics it is remarkable how little interested he was in their actual employment, as though the tinkering with them, both intellectually and materially, sufficed.

In the 1660s he continued his mathematical analysis of lenses by elaborating on a theory of spherical aberration, relating the amount of aberration to the properties of a lens. He then managed to determine a configuration of two lenses that mutually cancelled out their spherical aberration. 'Eureka', Huygens exclaimed on 1 February 1669 in Archimedean style — a perfect telescope made of ordinary, spherical lenses. Soon, the novice Newton would prove that his invention would never work in practice due to the nature of chromatic aberration. On 25 October 1672, Huygens crossed out his invention and wrote 'useless'. This debacle is probably the reason he never published his dioptrics: a theory without an impressive invention was futile.[18]



Left, Huygens's design of the lens (1669); right: the word Eureka is crossed out.

Mechanics

In mechanics things were different. Huygens gained fame with the invention in 1657 of the pendulum clock and this was a perfect stepping-stone for publishing his mathematics. That winter he invented a mechanism to regulate a clock with a pendulum. Mechanical clocks were grossly inaccurate at that time.

The motion of a pendulum was known to be very regular and used by astronomers and the like to measure time. The combination of these two elements yielded a clock that was accurate within a couple of seconds a day. In 1658 Huygens published *Horologium*, a description of his clock, that contained an additional invention. He hung the pendulum between little 'cheeks', so that its length diminished for larger displacements. A pendulum was, after all, not isochronous: the period increases slightly with the amplitude. Modifying the path of them both slightly would yield an isochronous pendulum and thus a perfect clock. However, Huygens did not know the exact path required (or the shape of the cheeks) and for this he needed a thorough understanding of pendulum motion.

Around the same time Huygens had begun to study free fall and in 1659, his annus mirabilis, he tackled a whole series of problems relating to accelerated motion.[19] The problem was to determine g, the constant of gravitational acceleration, or in 17th-century phrasing: the distance an object in free fall covers in one second. Several experiments had been performed without a conclusive outcome. Huygens himself tried to measure the distance by letting a seconds pendulum hit the wall and having a mass hit the ground at the same time. It did not yield any convincing results so he turned to a mathematical analysis of the situation. A pendulum is a body in circular motion and weight, he argued, is identical to the tension exerted by the body on the cord. Comparing a horizontally moving body deflected by gravity (a parabolic path) and by a chord (a circular path) and noting the identical centres of curvature, Huygens derived a measure for centrifugal acceleration equal to our modern mv^2/r. He then devised a set-up in which gravitational and centrifugal acceleration are in balance: a pendulum rotating in a horizontal plane. Knowing now how to calculate the centrifugal acceleration of this conical pendulum, Huygens could determine gravitational acceleration to be 9.79m/s^2 (in modern terms) — quite accurate indeed!

Huygens then attacked the problem of the isochronous pendulum and this shows all the features of his mathematical genius.[20] What is the ratio between an oscillation with a small amplitude EZ and free fall over the height of the pendulum TZ? Huygens compared the infinitesimals at E and B and constructed the parabola $AD\Sigma$ representing velocities along the path of the pendulum. He then considered the times to traverse the

