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An LMI approach to stabilization of linear port-controlled Hamiltonian systems

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Abstract

In this paper, it is shown that controllers for stabilizing linear port-controlled Hamiltonian (PCH) systems via interconnection and damping assignment can be obtained by solving a set of linear matrix inequalities (LMIs). Two sets of (almost) equivalent LMIs are proposed. In the first set, the interconnection and damping matrices do not appear explicitly, which makes it more difficult to directly manipulate those matrices. By requiring the system to have no uncontrollable pole at $s = 0$, the second set of LMIs, explicitly containing the interconnection and damping matrices, can be obtained. Taking into account the physical properties of the system, some prespecified structures can be imposed directly on those matrices. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The port-controlled Hamiltonian (PCH) approach has been proposed in [5] as a way for modelling of physical systems. It originates from the network modelling of energy-conserving lumped-parameter physical systems with independent storage elements. This kind of models encompasses a very large class of physical systems, containing the class of Euler–Lagrange models. Readers are referred to [12, Chapter 4] for a survey of results on this modelling approach.

Recently, stabilization of PCH systems via interconnection and damping assignment has been introduced in [7–9,11,2]. In this method stabilization is approached by shaping the energy and damping of the system, by also allowing for a modification of the internal *interconnection structure* of the system. An important feature of the method is that it stimulates a physical motivation and interpretation of the control action (insertion of e.g. virtual springs, dampers and constraints). Furthermore, since the controlled system is still a Hamiltonian system it enjoys some inherent robustness properties. Similar ideas have been expressed in the context of robotics or general physical systems under the name of virtual model control (see e.g. [10]) or impedance control [4].

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In this stabilization method of PCH systems, one starts by assigning some closed-loop system interconnection and damping structures; and then obtains a closed-loop energy function and a feedback law by solving a system of partial differential equations, which is governed by the choice of system interconnection and damping. Until now, however, there has been no general rule on how one should assign the system interconnection and damping, although physical considerations can usually be used in it. The procedure is made involved by the fact that the assigned system interconnection and damping should be expressed in terms of several undetermined parameters. Those parameters have to be determined later, in order to satisfy some conditions such as the positive definiteness of the Hessian of the closed-loop energy function (evaluated at the desired equilibrium). In general, this accounts for solving a set of nonlinear inequalities. Obviously, this is not an easy thing to do, especially for high-order systems. Some references that provide illustrations of this procedure are [6,7].

The purpose of this paper is to show that for a special class of PCH systems, namely *linear* PCH systems, the whole procedure described above can be combined into the process of solving systems of linear matrix inequalities (LMIs). The system interconnection and damping, the closed-loop energy function, and the feedback law can then be directly computed from the solution of these LMIs, simply by performing some matrix manipulations. For solving the LMIs, powerful algorithms (interior point methods [1]) implemented in several software packages (such as the LMI Control Toolbox for Matlab [3]) are available. Hence, this approach is numerically tractable and efficient.

In the next section, a brief overview of linear PCH systems will be given. Section 3 will elaborate on the stabilization of linear PCH systems by the usual, as well as by the LMI approach. In addition, sufficient and necessary conditions for the solvability of the LMIs will also be given there. Finally, a worked example of a mass-spring system (with negative spring constant) will be given in Section 4.

2. Linear port-controlled Hamiltonian systems

A port-controlled Hamiltonian (PCH) system is a representation of the form

$$\dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u, \tag{1}$$

$$y = g^{\mathrm{T}}(x)\frac{\partial H}{\partial x}(x) \tag{2}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$, respectively, the state, input, and output of the system; $J(x) = -J^{\mathrm{T}}(x) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ a skew-symmetric matrix which determines the interconnection structure of the system; $R(x) = R^{\mathrm{T}}(x) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ a symmetric positive semi-definite matrix which defines the dissipation; $g(x) : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ the port matrix; and $H(x) : \mathbb{R}^n \to \mathbb{R}$ the Hamiltonian (which represents the energy of the system). The rate of energy increase in the system is given by

$$\frac{\mathrm{d}H}{\mathrm{d}t} = y^{\mathrm{T}}u - \frac{\partial^{\mathrm{T}} H}{\partial x}(x)R(x)\frac{\partial H}{\partial x}(x). \tag{3}$$

If $H(x)$ is bounded from below, then the PCH system (1)–(2) is *passive*.

In this paper, we only consider *linear* PCH systems. These systems are represented by the following equations:

$$\dot{x} = (J - R)\frac{\partial H}{\partial x}(x) + Bu = (J - R)Qx + Bu, \tag{4}$$

$$y = B^{\mathrm{T}}\frac{\partial H}{\partial x}(x) = B^{\mathrm{T}}Qx, \tag{5}$$

where $J=-J^{\mathrm{T}}$, $R=R^{\mathrm{T}}$, and B are constant matrices, and $Q\in\mathbb{R}^{n\times n}$ is a symmetric matrix which defines the Hamiltonian of the system, i.e., the Hamiltonian is given by $H(x)=\frac{1}{2}x^{\mathrm{T}}Qx$. In line with general PCH systems, the linear PCH system (4)–(5) is *passive* if Q is *positive semi-definite*.

It should be noted that the state equations of stable or asymptotically stable linear systems can be expressed in the PCH form (4), with Q and R having some definiteness properties. This fact is stated in the proposition below.

Proposition 1. *The state equation of a stable (or asymptotically stable) linear system, $\dot{x}=Ax+Bu$, can be written in the PCH form (4), with a positive definite Q and a positive semi-definite (or, respectively, positive definite) R.*

Proof. Since the system under consideration is stable, there exists a Lyapunov function of the form $V(x)=\frac{1}{2}x^{\mathrm{T}}Qx$ with $Q=Q^{\mathrm{T}}>0$, such that $\dot{V}(x)=\frac{1}{2}x^{\mathrm{T}}(A^{\mathrm{T}}Q+QA)x$ is nonpositive (that is, $A^{\mathrm{T}}Q+QA\leqslant 0$). Define $J=\frac{1}{2}(AQ^{-1}-Q^{-1}A^{\mathrm{T}})$ and $R=-\frac{1}{2}(AQ^{-1}+Q^{-1}A^{\mathrm{T}})$. It is clear that $J=-J^{\mathrm{T}}$ and $R=R^{\mathrm{T}}$. The state equation of the system can now be rewritten as

$$\dot{x}=Ax+Bu=(J-R)Qx+Bu \tag{6}$$

and since $A^{\mathrm{T}}Q+QA\leqslant 0$ it follows that

$$R\geqslant 0. \tag{7}$$

For an asymptotically stable system, there exists a Lyapunov function $V(x)=\frac{1}{2}x^{\mathrm{T}}Qx$ with $Q=Q^{\mathrm{T}}>0$, such that $\dot{V}(x)=\frac{1}{2}x^{\mathrm{T}}(A^{\mathrm{T}}Q+QA)x$ is negative ($A^{\mathrm{T}}Q+QA<0$), implying that in this case $R>0$. □

