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Slotted High-T_c dc SQUID Magnetometers

A. B. M. Jansman, M. Izquierdo, J. Flokstra and H. Rogalla
Low Temperature Division, Department of Applied Physics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

***Abstract*—Recently, it was observed that the low frequency noise of slotted SQUIDs is not affected when cooling them down in magnetic fields up to 0.05 mT. This behavior is ascribed to the fact that magnetic vortices are expelled out of the narrow strips into the slots. Thus 1/f noise by hopping of vortices in the superconducting structure is eliminated. Here we present a systematic investigation on a series of slotted high-T_c dc SQUIDs. The number of slots varies from zero to eight. A model is developed that provides values for the effective area and inductance of the slotted washers and the current distribution within the slotted SQUIDs. The model can explain the experimentally observed maximum in the effective area for four slots very well. The observed agreement with respect to the modulation depth supports the theoretical values for the inductance. The white noise of the slotted SQUIDs is higher than expected, while the expected rise of low frequency noise for field cooled solid washer SQUIDs was not observed.**

I. INTRODUCTION

Some applications of high-T_c dc SQUIDs, like biomagnetism, require low field-noise values at frequencies in the order of 1 Hz. By proper modulation schemes one of the sources of 1/*f* noise, the out of phase variation in critical currents of the junctions can be eliminated. The other source, hopping of vortices in the superconducting material, is harder to get rid of. One can diminish the width of the superconducting washer to a value w. When cooled in a magnetic field B, lower than $\pi\Phi_0/4w^2$, vortices will not penetrate in the strip and thus will not cause 1/*f* noise [1]. By splitting the washer of a SQUID up into narrow strips, one can diminish the 1/*f* noise in an applied field while keeping the SQUID's effective area high and its inductance low [2].

In this work, we present a systematic investigation on slotted SQUIDs. A model is proposed accounting for the experimentally observed dependence of the effective area on the slot number. Other properties like the modulation depth and the white noise of slotted SQUIDs are fit to existing models with varying success. Finally, the flux noise of slotted SQUIDs at 1 Hz was determined, with and without an applied field. No clear dependence on the slot number is observed.

II. THEORY

A slotted SQUID washer can be regarded as a flux focusser consisting of a parallel set of inductances [3], [4]. The currents through the inductances are such that the total flux in each slot equals zero, except for the innermost slot which is opened through the junctions. If one knows the self inductances, mutual inductances between the strips and pick up area of the strips, one can calculate the total flux focussed in the SQUID hole. Unfortunately, since currents in superconductors are known to be far from homogeneous, classical formulas do not apply.

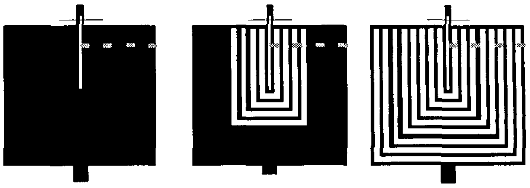

Fig. 1. Design of three slotted SQUIDs. The outer dimension of the washer is 204 × 182 μm. The slit width is 4 μm. Subsequently, slots of 8 μm width are cut out, leaving strips of 4 μm behind. The grey dotted lines are markers that will be referred to later. The thin solid lines represent the bicrystal grain boundary.

One can overcome this problem by dividing the structure up into many narrow concentric strips which can be assumed to carry a homogeneous current. By minimization of the total electromagnetic energy

$$E=\frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M}I_i\cdot M_{ij}I_j+\frac{1}{2}\sum_{i=1}^{M}L_{kin,i}\cdot I_i^2+\sum_{i=1}^{M}A_i\cdot I_i\cdot B, \quad (1)$$

under the restriction that the sum of currents ΣI_i equals zero since only an autonomous washer is considered, one comes to the result

$$\begin{aligned}&\sum_j M_{ij}I_j+L_{kin,i}\cdot I_i+A_i\cdot B=\beta,\\ &\sum_i I_i=0.\end{aligned}\quad (2)$$

Equation (2) manifests flux quantization in each coil i: the first term represents the flux caught up by coil i originating from a current I_j in coil j. The second term is the kinetic contribution to the flux while the third is the applied flux enclosed by coil i. From this it follows that β plays the role of the flux coupled into the innermost SQUID hole.