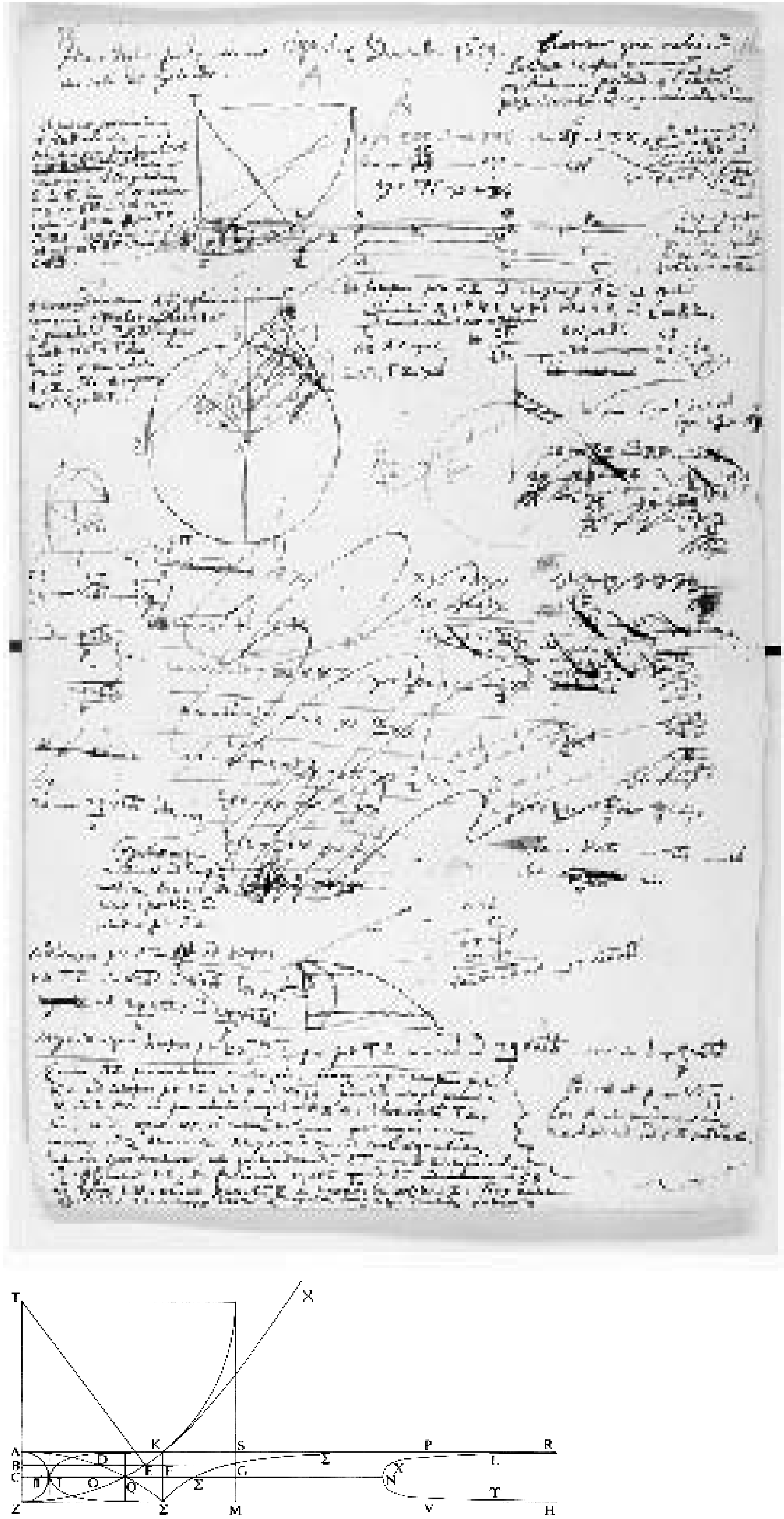


Figure 5 Top: the deduction of the isochronism of the cycloid (*Codex Hugeniorum* **26**, f.72r.). Bottom: diagram of this figure (construction taken from: Joella G. Yoder, *Unrolling Time. Christiaan Huygens and the Mathematization of Nature* (1989), Cambridge, Cambridge University Press)

infinitesimals and constructed curve LXN. Notice that the curves are constructed within the diagram and directly represent the mathematics of the motions. The time to traverse arc KEZ is equal to the indented rectangle between AZ and LXN, but this involved the uniform velocity over AZ. To get around that, Huygens approximated arc KEZ with a parabola $ZK\aleph$ and determined the time anew. He concluded that the time is independent of the amplitude, and so he established isochronism. However, this applied only to small amplitudes. Huygens then showed his mathematical ingenuity and his masterly eye in geometry. He went through his diagram again and found the solution by recognizing the properties of the curve needed. The radius TE is perpendicular to the circle ZEK but not to the parabola $ZK\aleph$ he had used as an approximation. He needed a new curve that did preserve the conditions of his derivation. And then, out of the blue, Huygens said: "I saw this was the cycloid because of the familiar method to draw its tangent". Thus he found out that an isochronous pendulum should follow a cycloidal path. And it turns out that the involute of a cycloid is also a cycloid. So the cheeks of his clocks should be cycloids. In 1673 Huygens published these results in his magnum opus *Horologium Oscillatorium*, in which the mathematics of motion was presented with the construction of his ingenious invention.