In the sequel, we shall assume that the *full state* of system (4)–(5) is available for feedback, thereby allowing to change the internal interconnection structure of the system, as well as its internal energetic and resistive structure, via a state feedback action along the input channels of the PCH system. Another, more restricted, approach to stabilization of PCH systems by interconnecting its input and output channels to a PCH controller system (that is, by dynamic output feedback) has been detailed in [6,7,9], where also the connections between both methods are described.

Furthermore, we shall assume that B has full column rank (if this is not the case then we can simply reduce the number of inputs), and we assume for clarity of presentation that $m<n$, i.e., the number of inputs is strictly less than the dimension of state. Indeed, as we shall see later, if m happens to be equal to n, then the stabilization problem is trivial, since the interconnection, damping, and energy of the system can be assigned and shaped as we wish.

Assumption 2. B has full column rank, and $m<n$.

3. Stabilization via interconnection and damping assignment

Stabilization of general PCH systems via interconnection and damping assignment has been discussed extensively in [7–9]. In this method, we shape the energy of the system by assigning some desired interconnection, such that the closed-loop energy function will have a minimum at the desired equilibrium. Then damping is added to the system to make the system asymptotically stable. The method is summarized in the following proposition.

Proposition 3 (Ortega et al. [7–9]). *Given $J(x)=-J^{\mathrm{T}}(x)$, $R(x)=R^{\mathrm{T}}(x)$, $H(x)$, $g(x)$ and a desired equilibrium to be stabilized at x_*. If we can find two $n\times n$ matrices $J_{\mathrm{d}}(x)$, $R_{\mathrm{d}}(x)$ and $H_{\mathrm{d}}(x):\mathbb{R}^n\to\mathbb{R}$, $\beta(x):\mathbb{R}^n\to\mathbb{R}^m$*

such that

(i) $[J_d(x) - R_d(x)](\partial H_d/\partial x)(x) = [J(x) - R(x)](\partial H/\partial x)(x) + g(x)\beta(x)$,
(ii) $J_d(x) = -J_d^T(x)$; *and* $R_d(x) = R_d^T(x) \geqslant 0$,
(iii) $(\partial H_d/\partial x)(x_*) = 0$; $(\partial^2 H_d/\partial x^2)(x_*) > 0$,

then the PCH system (1)–(2) *in closed loop with* $u = \beta(x) + v$ *will be a PCH system of the form*:[2]

$$\dot{x} = [J_d(x) - R_d(x)]\frac{\partial H_d}{\partial x}(x) + g(x)v, \tag{8}$$

$$y' = g^T(x)\frac{\partial H_d}{\partial x}(x). \tag{9}$$

Furthermore, the system will be stable if $v = 0$, *and* (*locally*) *asymptotically stable if the largest invariant set contained in* $\{x|\mathscr{B}\cap[(\partial H_d/\partial x)(x)]^T R_d(x)[(\partial H_d/\partial x)(x)]=0\}$ *is equal to* $\{x_*\}$ (*where* $\mathscr{B}$ *denotes a positively invariant set of the system containing* x_*).

Remark 4. We see that if the number of inputs is equal to the dimension of state, i.e., $m = n$, then Condition (i) can always be satisfied for any $J_d(x)$, $R_d(x)$, $H_d(x)$.

For linear PCH systems (4)–(5), the following corollary applies.

Corollary 5. *Given* $J = -J^T$, $R = R^T$, $Q = Q^T$, B *and the equilibrium to be stabilized at* $x_* = 0$. *If we can find* $J_d \in \mathbb{R}^{n\times n}$, $R_d \in \mathbb{R}^{n\times n}$, $Q_d \in \mathbb{R}^{n\times n}$ *and* $F \in \mathbb{R}^{m\times n}$ *such that*

(i) $[J_d - R_d]Q_d = [J - R]Q + BF$,
(ii) $J_d = -J_d^T$; *and* $R_d = R_d^T \geqslant 0$,
(iii) $Q_d = Q_d^T > 0$,

then the linear PCH system (4)–(5) *in closed loop with* $u = Fx + v$ *will be a PCH system of the form*

$$\dot{x} = [J_d - R_d]Q_d x + Bv, \tag{10}$$

$$y' = B^T Q_d x. \tag{11}$$

Furthermore, the closed-loop system will be stable with respect to $x_* = 0$ *if* $v = 0$, *and any trajectory of the system will converge to the largest invariant set contained in* $U = \{x|x^T Q_d R_d Q_d x = 0\}$.

Notice that if we have $R_d > 0$, then the closed-loop system will automatically be asymptotically stable. Even if R_d is only positive semi-definite, we may still obtain asymptotic stability, provided the largest invariant set contained in U is just the origin. As a matter of fact, if the system is asymptotically stable, then there exists $\tilde{J}_d = -\tilde{J}_d^T$, $\tilde{Q}_d = \tilde{Q}_d^T > 0$, $\tilde{R}_d = \tilde{R}_d^T > 0$ such that $(J_d - R_d)Q_d = (\tilde{J}_d - \tilde{R}_d)\tilde{Q}_d$. This is because all asymptotically stable closed-loop systems can be written in the PCH form with $R > 0$ and $Q > 0$ (cf. Proposition 1).

There are several ways to check if the closed-loop system (10)–(11) is asymptotically stable for $v(t) = 0$. Two ways to do it are by computing the eigenvalues of $(J_d - R_d)Q_d$, or by utilizing LMIs to compute a quadratic Lyapunov function for the system. Yet still another way to check the asymptotic stability is provided by the following proposition (which is actually quite similar to the eigenvalue test).

