High sensitivity double relaxation oscillation superconducting quantum interference devices

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Double relaxation oscillation superconducting quantum interference devices (SQUIDs) (DROSs) have been fabricated with estimated relaxation frequencies up to 14 GHz. Both the intrinsic flux noise and the performance in a flux locked loop with direct voltage readout have been studied. In flux locked loop, a noise level of 0.55 $\mu\phi_0/\sqrt{\text{Hz}}$ corresponding to an energy sensitivity of 34 h has been obtained for a DROS with a SQUID inductance of 29 pH. The intrinsic sensitivity improves with increasing relaxation frequency, leveling off to a value of 13 h at relaxation frequencies higher than about 3 GHz. © *1994 American Institute of Physics.*

A dc superconducting quantum interference device (SQUID) magnetometer is the most sensitive device to measure low frequency magnetic fields. Usually it is based on a nonhysteretic dc SQUID which transforms a magnetic flux to a voltage.[1,2] Unfortunately, the transfer from flux to voltage is generally too low to facilitate direct voltage readout by a room-temperature amplifier without a significant reduction of the sensitivity of the SQUID system. Complicated flux modulation techniques and impedance matching at 4.2 K are needed to achieve a high sensitivity.[3]

Several groups have proposed a second generation of SQUID sensors that facilitate readout by simplified electronics. By using a series array of SQUIDs the output voltage of the sensor can be increased.[4,5] Additional positive feedback (APF) also increases the output voltage to a level sufficiently high for direct voltage readout.[6,7] In this approach, only one SQUID per sensor is needed which makes them more easy to fabricate and operate than series SQUID arrays. APF, however, may decrease the intrinsic sensitivity of the dc SQUID.[8] Relaxation oscillation SQUIDs (ROSs) and double relaxation oscillation SQUIDs (DROSs) have also been demonstrated to facilitate readout by simple room-temperature electronics.[9,10] In a ROS, the applied flux is transformed to an oscillation frequency of 0.1–1 GHz, typically.[11–14] A DROS transforms an input flux to a voltage with a transfer coefficient that is two or three orders of magnitude larger than in comparable standard dc SQUIDs, which facilitates direct voltage readout.[15,16] In this letter, we will show that the intrinsic sensitivity of a DROS can be of the same order of magnitude as in comparable standard dc SQUIDS.

A DROS consists of two hysteretic dc SQUIDs in series shunted by an inducer L_{sh} and a resistor R_{sh} in series and is biased by a dc current I_b. At bias currents larger than the critical current of the SQUID, relaxation oscillations will occur if the produce I_bR_{sh} is not too large. The condition for stable operation of a DROS is given by

$$\left(\frac{I_b}{2I_0}\right)^2\beta_c^*\leqslant 1, \tag{1}$$

where $2I_0$ is the maximum critical current of the symmetric dc SQUID.[14] The effective McCumber parameter β_c^* is given by

$$\beta_c^*=\frac{2\pi(2I_0)R_{\text{sh}}^2C_{\text{sq}}}{\phi_0}, \tag{2}$$

where C_{sq} is the total SQUID capacitance.

In DROSs based on unshunted tunnel junctions, LC resonances between the SQUID capacitance and the shunt inductance can reduce the maximum relaxation frequency and may limit the sensitivity of a DROS.[16,17] By adding an extra resistor R_d parallel to the SQUID, these resonances can be damped.[14] The value of R_d can be obtained from

$$D=\frac{L_{\text{sh}}}{4C_{\text{sq}}R_d^2}\geqslant 1. \tag{3}$$

Numerical simulations have indicated that for $D=0.1$ the oscillations may also be sufficiently damped. This value of D enables a higher output voltage.

The maximum transfer $\delta V/\delta\phi$ and the sensitivity ϵ of a DROS are both limited by the noise on the critical current of the SQUID.[14,16,17] At relaxation frequencies f_r well below the plasma frequency, this noise is determined by thermal fluctuations. At high relaxation frequencies, the noise is caused by the spread in the phases across the junctions every time the oscillating current through the SQUID exceeds the critical current, forcing the SQUID to switch from the zero-voltage state to the voltage state.[18] In the thermal noise regime, the maximum transfer from flux to voltage can be calculated from

$$\frac{\delta V}{\delta\phi}=3.5I_0^{2/3}\,\frac{V_c}{\phi_0}, \tag{4}$$

with I_0 in μA for a symmetrical dc SQUID with $\beta_L=1$. The voltage V_c is typically equal to 0.5–0.75(I_bR_{sh}) and its maximum value can be obtained by Eq. (1). In the thermal noise regime, the maximum sensitivity is given by

$$\left(\frac{\epsilon}{h}\right)=\frac{64}{f_rI_0^{1/3}}, \tag{5}$$

where $h=6.63\times10^{-34}$ J s is Planck's constant and f_r and I_0 are in GHz and μA, respectively.[16] Numerical simulations

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TABLE I. Design parameters for the fabricated DROSs. The DROSs with $\tau_L=1$ ns have been designed with different values of the damping resistor R_d, corresponding to $D=0.1$, 1, or 10. In the DROSs with $\tau_L=0.1$ ns, D equals 0.1 or 1.