The self and mutual inductances can be obtained from classical formulas. The kinetic inductance of a coil is $\mu_0\lambda^2\cdot l/\sigma$, where l and σ are the length and the cross section of the coil. λ is the temperature dependent penetration depth of the superconducting film $\lambda(T)=\lambda(0)/\sqrt{(1-(T/T_c)^{\alpha}}$. Depending on the temperature range α can be either 2 or 4.

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A. B. M. Jansman a.b.m.jansman@tn.utwente.nl

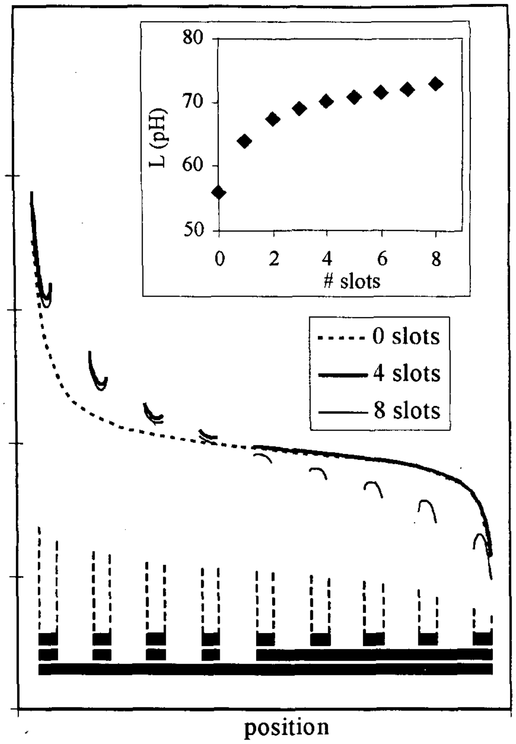

Fig. 2. Current distribution in the strips for slotted washers with 0, 4 and 8 slots in an applied field B. As a guide to the eye, a cross section of one half of the slotted SQUIDs is drawn. These cross sections are indicated in Fig. 1 as dotted lines. The inset shows the calculated inductance of the washer as a function of the number of slots. Here, $\lambda(0)$=267 nm.

Solving this set of equations provides us with the current distribution I_i and the effective area of the SQUID β/B. Given a choice of α only one free parameter is available, $\lambda(0)$. Fig. 2 displays the normalized current distribution in the SQUIDs with 0, 4 and 8 slots, respectively, for $\lambda(77.4\text{ K}) = 408$ nm. This corresponds to $\lambda(0) = 267$ nm if $\alpha=4$ or $\lambda(0) = 200$ nm if $\alpha=2$.

The inductances, L, of the slotted washers are found by solving the first of equations (2) while applying a net circular current through the washer: $\Sigma I_i = I_{appl}$ and taking the applied field equal to zero. Now the inductance of the washer can be calculated as $L=\beta/I_{appl}$.

III. EXPERIMENTAL

In order to verify our model, we fabricated two sets of slotted SQUIDs, with slot numbers varying from zero to eight. The 100 nm thick $YBa_2Cu_3O_{7-x}$ films were produced by pulsed laser deposition on two $SrTiO_3$ bicrystal substrates with 24° misorientation angle. The slotted SQUIDs with 2 μm wide junctions were structured by standard photolithography and room temperature Ar ion beam etching.

TABLE 1
GEOMETRICAL PROPERTIES AND PERFORMANCE OF SLOTTED SQUIDS AT 77 K.

Device	# slots	I_c (μA)[a]	R_n (Ω)[a]	$V_{mod,p\text{-}p}$ (μV)	A_{eff} (10^{-3}mm^2)	$S_\Phi^{1/2}$ (μΦ/√Hz)	L (pH)
WB 63	0	45	3.8	7	7.05	-	80
WB 63	0	100	2.8	6	6.81	-	80
WB 63	1	60	2.8	5	8.14	-	88
WB 63	2	61	2.6	3.5	8.64	-	92
WB 63	4	47	3.7	3	8.70	-	94
WB 63	6	67	2.7	3	8.43	-	96
WB 63	8	66	3	3.5	8.34	-	97
CR 12	0	41	7.5	23	6.48	59	80
CR 12	8	46	7.0	13	7.52	41	97
CR 12	0	157	2.1	11	-	57	80
CR 12	2	49	6.4	8.5	-	-	92
CR 12	8	28	9.9	13	-	-	97
CR 12	6	1.0	-	-	-	-	96
CR 12	6	60	5.2	6.5	8.57	54	96
CR 12	3	68	4.6	6	8.23	63	93
CR 12	4	39	3.3	7	8.10	-	94
CR 12	1	8.5	6.7	6	8.30	120	88
CR 12	5	19	5.1	5.5	8.41	80	95