[2] We use y' as the new output in order to obtain a conjugate input–output pair. Notice that by doing this, we have $\mathrm{d}H_d/\mathrm{d}t = (y')^T v - (\partial^T H_d/\partial x)(x)R_d(x)(\partial H_d/\partial x)(x)$.

Proposition 6. *Suppose that the closed-loop system* (10)–(11) *with* $v(t)=0$ *is stable with* $R_{\rm d}\geqslant 0$, *and the kernel of* $R_{\rm d}$ *is spanned by* $\{r_1,r_2,\ldots,r_k\}$. *This system is asymptotically stable iff* $[sI-Q_{\rm d}J_{\rm d}][r_1|r_2|\ldots|r_k]$ *has rank* k *for every* $s={\rm j}\omega$, $\omega\in\mathbb{R}$.

Proof. According to LaSalle's invariance principle, any trajectory of the system will converge to the largest invariant set contained in $U=\{x\,|\,x^{\rm T}Q_{\rm d}R_{\rm d}Q_{\rm d}x=0\}$. We shall look first at the set of all system trajectories in U. Since $R_{\rm d}$ is a symmetric positive semi-definite matrix, the steady-state trajectory $x_{\rm ss}(t)$ must satisfy $R_{\rm d}Q_{\rm d}x_{\rm ss}(t)=0$, or in other words, $Q_{\rm d}x_{\rm ss}(t)\in\ker R_{\rm d}$, for every t. Therefore $Q_{\rm d}x_{\rm ss}(t)$ can be written as

$$Q_{\rm d}x_{\rm ss}(t)=r_1c_1(t)+\cdots+r_kc_k(t), \tag{12}$$

for some periodic or constant scalar functions $c_1(t),\ldots,c_k(t)$. Premultiplying the state equation $\dot{x}=(J_{\rm d}-R_{\rm d})Q_{\rm d}x$ by $Q_{\rm d}$, and substituting $Q_{\rm d}\dot{x}_{\rm ss}(t)$ as well as $Q_{\rm d}x_{\rm ss}(t)$ into the equation, we get

$$\begin{aligned} r_1\dot{c}_1(t)+\cdots+r_k\dot{c}_k(t)&=Q_{\rm d}(J_{\rm d}-R_{\rm d})(r_1c_1(t)+\cdots+r_kc_k(t)),\\ &=Q_{\rm d}J_{\rm d}(r_1c_1(t)+\cdots+r_kc_k(t)) \end{aligned} \tag{13}$$

Since $c_1(t),\ldots,c_k(t)$ are bounded, the Laplace transform of (13) exists. After applying the Laplace transform and rearranging the terms in the equation, we obtain

$$[sI-Q_{\rm d}J_{\rm d}][r_1|r_2|\ldots|r_k]\begin{bmatrix}c_1(s)\\ \vdots\\ c_k(s)\end{bmatrix}=0. \tag{14}$$

Since we are interested in periodic or constant $c_1(t),\ldots,c_k(t)$, we just have to focus our attention on $s\in{\rm j}\omega$. Eq. (14) has a unique solution $c_1(s),\ldots,c_k(s)$ all equal to zero (which implies that $c_1(t)=\cdots=c_k(t)=0$) iff $[sI-Q_{\rm d}J_{\rm d}][r_1|r_2|\ldots|r_k]$ has rank k for every $s\in{\rm j}\omega$. It means that the invariant set contained in U is just the origin, and hence the system is asymptotically stable, iff $[sI-Q_{\rm d}J_{\rm d}][r_1|r_2|\ldots|r_k]$ has rank k for every $s\in{\rm j}\omega$. □

We are now ready to present the main results of this paper. That is, we shall look at ways to find $J_{\rm d},R_{\rm d},Q_{\rm d},F$ satisfying all conditions mentioned in Corollary 5. They will be formulated in sets of LMIs.

Proposition 7. *Denote by* $B_\perp$ *a full row rank* $(n-m)\times n$ *matrix that annihilates* B, *i.e.*, $B_\perp B=0$. *Let us also denote* $B_\perp(J-R)Q$ *by* $E_\perp$. *There exist matrices* $J_{\rm d},R_{\rm d},Q_{\rm d},F$ *that satisfy all conditions in Corollary* 5 *iff we can find a solution* $X=X^{\rm T}\in\mathbb{R}^{n\times n}$ *of the following LMIs*:

$$X>0, \tag{15}$$

$$-[E_\perp XB_\perp^{\rm T}+B_\perp XE_\perp^{\rm T}]\geqslant 0. \tag{16}$$

Given such an X, *compute* $S_{\rm d}$ *as follows*:

$$S_{\rm d}=\begin{bmatrix}B_\perp\\ B^{\rm T}\end{bmatrix}^{-1}\begin{bmatrix}E_\perp X\\ -B^{\rm T}XE_\perp^{\rm T}(B_\perp B_\perp^{\rm T})^{-1}B_\perp\end{bmatrix}, \tag{17}$$

then matrices $J_{\rm d},R_{\rm d},Q_{\rm d},F$ *that satisfy all conditions in Corollary* 5 *are*, *for instance*, *given by*

$$J_{\rm d}=\tfrac{1}{2}(S_{\rm d}-S_{\rm d}^{\rm T}), \tag{18}$$

$$R_{\rm d}=-\tfrac{1}{2}(S_{\rm d}+S_{\rm d}^{\rm T}), \tag{19}$$

$$Q_d = X^{-1}, \tag{20}$$

$$F = (B^T B)^{-1} B^T (S_d X^{-1} - (J - R)Q). \tag{21}$$

Proof. First, we define S_d as follows:

$$S_d \triangleq J_d - R_d. \tag{22}$$

Using this variable, it can be straightforwardly verified that all conditions in Corollary 5 can be rewritten as

$$Q_d = Q_d^T > 0, \tag{23}$$

$$2R_d = (R_d + R_d^T) = -(S_d + S_d^T) \geqslant 0, \tag{24}$$

$$S_d Q_d = (J - R)Q + BF. \tag{25}$$

Hence, we only need to prove that there exist[3] Q_d, S_d, and F which satisfy (23)–(25) if and only if there exists $X = X^T$ such that the LMIs (15)–(16) are satisfied.