The Huygens principle

Huygens' work on the kinematics of fall, circular and pendulum motion has all the characteristics of his mathematics: exact, rigorous, visual and concrete. It was a brilliant specimen of the new Galilean science of motion. It was not revolutionary, though. Huygens did not break new grounds but stayed well within the established domains of 17th century mathematics. He consciously did so. In mechanics he explicitly rejected non-kinematic concepts like force. And in optics too, he kept to the beaten track when he said that the coloured fringes that disturbed his telescopic images exceeded mathematical analysis.[21] Traditionally colours were not subject to mathematical analysis and Huygens was not someone to change that. Newton would turn the science of colours 'mathematical' with his prism experiments. And he would turn mechanics into dynamics by introducing a new concept of force. Still, Huygens would go on to make a revolutionary step beyond the established borders of mathematical inquiry; however, he did not truly realise he was doing so. In the 1670s, Huygens tackled an intriguing optical phenomenon: the strange refraction of Iceland crystal.[22] This crystal displays a double refraction and one of these does not follow the sine law of refraction: a perpendicular ray is broken and an oblique ray passes unrefracted. Huygens was by then convinced that light consists of waves in a material ether but he could not figure out how a wave traversing a refracting surface in a straight direction was diverted to produce a refracted ray. The flash of genius came in August 1677 and the way it came corroborates the thorough mathematical character of Huygens' work. He was working on a sophisticated dioptrics topic that also seemed to defy his understanding of waves: caustics, the bright curves produced when light passes through lenses and other curved bodies (like a glass of water in the sun). Trying to construct geometrically a wave propagating after traversing the curved boundary, Huygens hit upon an ingenious idea that we now know as his principle of wave propagation. At this time it was only a tiny sketch jotted down to note the method he used in constructing caustics by means of tangents to wavelets. And then to tackle strange refraction, without pausing to elaborate on his idea, Huygens went on to construct a wave refracted in Iceland crystal. He had it propagate unequally in the crystal producing an elliptical instead of a circular wave and this did the trick. The tangent to these ellipses was parallel to the refracting surface but propagated somewhat askew and thus the ray was refracted. 'Eureka', Huygens noted again and this time nobody would take it away from him.

Unwittingly, Huygens took a revolutionary step when he tackled strange refraction. His principle of wave propagation made mathematical the properties of hypothetical entities in the unobservable realm of the ether; he had transgressed the borders of mathematics, for mathematics did not treat the unobservable motions of unobservable matter that explained natural phenomena. Huygens thus transferred mathematical inquiry to the domain of natural philosophy and he was the first to do so. Yet to Huygens this went without saying: of course the motions of hypothetical waves in the ether ought to be discussed in the same exact manner as the motions of rays, pendula and billiard balls. Bear in mind that the principle came from mathematics rather than physics. Huygens devised a way to geometrically construct caustics and strangely refracted rays; he did not consider the nature of light waves at this point. He would do so only afterwards when preparing a presentation of his account for the Académie, which he eventually published in 1690 as *Traité de la Lumière*. It presents the principle of wave propagation as a much better means of deriving the laws of optics and as a unique way of enabling the penetration of strange refractions. But it does not present it as we see it: a revolutionary transformation of mathematics (and natural philosophy). If Huygens was a revolutionary, he was sleepwalking.

Huygens' wave theory was part of his dioptrics. Although he never published his theory of the telescope, he cut the umbili-

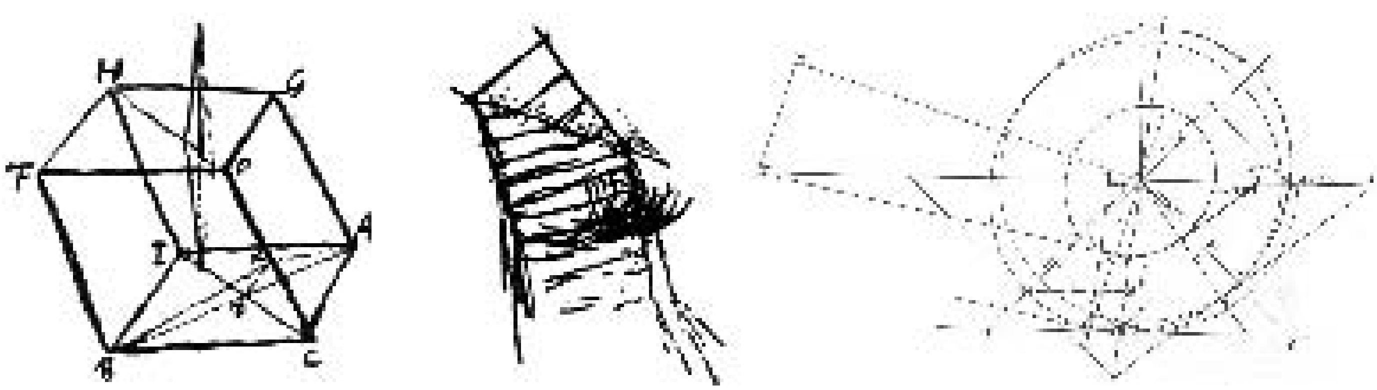


Figure 6 Strange refraction. On the left, Huygens's sketch of this phenomenon (1672); middle: his sketch of the 'Huygens principle' (1677); right: the solution to the problem