[a] I_c and R_n for one junction

The effective area of the SQUIDs was determined by applying a homogeneous field from a calibrated Helmholtz coil set and a calibrated solenoid for WB 63 and CR12, respectively. The system was surrounded by μ-metal to shield the system from the environmental magnetic field. Current-voltage, voltage-flux and noise characteristics were measured in a He flow cryostat, in which the SQUIDs were surrounded by a superconducting shield. A field could be applied by means of a wire wound coil connected to a low noise current source. Table 1 shows the measured critical current I_c, normal resistance R_n of the junction, the modulation depth $V_{mod,\ p\text{-}p}$ and the white flux noise $S_\Phi^{1/2}$. The critical current of a junction was determined as $I_c=I_b/2 + k_BT/\Phi_0 \cdot(1+\sqrt{(1+I_b\cdot\Phi_0/k_BT)})$, where I_b is the SQUID's bias current for maximum modulation [5]. We see a large spread in the critical current and normal resistance values, while their product is more or less

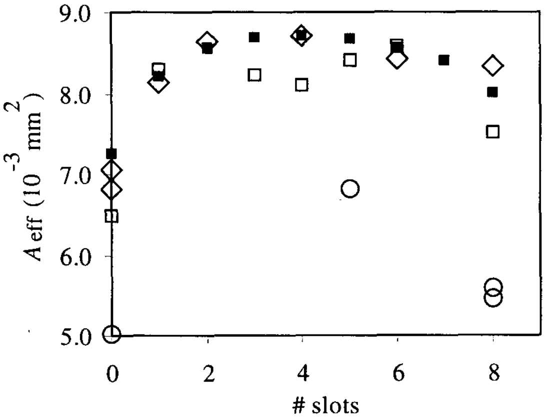

Fig. 3 Effective area for slotted SQUIDs at 77.4 K as it follows from our experiments (WB 63 ◊, CR12 □), the model with λ_0=267 nm (+) and the experiments from [2] (O).

constant for each substrate. Obviously, the effective junction width is not constant, although the physical width does not show this much variation.

The last column in the table shows the estimated inductance, being the sum of the washer's inductance as described above and a contribution from the junction striplines.

A. Effective area

The effective areas of the slotted SQUIDs are displayed in Fig. 3. Also displayed is a best fit obtained for $\lambda(0)$=267 nm and α=4, for which the best agreement with WB 63 is found. For CR 12 a somewhat lower value for $\lambda(0)$, 240 nm, leads to a better result. In further calculations $\lambda(0)$ = 267 nm and α=4 will be used. The results from [2] could not be fitted with a single value for $\lambda(0)$ = 267 nm. Even when the kinetic inductance contribution is omitted theory predicts higher effective area values than the experimental ones in Fig. 3. The authors cannot give an explanation for this.

We found that, independent of the value $\lambda(0)$, theory predicts a maximum in effective area for four slots. In Fig. 2 we see that the current distribution in the outermost part of the washer is similar for the solid and the four slot SQUID. The same is valid for the current distribution in the innermost part of the SQUIDs with four and eight slots, respectively. The difference in effective area must be explained by examining the innermost part of the SQUIDS in the first case and the outermost part in the second case, respectively. One can conclude that obviously, the current couples more effectively into the innermost hole when its trajectory is split up in different strips. For this reason there is an optimum in the effective area for four slots: the positive currents at the interior of the fourth slot couple effectively, while the current in the non-slotted outer structure couples less flux into the central hole. Although the values from [2] could not be fitted with a single value for $\lambda(0)$, rather good agreement is obtained for the SQUIDs on both bicrystal substrates. The best fit to our experimental results leads to $\lambda(0)$=267 and 240 nm for the two devices. These values are common for $YBa_2Cu_3O_{7-x}$ films indicating the applicability of the model [4], [6].