To prove the "only if" part, we proceed as follows. Suppose that Q_d, S_d, and F which satisfy (23)–(25) exist. Define $X \triangleq Q_d^{-1}$. Obviously $X = X^T$ and $X > 0$. Next, postmultiply Eq. (25) by X to obtain

$$S_d = (J - R)QX + BFX \tag{26}$$

and substitute this equation into (24). As a result, we have

$$-[(J - R)QX + BFX + XQ(J - R)^T + XF^T B^T] \geqslant 0. \tag{27}$$

Finally, premultiply the last inequality by the full row rank matrix $B_\perp$ and postmultiply by $B_\perp^T$ to obtain

$$-[B_\perp (J - R)QXB_\perp^T + B_\perp XQ(J - R)^T B_\perp^T] \geqslant 0, \tag{28}$$

which is nothing but the LMI (16). Therefore, we conclude that there exists $X = X^T$ such that the LMIs (15)–(16) are satisfied.

Now we shall prove the "if" part. Suppose that the matrix $X = X^T$ which satisfies the LMIs (15)–(16) exists. Use X^{-1} as Q_d, then we immediately have $Q_d = Q_d^T > 0$. In addition, choose S_d as in Eq. (17). With this choice of S_d, we obtain

$$-\begin{bmatrix} B_\perp \\ B^T \end{bmatrix} [S_d + S_d^T] \begin{bmatrix} B_\perp \\ B^T \end{bmatrix}^T = -\begin{bmatrix} E_\perp XB_\perp^T + B_\perp XE_\perp^T & 0 \\ 0 & 0 \end{bmatrix}. \tag{29}$$

Since $-(E_\perp XB_\perp^T + B_\perp XE_\perp^T) \geqslant 0$, it follows that

$$-\begin{bmatrix} B_\perp \\ B^T \end{bmatrix} [S_d + S_d^T] \begin{bmatrix} B_\perp \\ B^T \end{bmatrix}^T \geqslant 0. \tag{30}$$

Notice that since B and $B_\perp^T$ have full column rank, $B^T B$ and $B_\perp B_\perp^T$ are invertible. Furthermore,

$$\begin{bmatrix} B_\perp \\ B^T \end{bmatrix}$$

[3] This is equivalent to the existence of J_d, R_d, Q_d, and F that satisfy all conditions in Corollary 5.

is also an invertible matrix, with the inverse given by

$$[\,B_\perp^{\mathrm T} \quad B\,]\begin{bmatrix} B_\perp B_\perp^{\mathrm T} & 0 \\ 0 & B^{\mathrm T}B \end{bmatrix}^{-1}.$$

Inequality (30) and the invertibility of

$$\begin{bmatrix} B_\perp \\ B^{\mathrm T} \end{bmatrix}$$

imply that $-(S_{\mathrm d}+S_{\mathrm d}^{\mathrm T}) \geqslant 0$. Hence, the only task left at this point is to show that Eq. (25) can be satisfied by a proper choice of F. For this purpose, choose F as in Eq. (21), and observe that

$$\begin{aligned}\begin{bmatrix} B_\perp \\ B^{\mathrm T} \end{bmatrix}[S_{\mathrm d}Q_{\mathrm d}-(J-R)Q-BF] &= \begin{bmatrix} B_\perp S_{\mathrm d}Q_{\mathrm d}-B_\perp(J-R)Q \\ B^{\mathrm T}S_{\mathrm d}Q_{\mathrm d}-B^{\mathrm T}(J-R)Q-B^{\mathrm T}BF \end{bmatrix} \\ &= \begin{bmatrix} E_\perp XQ_{\mathrm d}-B_\perp(J-R)Q \\ B^{\mathrm T}S_{\mathrm d}Q_{\mathrm d}-B^{\mathrm T}(J-R)Q-B^{\mathrm T}BF \end{bmatrix} = 0. \end{aligned} \tag{31}$$

Since

$$\begin{bmatrix} B_\perp \\ B^{\mathrm T} \end{bmatrix}$$

is invertible, it immediately follows that $S_{\mathrm d}Q_{\mathrm d}-(J-R)Q-BF=0$, or in other words, Eq. (25) has been satisfied. This ends the proof of the "if" part.

Finally, from Eq. (22) and the proof of the "if" part, it is straightforward to see that $J_{\mathrm d}$, $R_{\mathrm d}$, $Q_{\mathrm d}$, and F as in Eqs. (18)–(21) satisfy all conditions in Corollary 5. □

Remark 8. If we want to enforce the asymptotic stability of the closed-loop system, then we can simply replace the nonstrict inequality in (16) by a strict inequality and use for instance

$$S_{\mathrm d} = \begin{bmatrix} B_\perp \\ B^{\mathrm T} \end{bmatrix}^{-1} \begin{bmatrix} E_\perp X \\ -B^{\mathrm T}XE_\perp^{\mathrm T}(B_\perp B_\perp^{\mathrm T})^{-1}B_\perp-\gamma B^{\mathrm T} \end{bmatrix}, \tag{32}$$

where γ is any positive constant. This will ensure that $R_{\mathrm d}=-\frac{1}{2}[S_{\mathrm d}+S_{\mathrm d}^{\mathrm T}]>0$, which implies that the system is asymptotically stable.

As an additional result, another necessary and sufficient condition for a solution of the LMIs (15)–(16) to exist will be stated in the following proposition.

Proposition 9. *The set of LMIs* (15)–(16) *is solvable iff the system* $((J-R)Q,B)$ *is* (*weakly*) *stabilizable, in the sense that the uncontrollable part of* $((J-R)Q,B)$ *is stable.*

Proof. The necessity part is apparent, i.e., if $((J-R)Q,B)$ is not (weakly) stabilizable, then $J_{\mathrm d},R_{\mathrm d},Q_{\mathrm d},F$ which satisfy the conditions of Corollary 5 cannot exist, and obviously the LMIs (15)–(16) have no solution (for if these LMIs have a solution, then $J_{\mathrm d},R_{\mathrm d},Q_{\mathrm d},F$ exist—a contradiction).