cal cord only at the very last. Until 1690, he planned a book on optics that included both, eventually deciding to publish his wave theory separately [23]. Thus this cluster of activities was pulled apart, obscuring the coherence of telescopes and waves. Light waves were part of Huygens' mathematical consideration of telescopes that had first produced his dioptrical analysis of lenses.

In Huygens we see a similar combination of 'spiegheling en daet' as in Stevin. Yet the nature and the context of his mathematics was entirely different. Huygens was a 'vernufteling' like Stevin, but in the learned world of instruments, academies and letters, rather than the artisan's world of mills, bookkeeping and fortifications. Huygens tinkered with instruments for the sake of tinkering with them, as an amateur pastime. He could be called a mathematical virtuoso but Huygens may have been somewhat ill at ease with such a qualification. He saw himself as a physicomathematician like Galileo; consider the evaluation of 17th century savants he wrote at the end of his life [24]. His father would surely have balked at the idea. He once received a letter that praised his 'mathématicien' son. Constantijn replied irritably that he did not know that he had craftsmen in his family. A *learned* mathematician was a 'géomètre' after all. For him, Christiaan always remained his little Archimedes.

Conclusion

Stevin and Huygens were children of their times and so were their mathematics. The times were changing and so were their mathematics. Early modern mathematics was a much different affair to mathematics today. Considering the pursuits of Stevin and Huygens from this historical perspective gives both an insight into their achievements and reinforces their ingenuity. Such are the fruits of recent developments in the historiography of mathematics. Inquiry into the culture of early modern mathematics raises exciting questions, for example on the role mathematicians played in the formation of the Dutch republic and the way patrician amateurs paved the way for the mathematization of natural philosophy. It promises to yield a better understanding of the way mathematicians began shaping our modern world.

Referenties

1 Apart from my own work, some examples are: Peter Dear, *Discipline and Experience. The Mathematical Way in the Scientific Revolution* (Chicago, 1995); Andrew Warwick, *Masters of Theory. Cambridge and the Rise of Mathematical Physics* (Chicago, 2003); Eric Ash, *Power, Knowledge, and Expertise in Elizabethan England* (Baltimore, 2004); Wolfgang Lefèvre (ed.), *Picturing Machines. 1400-1700* (Boston, 2004); Volker Remmert, *Widmung, Welterklärung und Wissenschaftslegitimierung: Titelbilder und ihre Funktionen in der Wissenschaftlichen Revolution* (Wiesbaden, 2005); Matthew Jones, *The Good Life in the Scientific Revolution. Descartes, Pascal, Leibniz, and the Cultivation of Virtue* (Chicago, 2006).

2 "With regard to their Object, *Mathematics* are divided into *pure* or *abstract*; and *mix'd*. Pure *Mathematics* consider Quantity, abstractedly; and without any relation to Matter: Mix'd *Mathematics* consider Quantity as subsisting in material Beings, and as continually interwove." Chambers, Ephraim, *Cyclopædia, or, 'An universal dictionary of arts and sciences : containing the definitions of the terms, and accounts of the things signify'd thereby, in the several arts, both liberal and mechanical, and the several sciences, human and divine : the figures, kinds, properties, productions, preparations, and uses, of things natural and artificial : the rise, progress, and state of things ecclesiastical, civil, military, and commercial : with the several systems, sects, opinions, &c : among philosophers, divines, mathematicians, physicians, antiquaries, critics, &c : the whole intended as a course of ancient and modern learning'.* The Second Volume (1728), 509.

3 Berghe, Guido Vanden, 'Simon Stevin. Een leven in de schaduw van de macht', pp. 19–25 in H. Elkadem (red.), *Simon Stevin 1548-1620. De geboorte van de nieuwe wetenschap*.