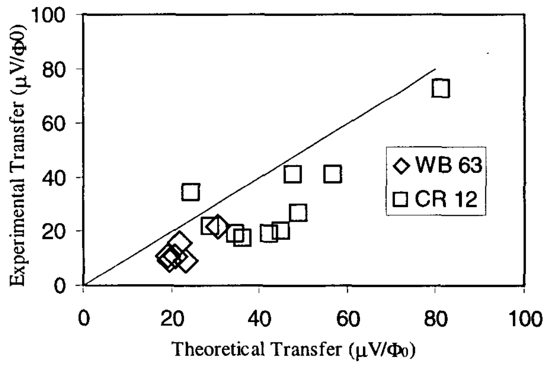

Fig. 4. Experimental vs. theoretical transfer function V_Φ. The dotted line represents perfect agreement between theory and experiment

B. Inductance and modulation depth

The dependence of the inductance on the geometry leads to a variation in the modulation depth. Using the inductances, critical currents and normal resistances at 77 K from Table 1 one can estimate the flux-to-voltage transfer, V_Φ, as [7], [8]

$$V_\Phi = K\cdot\frac{I_cR_n}{1+\beta_L}\cdot\left(1-3.57\cdot\frac{\sqrt{k_B\cdot T\cdot L}}{\Phi_0}\right), \tag{3}$$

where $\beta_L=2I_cL/\Phi_0$, k_B is Boltzmann's constant and Φ_0 is the flux quantum. The prefactor K can be either $7/\pi$ or 4. Fig. 4 shows the modulation depth for the SQUIDs for $K=7/\pi$. The experimental values are slightly smaller than predicted. For K=4, one needs a constant scaling factor to get agreement. The scaling with theory over a broad range of V_Φ values indicates that the inductance values following from the model are good. For SQUIDs with β_L values much higher than the optimum 1, the deviation from (3) grows as is also seen for other SQUIDs [7].

C. White noise

The flux noise of the SQUIDs was measured with standard flux locked loop electronics. Being surrounded by a BiSCCO superconducting shield, the SQUIDs were cooled down in a He flow cryostat to 77 K. By means of a copper wire wound 150 windings coil connected to a low noise current source, we could apply a magnetic field as high as 100 μT during cool down and measurement. In spite of its simplicity, this method did not affect the measured noise. With the coil connected, but not carrying a current, only an excess noise contribution at 50 Hz could be seen, due to the wiring between

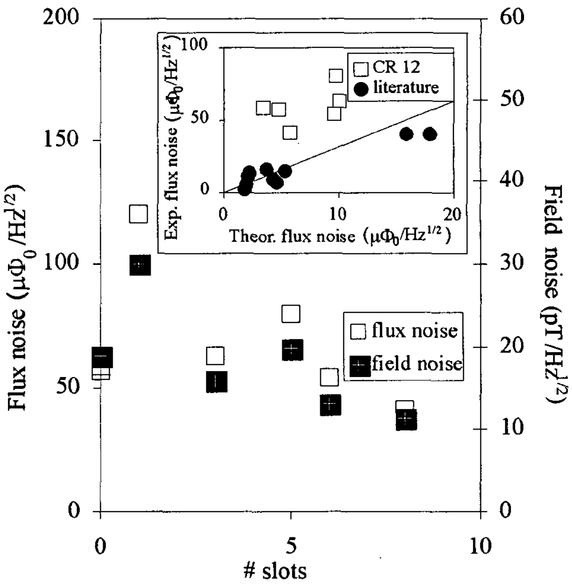

Fig. 5. Flux and field noise of slotted SQUIDs. In the inset a comparison to theory and some literature values [7] are plotted.