To prove the sufficiency part, we use the fact that if $((J-R)Q,B)$ is (weakly) stabilizable, then there exists F such that the closed-loop matrix $\tilde{A}=(J-R)Q+BF$ is stable. This implies the existence of a symmetric positive definite matrix Q_d, a skew-symmetric matrix J_d, and a symmetric matrix $R_d \geqslant 0$ such that $\tilde{A} = (J_d - R_d)Q_d$ (cf. Proposition 1). Now define $S_d = (J_d - R_d)$. Obviously $-(S_d + S_d^T) \geqslant 0$. In addition, we have the equality $\tilde{A} = S_d Q_d = (J - R)Q + BF$. All of these imply that there exist Q_d, S_d, and F which satisfy (23)–(25), or equivalently, the set of LMIs (15)–(16) is solvable. □

Remark 10. If we use a strict inequality in (16), then the LMIs are solvable iff $((J - R)Q, B)$ is stabilizable, in the usual sense that the uncontrollable part of $((J - R)Q, B)$ is asymptotically stable. The proof is similar to the nonstrict inequality case; we only need to replace "stable" by "asymptotically stable" and nonstrict inequalities by strict inequalities.

Although we can use Proposition 7 to obtain a stabilizing controller, notice that the matrix S_d does not appear explicitly in the LMIs (15)–(16). This has a disadvantage of us not being able to directly determine which part of the interconnection matrix should be modified and where the damping should be added. Sometimes we may want to enforce S_d to have certain properties. For example, we may want some entries in S_d to be equal to zero. Hence, it may be of interest to see if we can find a set of LMIs containing S_d explicitly, and equivalent to (15)–(16). In case the system is controllable at $s = 0$ (has no uncontrollable pole at $s = 0$), which is equivalent to $[B\ (J - R)Q]$ having rank n, such LMIs can indeed be found. The following lemma is useful in obtaining those LMIs.

Lemma 11. *If the system $((J - R)Q, B)$ is controllable at $s = 0$, then $E_\perp = B_\perp(J - R)Q \in \mathbb{R}^{(n-m)\times n}$ has full row rank.*

Proof. Since the system is controllable at $s = 0$, we know from the controllability rank test that rank $[B\ sI - (J - R)Q] = n$ for $s = 0$, or equivalently rank $[B\ (J - R)Q] = n$. Premultiply $[B\ (J - R)Q]$ by the invertible matrix

$$\begin{bmatrix} B_\perp \\ B^T \end{bmatrix}.$$

This does not change the rank, i.e.,

$$\text{rank} \begin{bmatrix} B_\perp \\ B^T \end{bmatrix} [B \quad (J - R)Q] = \text{rank} \begin{bmatrix} 0 & B_\perp(J - R)Q \\ B^T B & B^T(J - R)Q \end{bmatrix} = n. \tag{33}$$

But $B^T B$ is an $m \times m$ invertible matrix, so rank$[B_\perp(J - R)Q] = n - m$. Since $E_\perp = B_\perp(J - R)Q$, we can immediately conclude that $E_\perp$ is a full row rank matrix. □

Now, a set of LMIs explicitly containing S_d is provided by the following proposition.

Proposition 12. *Suppose that the system $((J - R)Q, B)$ is controllable at $s = 0$. Denote by $B_\perp$ a full row rank $(n - m) \times n$ matrix that annihilates B, and also denote $B_\perp(J - R)Q$ by $E_\perp$. Furthermore, let $E \in \mathbb{R}^{n\times m}$ be a full column rank matrix that is annihilated by $E_\perp$, i.e., $E_\perp E = 0$. There exist matrices J_d, R_d, Q_d, and F that satisfy all conditions in Corollary 5 iff we can find a solution $S_d \in \mathbb{R}^{n\times n}$ of the following LMIs*:

$$[B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T] > 0, \tag{34}$$

$$-[S_d + S_d^T] \geqslant 0, \tag{35}$$

together with a linear constraint

$$B_\perp S_d E_\perp^T - E_\perp S_d^T B_\perp^T = 0. \tag{36}$$

Given such an S_d, *then matrices* J_d, R_d, Q_d, F *that satisfy all conditions in Corollary* 5 *are for instance provided by*

$$J_d = \tfrac{1}{2}(S_d - S_d^T), \tag{37}$$

$$R_d = -\tfrac{1}{2}(S_d + S_d^T), \tag{38}$$

$$Q_d = X^{-1}, \tag{39}$$

$$F = (B^T B)^{-1} B^T (S_d X^{-1} - (J - R)Q), \tag{40}$$

where X *is*

$$X = \frac{1}{2}\left(\begin{bmatrix} E_\perp \\ E^T \end{bmatrix}^{-1} \begin{bmatrix} B_\perp S_d \\ Z \end{bmatrix} + \begin{bmatrix} B_\perp S_d \\ Z \end{bmatrix}^T \begin{bmatrix} E_\perp \\ E^T \end{bmatrix}^{-T} \right) \tag{41}$$

with

$$Z = [E_\perp^T (E_\perp E_\perp^T)^{-1} (B_\perp S_d E) + \gamma E]^T. \tag{42}$$

Here γ *is a positive constant, which has to be large enough in order to get a positive definite* X (*this will be made clear in the proof of the proposition*).

Proof. Since the system is controllable at $s=0$, by Lemma 11 we know that $E_\perp$ is a full row rank $(n-m)\times m$ matrix. This implies that

$$\begin{bmatrix} E_\perp \\ E^T \end{bmatrix}$$

is nonsingular. Following the same path as in the proof of Proposition 7, we shall show that there exist Q_d, S_d, and F which satisfy (23)–(25) if and only if there exists S_d such that (34)–(36) are satisfied.

To prove the necessity part, we proceed as follows. Suppose that Q_d, S_d, and F which satisfy (23)–(25) exist. Automatically the LMI (35) is satisfied. Now premultiply Eq. (25) by $B_\perp$ to obtain

$$B_\perp S_d Q_d = B_\perp (J - R) Q = E_\perp. \tag{43}$$

Since $E_\perp$ has full row rank and $Q_d > 0$, it follows that $B_\perp S_d$ also has full row rank. Furthermore, since Q_d is positive definite, we have $Q_d + Q_d^T > 0$, which in turn implies that $B_\perp S_d (Q_d + Q_d^T) S_d^T B_\perp^T > 0$, or

$$B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T > 0. \tag{44}$$

Hence, the LMI (34) is satisfied. Finally, we only have to show that Eq. (36) is also fulfilled. Since $Q_d = Q_d^T$, we have

$$0 = B_\perp S_d (Q_d - Q_d^T) S_d^T B_\perp^T = B_\perp S_d E_\perp^T - E_\perp S_d^T B_\perp^T. \tag{45}$$

In other words, the linear constraint (36) holds.