4 *Tafelen* was published by Plantijn in Antwerp (dedicated to the aldermen of Leiden), *Nieuwe Inventie* by Hendricsz in Delft (dedicated to the Amsterdam burgomasters). *Nieuwe Inventie* has only been recently discovered and now turns out to be Stevin's first publication. Heirwegh, Jean-Jacques, and Frédéric Métin, 'Simon Stevin en de financiële wereld', pp. 73–81 in *Simon Stevin*.

5 Dijksterhuis, E. J., *Simon Stevin. Science in The Netherlands around 1600*, Den Haag, 1970, pp. 93–98.

6 In the *Wisconstighe Ghedachtenissen* Stevin later elaborated his ideas about the 'mixing of reflection and action' (part of the introduction of the *Eertclootschrift* in the *Weereltschrift*, part 1 of the *Memoirs*). "Spiegheling is een verdochten handel sonder natuerlicke stof, ghelijck onder anderen sijn de Spieghelinghen des Spieghelaers *Euclides*, handelende deur stelling (per hypotesim) van grootheden en ghetalen, maer elck ghescheyden van natuerlicke stof. Daet is een handel die wesentlick met natuerlicke stof gheschiet, als lant en wallen meten, de menichte der roen of voeten tellen dieder in sijn, en dierghelijcke. T' besluyt vande voorstellen der Spieghelingh is volcommen, maer der daet onvolcommen: ..."

7 Weeghdaet, 3. "Alsoo is de spiegheling (theoria) inde beghinselen der consten verloren arbeydt, daer t'einde totte deat (effectum) niet en strect."

8 Weeghconst, 1.

9 "Mathematics are distinguish'd with regard to their End, into Speculative, which rest in the bare Contemplation of the Properties of Things; and Practical, which apply the Knowledge of those Properties to some Uses in Life." Chambers, *Cyclopædia*, 509.

10 Molhuysen, *Bronnen*, 390*. "Het meten des rondts mette gedeelten van dien aengaende, voerts het vlack des cloots, de formen genaemt ellipsis, parabola, hyperbole ende diergelijcke, dat en is hyer nyet nodich, wantet den ingenieurs seer selden te voeren compt, sulcke metinge te moeten doen; maer alleenlyck sullense leeren met rechtlinige platten, daer na cromlinige landtmetersche wijse, metende alsoe een plat deur versceyde verdeelinge, als in dryehoucken of ander platten om te syen hoe t'een besluyt met het ander overcompt."

11 After his father Willem van Oranje had been assassinated in 1584, Maurits became Stadholder of Holland and Zeeland in 1585, at his 18th birthday. Although he used the title 'Prince of Orange' from 1584, he officially became prince only in 1618.

12 Berghe, *Simon Stevin*, 23.

13 Heuvel, Charles van den, 'Wisconstighe Ghedachtenissen. Maurits over de kunsten en wetenschappen in het werk van Stevin.', pp. 107–121 in Kees Zandvliet, Maurits Prins van Oranje (Zwolle, 2000).

14 See: Dijksterhuis, F. J., 'Duytsche Mathematique and the building of a new society: pursuits of mathematics in the seventeenth-century Dutch republic' in L. Cormack (ed.), *Mathematical Practitioners and the Transformation of Natural Knowledge in early modern Europe (forthcoming)*.

15 A good, concise biography in Dutch is Vermij, R., *Christiaan Huygens. De Mathematisering van de Werkelijkheid*, Diemen, 2004. A highly readible, although somewhat fictitious biography in English is Andriesse, C. D., Huygens. *The Man behind the Principle*, Cambridge, 2005. Vermij does not go into the mathematics while Andriesse does in enlightening and inspiring ways.

16 Huygens, Chr., *Oeuvres Complètes de Christiaan Huygens*. 22 vols, The Hague, 1888-1950: vol. 1, 47.

17 Dijksterhuis, F. J., *Lenses and Waves. Christiaan Huygens and the Mathematical Science of Optics in the Seventeenth Century*, Dordrecht, 2004, pp. 11–24.

18 Dijksterhuis, *Lenses and Waves*, pp. 67–91.

19 Yoder, J. G., *Unrolling Time. Christiaan Huygens and the Mathematization of Nature*, Cambridge, 1989, pp. 9–32.

20 Yoder, *Unrolling Time*, pp. 44–63.

21 Huygens, *Oeuvres Complètes*, vol. 17, p. 359.

22 Dijksterhuis, *Lenses and Waves*, pp. 169–184.

23 Dijksterhuis, *Lenses and Waves*, pp. 219–223.

24 Huygens, *Oeuvres Complètes*, vol. 10, pp. 399–406.