				$\tau_L=1$ ns		$\tau_L=0.1$ ns	
$(I_b/2I_0)^2\beta_c^*$ at $I_b=150$ μA	R_{sh} (Ω)	$I_{b,max}$ (μA)	$\delta V/\delta\phi$ (mV/ϕ_0)	$f_{r,max}$ (GHz)	ϵ (in h)	$f_{r,max}$ (GHz)	ϵ (in h)
0.3	0.9	274	6–9	2.7	6	27	<6
1.0	1.7	150	6–9	1.2	14	12	<14
3.0	3.0	87	6–9	0.5	35	5	<35

have shown that for our SQUIDs the crossover between the thermal noise and the ''random phase noise'' is at a relaxation frequency of about 2 or 3 GHz. Above this frequency, Eqs. (4) and (5) no longer hold.

The design parameters of the fabricated DROSs are listed in Table I. The designed DROSs are based on 2×2 μm^2 Nb/Al, AlO_x/Nb tunnel junctions with a critical current I_0 of 50 μA and a capacitance of about 0.25 pF each. The DROSs have three different design values of $(I_b/2I_0)^2\beta_c^*$: 0.3, 1.0, or 3.0 at $I_b=150$ μA. In the design of the DROSs with $\tau_L=L_{sh}/R_{sh}=1$ ns, three values of D have been included: $D=0.1$, 1, or 10. In the DROSs with $\tau_L=0.1$ ns, D is equal to 0.1 or 1. A damping value of 10 would make the dc SQUIDs in these DROSs nonhysteretic and therefore unsuitable. So, nine different DROSs with $\tau_L=1$ ns and six different DROSs with $\tau_L=0.1$ ns have been designed and fabricated in total. Damping of LC resonances is provided by two resistors with a value of $2R_d$, located symmetrically on both sides of the SQUIDs in order to minimize the mutual inductance between these resistors and the SQUIDs. The design values of R_d range from 7 up to 123 Ω. The shunt inductances consist of planar single-turn loops. The SQUIDS are of square-washer type with an integrated half-turn input coil on top.

The maximum relaxation frequency $f_{r,max}$ and the theoretical values of the transfer coefficient and the sensitivity are also listed in Table I. The maximum frequency has been calculated at the theoretical maximum bias current and at an applied flux of $\phi_0/4$. For the DROSs with $\tau_L=0.1$ ns the sensitivity cannot be calculated by Eq. (5) because of the high relaxation frequency. However, their sensitivity is expected to be at least equal to the sensitivity of the DROSs with $\tau_L=1$ ns.

The DROS output voltage was preamplified by a low-noise dc amplifier[8] with an input noise of 1.5 nV/$\sqrt{\text{Hz}}$ at 1 kHz. All flux locked loop (FLL) measurements were based on direct voltage readout, without flux modulation. The intrinsic sensitivity was obtained in open loop, after substraction of the amplifier noise. The DROSs were shielded by a Nb tube. The maximum critical current of the SQUIDs is about 70 μA. The SQUID inductances range from 29 up to 34 pH, as obtained from the critical current modulation. Due to the small outer dimensions and the layout of the washer these inductances are larger than the intended value of 20 pH.

Experimental characteristics of the DROSs with τ_L values of about 1 ns and $D=10$ are listed in Table II(a). The highest sensitivities were obtained in DROSs with this value of D. Typically, the maximum sensitivity decreases by a factor of 2–3 if the damping parameter decreases from 10 to 0.1. The best intrinsic noise level obtained in these DROSs is 0.4 $\mu\phi/\sqrt{\text{Hz}}$ at 1 kHz for a DROS with $L_{sh}=0.9$ nH, $R_{sh}=0.9$ Ω, and a damping resistor of 7 Ω ($D=10$). This corresponds to an intrinsic sensitivity ϵ_{intr} of 18 h. When operated in FLL, the best noise level of this DROS is 0.55 $\mu\phi/\sqrt{\text{Hz}}$ (at 1 kHz), corresponding to an energy sensitivity ϵ_{FLL} of 34 h. At 1 Hz the noise level is 4 $\mu\phi/\sqrt{\text{Hz}}$ due to $1/f$ noise with a corner frequency of 20–30 Hz. The transfers $\delta V/\delta\phi$ listed in Table II have been measured at the bias point with maximum sensitivity in FLL operation. The bias currents at these optimum operation points are typically about 50% of the maximum value that can be calculated by Eqs. (1) and (2). This indicates that the relaxation frequencies are smaller than the designed values which partly explains why the intrinsic sensitivities are lower than expected. Due to the relatively low optimum bias current, the transfer coefficients are also lower than expected.