We shall continue with proving the sufficiency part. Suppose that S_d which satisfies (34)–(36) exists. Then (24) automatically holds. Next, define Z and X as in Eqs. (42) and (41). For these choices of Z and X, we have

$$\begin{bmatrix} E_\perp \\ E^T \end{bmatrix} X \begin{bmatrix} E_\perp \\ E^T \end{bmatrix}^T = \frac{1}{2}\begin{bmatrix} B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T & B_\perp S_d E + E_\perp Z^T \\ E^T S_d^T B_\perp^T + Z E_\perp^T & ZE + E^T Z^T \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T & 2B_\perp S_d E \\ 2E^T S_d^T B_\perp^T & 2\gamma E^T E \end{bmatrix}. \tag{46}$$

Using the Schur complements, we know that (46) is positive definite iff the following two inequalities are fulfilled:

$$B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T > 0, \tag{47}$$

$$2\gamma E^T E - (2B_\perp S_d E)(B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T)^{-1}(2E^T S_d^T B_\perp^T) > 0. \tag{48}$$

The first inequality already holds, because of (34). Moreover, since E has full column rank, we can make the second inequality to be fulfilled by choosing γ sufficiently large. Therefore (46) can be made positive definite. This will further imply that $X > 0$. Use X^{-1} as Q_d and obviously we have $Q_d = Q_d^T > 0$.

Next, we shall show that Eq. (25) can be satisfied. For this purpose, choose F as in Eq. (40) and observe that

$$\begin{bmatrix} B_\perp \\ B^T \end{bmatrix} (S_d - (J-R)QX - BFX) \begin{bmatrix} E_\perp \\ E^T \end{bmatrix}^T$$

$$= \begin{bmatrix} B_\perp S_d - E_\perp X \\ 0 \end{bmatrix} \begin{bmatrix} E_\perp \\ E^T \end{bmatrix}^T$$

$$= \begin{bmatrix} B_\perp S_d E_\perp^T - E_\perp X E_\perp^T & B_\perp S_d E - E_\perp X E \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(B_\perp S_d E_\perp^T - E_\perp S_d^T B_\perp^T) & \frac{1}{2}(B_\perp S_d E - E_\perp Z^T) \\ 0 & 0 \end{bmatrix}$$

$$= 0, \tag{49}$$

where the last equality follows because of Eq. (36). Since

$$\begin{bmatrix} B_\perp \\ B^T \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} E_\perp \\ E^T \end{bmatrix}^T$$

are invertible, we can conclude that $S_d - (J-R)QX - BFX = 0$. Postmultiply it by $X^{-1} = Q_d$ to obtain $S_d Q_d - (J-R)Q - BF = 0$, which is nothing but Eq. (25). This ends the proof of the sufficiency part.

At the end, it is straightforward to show that J_d, R_d, and Q_d are given by Eqs. (37)–(39). □

For a system that is controllable at $s=0$ (as required by Proposition 12), the LMIs (34)–(35) together with the linear constraint (36) are equivalent to the set of LMIs (15)–(16). Hence, we can infer from Proposition 9 that the set of LMIs (34)–(35) with constraint (36) is solvable iff the system is stabilizable, in addition to being controllable at $s=0$. However, if we impose some conditions on S_d (e.g. by enforcing some entries to be equal to zero), then those LMIs may no longer be solvable even if the system is stabilizable.

In several software packages, it is often not possible to directly solve the LMIs (34)–(35) with the linear constraint (36). To overcome this obstruction we can either parametrize S_d so that Eq. (36) is satisfied, or recast the problem as a generalized eigenvalue problem (GEVP, see [1]), which can be efficiently solved. One way to do it is summarized in the following remark.

Remark 13. The problem of finding S_d such that (34)–(36) are satisfied can be recast as: find S_d that minimizes δ, subject to

$$[B_\perp S_d E_\perp^T + E_\perp S_d^T B_\perp^T] > 0, \tag{50}$$

$$-[S_d + S_d^T] \geqslant 0, \tag{51}$$

$$\begin{bmatrix} I & B_\perp S_d E_\perp^T - E_\perp S_d^T B_\perp^T \\ E_\perp S_d^T B_\perp^T - B_\perp S_d E_\perp^T & \delta I \end{bmatrix} > 0. \tag{52}$$

Indeed, there exists a solution of (34)–(36) if and only if the GEVP of $\min \delta$ over S_d subject to the above LMIs has a solution $\delta = 0$.

4. An example: mass–spring system

Consider a system of three masses and two springs, connected in series, with one of the spring constants being *negative*. The motion of such a system is governed by the following set of equations (in PCH form):

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_3 \\ -I_3 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H}{\partial q}(q,p) \\ \dfrac{\partial H}{\partial p}(q,p) \end{bmatrix} + \begin{bmatrix} 0 \\ B_p \end{bmatrix} u, \tag{53}$$

$$y = [0 \quad B_p^T] \begin{bmatrix} \dfrac{\partial H}{\partial q}(q,p) \\ \dfrac{\partial H}{\partial p}(q,p) \end{bmatrix}, \tag{54}$$

where $q = (q_1, q_2, q_3)^T$ is the vector of generalized coordinates, which corresponds to the position of Mass I, Mass II, and Mass III; $p = (p_1, p_2, p_3)^T$ is the vector of generalized momenta; and I_3 is the 3×3 identity matrix. If we assume that the system is actuated by a force acting on Mass I, then the input u is a scalar (the force), the conjugate output y is also a scalar (the velocity of Mass I), and $B_p = [1\ 0\ 0]^T$. The Hamiltonian of the system is given by

$$H(q,p) = \frac{1}{2}\left[\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} + \frac{p_3^2}{m_3} + k_1(q_1 - q_2)^2 + k_2(q_2 - q_3)^2\right] \tag{55}$$

with m_1, m_2, m_3, respectively, the masses of Masses I, II, and III; and k_1, k_2 the spring constants of Springs I and II. We assume that all m's and k_1 are positive, but k_2 is negative.