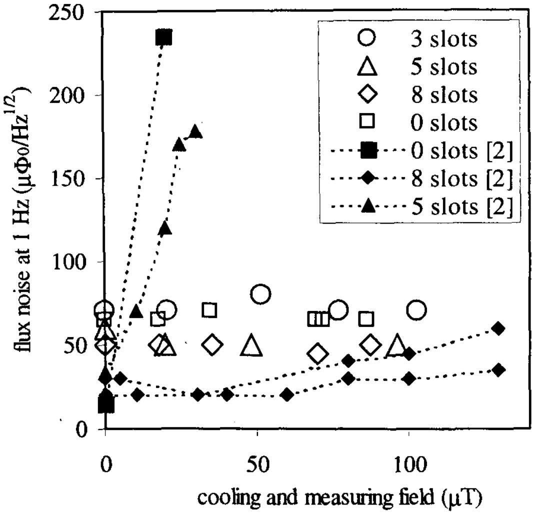

Fig. 6. Flux noise measured at 1 Hz. The slotted SQUIDs were cooled and measured in a field. Also plotted are the noise values for the slotted SQUIDs from [2].

current source and measuring probe. In order to eliminate the $1/f$ noise contribution from the fluctuations of the critical currents of the junctions, an ac bias current scheme was applied.

In Fig. 5 the flux and field noise as a function of the number of slots are shown. The inset shows a comparison between the experimental and theoretical flux noise $S_\Phi^{1/2}$. The latter is obtained from a numerical model as [7]

$$S_\Phi = \left(1+\exp(1.23-4.82\Gamma)\right)\cdot L^2\left(\frac{2k_BT}{R_n}\right)\left[1+\left(\frac{R_n}{LV_\Phi}\right)^2\right]. \quad (4)$$

For V_Φ, the experimental value is used. $\Gamma=2\pi k_BT/\Phi_0$ is the noise parameter. Clearly, the experimental noise values are much higher than expected, even when one takes the usual deviation with a factor of 10 into account. The noise behavior may be strongly affected by the LC-resonances [9] clearly present in the current-voltage characteristics. In spite of their higher inductance values, the SQUIDs with many slots show low flux noise values. These SQUIDs also turn out to follow the prediction best, probably because the concentric current paths assumed in the model are most appropriate in these cases. The noise measurement for the one slot SQUID was most probably limited by the electronics, although their influence has not been quantified yet.

D. 1/f noise

The original purpose of slotted SQUIDs is the suppression of $1/f$ noise when they are cooled through their transition temperature in a magnetic field. One may expect flux penetration and an accompanying increase of the $1/f$ noise of a SQUID when the magnetic field exceeds the threshold value $\pi\Phi_0/4w^2$. The flux noise at 1 Hz for various SQUIDs as a function of the magnetic field they were cooled in, is depicted in Fig. 6. Also depicted are the values found in [2]. Clearly, the $1/f$ noise of our SQUIDs does not show the expected dependence; for all cooling fields, the noise at 1 Hz equals the white noise.

Can this behavior be explained with the theory of hopping vortices? Within the frame of this model, two explanations exist: the vortices are not there or they do not hop. The first explanation is difficult to defend; the relation between the stripline dimensions and the threshold value for penetration reflects the fundamental type-II nature of an $YBa_2Cu_3O_{7-x}$ film. Alternatively, several causes exist for flux pinning: imperfect films, rough stripline edges. If the thermal energy is too low to activate them, no excess $1/f$ noise will be found.

III. CONCLUSIONS

Slotted SQUIDs were investigated both theoretically and experimentally. A model considering a slotted SQUID as a parallel set of inductances provides a prediction for the effective areas, current distributions and inductances of the slotted SQUIDs. The first of these properties shows good agreement with the experiment and although the latter two could not be measured directly at least qualitative agreement was reached for the modulation depths. This supports the validity of the model.

The white noise values obtained for the slotted SQUIDs are higher than predicted by theory, even when one takes the usual spread in white noise values into account. LC-resonances might play a role in this frame. The expected dependency of the low frequency noise on the number of slots could not be observed. For all SQUIDs, the noise at 1 Hz equals the white noise values up to cooling fields of 100 μT. Probably, the vortices in the film are pinned stronger than expected due to rough edges or nonperfect film quality. When a SQUID washer is shunted by a pick up loop, the circulating current through washer and loop may act as an additional driving force for vortex hopping and slots will prove their usefulness.

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