Experimental characteristics of the DROSs with τ_L values of about 0.1 ns and $D=1$ are listed in Table II(b). The characteristics of the DROSs with $D=0.1$ are almost the same as for the DROSs with $D=1$. The intrinsic sensitivity of these DROSs are all equal to 13–14 h. In Fig. 1 the intrinsic flux noise spectral density of one of the DROSs is depicted, showing a noise level of 0.36 $\mu\phi/\sqrt{\text{Hz}}$ at 1 kHz which corresponds to a sensitivity of 13 h. The equal values of the intrinsic sensitivities indicate that at these relaxation frequencies, which are estimated to be ranging from 3 to 14 GHz, the spread in the critical current and therefore the intrinsic sensitivity is limited by the plasma frequency. This could also explain the relatively low values of $\delta V/\delta\phi$ in these DROSs. Due to this low transfer the energy sensitivity

TABLE II. Experimental characteristics of six different DROSs with (a) $\tau_L\approx1$ ns, $D=10$, and (b) $\tau_L\approx0.1$ ns, $D=1$. The values of L_{sh} are design values.

	L_{sh} (nH)	R_{sh} (Ω)	$\delta V/\delta\phi$ (mV/ϕ_0)	ϵ_{FLL} (in h)	ϵ_{intr} (in h)
(a)	0.9	0.9	3.8	34	18
	1.7	1.8	7.0	34	29
	3.0	2.3	4.4	71	60
(b)	0.09	0.7	2.0	73	13
	0.17	1.2	2.6	56	14
	0.3	1.4	2.7	48	13

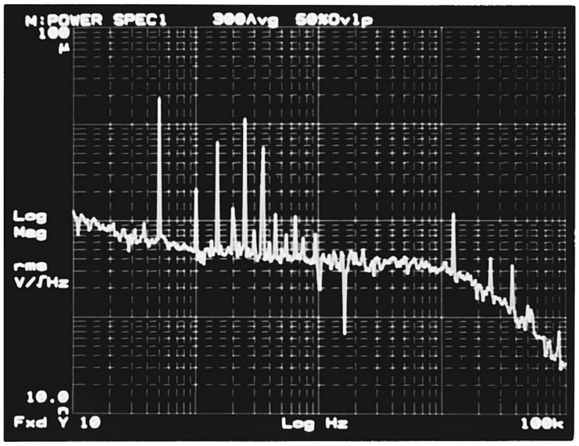

FIG. 1. Intrinsic flux noise spectral density of a DROS with $L_{sh}=90$ pH (design value), $R_{sh}=0.7\ \Omega$, $D=1$, and a SQUID inductance of 33 pH. The estimated relaxation frequency is 14 GHz. The horizontal axis ranges from 10 Hz to 100 kHz. The vertical axis ranges from 10 nV/$\sqrt{\text{Hz}}$ to 100 μV/$\sqrt{\text{Hz}}$, where 1 V corresponds to 1 ϕ_0.

in FLL is not better than for the DROSs with a time constant τ_L of about 1 ns, despite the higher intrinsic sensitivity.

The maximum theoretical sensitivity of a comparable standard dc SQUID with $L_{sq}=30$ pH and $2I_0=70$ μA can be estimated to be 6 h for a SQUID with $\beta_c=0.3$ (Ref. 1). This is very close to the experimental DROS result of 13 h, considering that a DROS is based on two dc SQUIDs. The voltage noise of this comparable standard dc SQUID, however, would be as low as 0.1 nV/$\sqrt{\text{Hz}}$, which is much too low for operation in the FLL with direct voltage readout used here.

Summarizing, double relaxation oscillation SQUIDs (DROSs) have been fabricated with estimated relaxation frequencies ranging from several hundred MHz up to 14 GHz. The intrinsic sensitivity improves with increasing relaxation frequency, leveling off to 13 h at relaxation frequencies higher than about 3 GHz. This sensitivity is very close to the theoretical maximum sensitivity of a comparable standard dc SQUID. In FLL operation with direct voltage readout a sensitivity of 34 h has been measured. This is comparable with the best results obtained for standard type dc SQUIDs operated in a FLL based on flux modulation and impedance matching.

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