To this end, we assign some numerical values as follow: $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, $k_1 = 1$, $k_2 = -2$. Notice that the Hessian Q of $H(q,p)$ is not positive definite, which implies that the uncontrolled system is unstable. Also notice that the system is controllable. Therefore, we may use either Proposition 7 or 12 in order to stabilize the system with respect to the origin.

First, we use Proposition 7. The following 5×6 matrix is used as $B_\perp$:

$$B_\perp = \begin{bmatrix} I_3 & 0 & 0 \\ 0 & 0 & I_2 \end{bmatrix}. \tag{56}$$

If we straightforwardly solve the LMIs (15)–(16), then we may obtain a matrix X that is close to singular. Besides, it may also happen that the eigenvalues of X are quite large, such that in turn Q_d is close to singular. To prevent this, (15) should be slightly modified. For example, we could set the following condition: $0.01 \times I_6 < X < 50 \times I_6$. In addition, to obtain sufficient but not excessive damping, we could modify (16) to $0.001 \times I_5 < -[E_\perp X B_\perp^T + B_\perp X E_\perp^T] < 10 \times I_5$.

The LMIs can be solved using the LMI Control Toolbox for Matlab [3]. After performing the necessary manipulations mentioned in Proposition 7 and using S_d as in Eq. (32) with $\gamma = 1$, we arrive at these results:

$$J_d = \begin{bmatrix} 0 & 1.3502 & 1.1780 & 34.6667 & 3.2513 & 8.9287 \\ -1.3502 & 0 & 0.2264 & -1.5073 & 2.9098 & 5.5749 \\ -1.1780 & -0.2264 & 0 & 2.8204 & 3.9811 & 7.8331 \\ -34.6667 & 1.5073 & 2.8204 & 0 & -0.0473 & 3.2285 \\ 3.2513 & -2.9098 & -3.9811 & 0.0473 & 0 & 0.5891 \\ 8.9287 & -5.5749 & -7.8331 & -3.2285 & -0.5891 & 0 \end{bmatrix},$$

$$R_d = \begin{bmatrix} 2.0329 & 0.1782 & 0.0296 & 0 & -0.2368 & -0.4674 \\ 0.1782 & 0.8566 & 0.4636 & 0 & -0.8139 & 0.3371 \\ 0.0296 & 0.4636 & 0.8690 & 0 & 0.4893 & -0.3163 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 \\ -0.2368 & -0.8139 & 0.4893 & 0 & 2.4778 & -0.4845 \\ -0.4674 & 0.3371 & -0.3163 & 0 & -0.4845 & 2.0895 \end{bmatrix},$$

$$Q_d = \begin{bmatrix} 0.4049 & -2.9896 & 2.5308 & 0.0883 & -1.1969 & 0.6201 \\ -2.9896 & 26.6066 & -22.6899 & -0.7271 & 10.9884 & -5.5696 \\ 2.5308 & -22.6899 & 19.4175 & 0.6181 & -9.3946 & 4.7636 \\ 0.0883 & -0.7271 & 0.6181 & 0.0521 & -0.3032 & 0.1654 \\ -1.1969 & 10.9884 & -9.3946 & -0.3032 & 4.9011 & -2.4537 \\ 0.6201 & -5.5696 & 4.7636 & 0.1654 & -2.4537 & 1.2777 \end{bmatrix},$$

$$F = [-8.4339 \quad 61.9730 \quad -52.9627 \quad -1.9163 \quad 23.7081 \quad -12.3810].$$

The eigenvalues of R_d are 3.2016, 2.4738, 1.3594, 1.2860, 1.0000, and 0.0050; while the eigenvalues of Q_d are 52.0974, 0.3852, 0.0790, 0.0443, 0.0306, and 0.0234. These results indicate that we have managed to asymptotically stabilize the system.

However, in the results above, we see that J_d and R_d have quite irregular structures. Suppose we want to impose some prespecified structures on J_d and R_d. Let us say that we shall give some coupling on the lower-left and upper-right 3×3 submatrices of the interconnection matrix, and add damping only on the diagonal entries. This corresponds to J_d and R_d of the form (notice the skew-symmetry of J_d):

$$J_d = \begin{bmatrix} 0 & 0 & 0 & *_1 & *_2 & 0 \\ 0 & 0 & 0 & *_3 & *_4 & *_5 \\ 0 & 0 & 0 & 0 & *_6 & *_7 \\ -*_1 & -*_3 & 0 & 0 & 0 & 0 \\ -*_2 & -*_4 & -*_6 & 0 & 0 & 0 \\ 0 & -*_5 & -*_7 & 0 & 0 & 0 \end{bmatrix}, \quad R_d = \begin{bmatrix} *_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & *_9 & 0 & 0 & 0 & 0 \\ 0 & 0 & *_{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & *_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & *_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & *_{13} \end{bmatrix}.$$

Here we use Proposition 12 to obtain a stabilizing feedback. We use the same $B_\perp$ as in the previous calculation, whereas $E = [1\ 1\ 1\ 0\ 0\ 0]^{\mathrm{T}}$. To ensure that we get some but not excessive damping, the LMI (35) is slightly modified to $0 < -[S_d + S_d^{\mathrm{T}}] < I_6$. Furthermore, to prevent Q_d and Q_d^{-1} from being too close to singular, we also change the LMI (34) to $0.01 E_\perp E_\perp^{\mathrm{T}} < [B_\perp S_d E_\perp^{\mathrm{T}} + E_\perp S_d^{\mathrm{T}} B_\perp^{\mathrm{T}}] < 100 E_\perp E_\perp^{\mathrm{T}}$.

Since the LMI Control Toolbox for Matlab cannot solve (34)–(36) directly, we use the method described in Remark 13 to obtain S_d. The resulting δ is 2.6836×10^{-11}, which is small enough to be considered equal to zero. In addition, we choose $\gamma = 500$ in order to get a positive definite X. At the end, we arrive at the following J_d, R_d, Q_d, and F:

$$J_d = \begin{bmatrix} 0 & 0 & 0 & 32.1066 & 2.1567 & 0 \\ 0 & 0 & 0 & 1.0783 & 5.0371 & 8.3407 \\ 0 & 0 & 0 & 0 & 5.5605 & 9.5192 \\ -32.1066 & -1.0783 & 0 & 0 & 0 & 0 \\ -2.1567 & -5.0371 & -5.5605 & 0 & 0 & 0 \\ 0 & -8.3407 & -9.5192 & 0 & 0 & 0 \end{bmatrix},$$

$$R_d = \begin{bmatrix} 0.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 25.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0000 \end{bmatrix},$$

$$Q_d = \begin{bmatrix} 0.5577 & -3.7483 & 3.1792 & 0.0000 & -0.0000 & 0.0000 \\ -3.7483 & 29.7794 & -25.8825 & -0.0000 & 0.0004 & -0.0002 \\ 3.1792 & -25.8825 & 22.5730 & 0.0000 & -0.0003 & 0.0002 \\ 0.0000 & -0.0000 & 0.0000 & 0.0555 & -0.3626 & 0.2118 \\ -0.0000 & 0.0004 & -0.0003 & -0.3626 & 5.3975 & -3.1528 \\ 0.0000 & -0.0002 & 0.0002 & 0.2118 & -3.1528 & 1.8767 \end{bmatrix},$$

$$F = [-12.8652 \quad 88.2343 - 75.1641 \quad -1.3876 \quad 9.0651 \quad -5.2952].$$

We can immediately observe that the prespecified structures have been successfully imposed on J_d and R_d. The eigenvalues of Q_d are 52.7709, 7.2725, 0.1373, 0.0312, 0.0259, and 0.0020. It can be verified that the closed-loop system is asymptotically stable. This is because the eigenvalues of $S_d Q_d$ are -0.5726 ± 3.3804i, -0.0989 ± 0.6844i, and -0.0223 ± 0.1080i.

5. Conclusions

In the preceeding sections, it has been shown that controllers for stabilizing linear PCH systems via interconnection and damping assignment can be obtained by solving a set of LMIs. Two sets of (almost) equivalent LMIs have been proposed for this purpose. In the first set, the interconnection and damping matrices do not appear explicitly, which makes it difficult to impose a prespecified structure on those matrices, but on the other hand allows for an easy manipulation of the shaped energy matrix. This set of LMIs is solvable iff the system is stabilizable. By requiring the system to be controllable at $s = 0$, the second set of LMIs can be obtained. In this alternative set, the interconnection and damping matrices appear explicitly, hence some prespecified structures can be imposed directly on those matrices.

An open problem, which we did not address in the current paper, is to fully characterize the "minimal" change in the interconnection structure matrix J that is necessary to stabilize the PCH system (although for a specific example this can be quite easily checked by substitution of the "required" J_{d} or S_{d} in the LMIs of Proposition 12). This is of importance since the interconnection structure captures the "topology" of the physical system under consideration [2,5,12,11], and in many cases we would like to remain as closely as possible to the original topology of the system. For example, if the original interconnection structure determines dynamical invariants for the original system, then it is certainly of interest to maintain these dynamical invariants (which usually have a clear physical interpretation) as much as possible.

As remarked in Proposition 1, any stable linear dynamics $\dot{x} = (A + BF)x$ for some feedback matrix F can be written in the form $\dot{x} = (J_{\mathrm{d}} - R_{\mathrm{d}})Q_{\mathrm{d}}x$ for some positive definite symmetric matrix Q_{d}, some skew-symmetric matrix J_{d} and some positive semi-definite matrix R_{d}. Hence, the stabilization procedure via interconnection and damping assignment for linear PCH systems can be also interpreted as an alternative way of *pole-placement* by state feedback, in much the same way as other control methodologies (such as LQ-control or H_∞-control) can be seen as a way of parametrizing (a subset of) the class of stabilizing linear state feedbacks F. An advantage of stabilization by interconnection and damping assignment is that we can keep track of the required changes in the interconnection and damping structures, so that the closed-loop system still can be given a physical interpretation and the stabilizing feedback does not completely "destroy" the physical characteristics of the system.

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References

[1] S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Studies in Applied Mathematics, Vol. 15, SIAM, Philadelphia, PA, 1994.

[2] M. Dalsmo, A.J. van der Schaft, On representations and integrability of mathematical structures in energy-conserving physical systems, SIAM J. Control Optim. 37 (1) (1999) 54–91.

[3] P. Gahinet, A. Nemirovski, A.J. Laub, M. Chilali, LMI Control Toolbox, The Mathworks, Inc., Natick, MA, 1995.

[4] N.H. Hogan, Impedance control: an approach to manipulation. Part I—theory; Part II—implementation, Part III—applications, J. Dyn. Systems Measurements Control 107 (1985) 1–24.

[5] B.M. Maschke, A.J. van der Schaft, Port-controlled Hamiltonian systems: modelling origins and system theoretic properties, Proceedings of the Second IFAC Symposium on Nonlinear Control Systems Design, NOLCOS92, Bordeaux, France, 1992, pp. 282–288.

[6] R. Ortega, M.W. Spong, Stabilization of underactuated mechanical systems via interconnection and damping assignment, in: N.E. Leonard, R. Ortega (Eds.), IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control, Princeton, NJ, March 2000, Elsevier, Amsterdam, pp. 74–79.

[7] R. Ortega, A.J. van der Schaft, B. Maschke, G. Escobar, Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems, Automatica J. IFAC, to appear.

[8] R. Ortega, A.J. van der Schaft, B.M. Maschke, Stabilization of port-controlled Hamiltonian systems via energy balancing, in: D. Aeyels, F. Lamnabhi-Lagarrigue, A.J. van der Schaft (Eds.), Stability and Stabilization of Nonlinear Systems, Lecture Notes in Control and Information Science, Vol. 246, Springer, London, 1999, pp. 239–260.

[9] R. Ortega, A.J. van der Schaft, I. Mareels, B.M. Maschke, Putting energy back in control, Control Systems Mag. 21 (2001) 18–33.

[10] G.A. Pratt, Legged robots at MIT: What's new since Raibert, IEEE Robot. Automat. Mag. 7 (3) (2000) 15–19.

[11] S. Stramigioli, B. Maschke, A.J. van der Schaft, Passive output feedback and port interconnection, in: H.J.C. Huijberts, H. Nijmeijer, A. van der Schaft, J. Scherpen (Eds.), Proceedings of the Fourth IFAC Symposium on Nonlinear Control Systems Design, NOLCOS98, Enschede, The Netherlands, 1998, pp. 613–618.

[12] A.J. van der Schaft, L_2-Gain and Passivity Techniques in Nonlinear Control, Springer, London, 2000.