

THE STATISTICAL CHARACTER OF PACKED-BED HEAT TRANSPORT PROPERTIES

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Abstract—Packed beds are essentially heterogeneous on a pellet scale. For random packed beds this heterogeneity causes a statistical character both on a pellet and bed scale. We discuss experimental results which deal with bed-scale statistics. For packed beds on a laboratory scale, the results indicate that the bed-scale statistical behaviour introduces a spread up to a factor of three in correlations for the effective radial heat conductivity, λ_{eff} , and the heat transfer coefficient at the wall, α_w . This spread of a factor of three roughly equals the spread between correlations available in the literature. For cooled tubular reactors the statistical spread in the behaviour of the individual tubes will have a large influence on reactor operation.

INTRODUCTION

Packed-bed properties are distributed both in space and in time. Distribution in time could lead to chaotic behaviour. Chaotic behaviour was studied by Kirillov *et al.* (1972), who have visualized the flow of gases and liquids in packed beds. Within the voids between the pellets, a flow-through zone exists in which the fluid flows along tortuous paths in the form of blending and separating streams. Further, a non-flow zone exists in the vicinity of the pellet contact points. A sharp separation boundary is formed between the two zones. This separation boundary is constantly being deformed by the existence of rotating and pulsating vortices in the non-flow zone. The pulsation frequency, f , is directly proportional to the local velocity of the streams and inversely proportional to the pellet size, so that the Strouhal number $Sr = f d_p / v_0 \approx 1/2$ for $20 < Re < 1000$. For packed beds, as used in the chemical industry, this means that the jets erupting from the voids between the pellets are pulsating and vibrating with frequencies from 10 to 100 Hz. Thus, for industrial packed beds, this chaotic behaviour becomes important only if phenomena on a pellet scale with a characteristic time of 0.1 to 0.01 s play a role. Typically, characteristic times on a pellet scale for reaction or intra-particle diffusion lie in the order of 1 to 10 s. Therefore, the mentioned chaotic behaviour will usually be unimportant.

Therefore, we will focus on statistical distributions of packed-bed properties in space. If the packing of the bed is random, the local porosity, and hence also the velocity and temperature, will, to a certain extent, be randomly distributed in space (see Fig. 1). Because of the mathematical difficulties in accounting for this random distribution in space, packed beds are usually

described by homogeneous or pseudo-homogeneous models, i.e. the solid phase and the gas phase are regarded as one single phase with average properties, like an average bulk porosity, an average gas velocity, and hence a smooth temperature profile. For a homogeneous model we can vary the average bed properties as a function of the place in the packed bed in any way we want, as long as that function is deterministic, e.g. we can choose a uniform porosity profile and neglect the wall effects, or we can account for wall effects and vary the porosity from unity, at the wall, to lower values further away from the wall (see Fig. 1).

Here we shall show that this statistical distribution in space can cause a statistical distribution of packed-bed heat transport properties.

TEMPERATURE PROFILES IN PACKED BEDS

Part of the scatter in temperature measurements in packed beds arises from the fact that the packed bed is heterogeneous on a pellet scale, i.e. if a radial profile is followed at a single axial position, the local porosity is a Boolean variable (see Fig. 1). Another part arises from the fact that gas streams within the voids are directed in space in a statistical manner. As a result, measured temperature profiles also exhibit a certain amount of scatter (see Fig. 2).

This is confirmed by Cresswell (1986) who measured radial temperature profiles in a packed bed and found a 10°C spread between angular replicates for a total gradient of 40°C. We measured the radial temperature profiles in a packed bed through which a cold gas (temperature 20°C) flows and which is heated at the wall (temperature 60°C). The results are indicated in Fig. 3. We repeated the temperature profile measurements several times; in between the experiments the packed bed was fluidized for a few seconds so that a new random packing was obtained. The thin lines in Fig. 3 represent the individual temperature profiles; the bold line gives the average temperature

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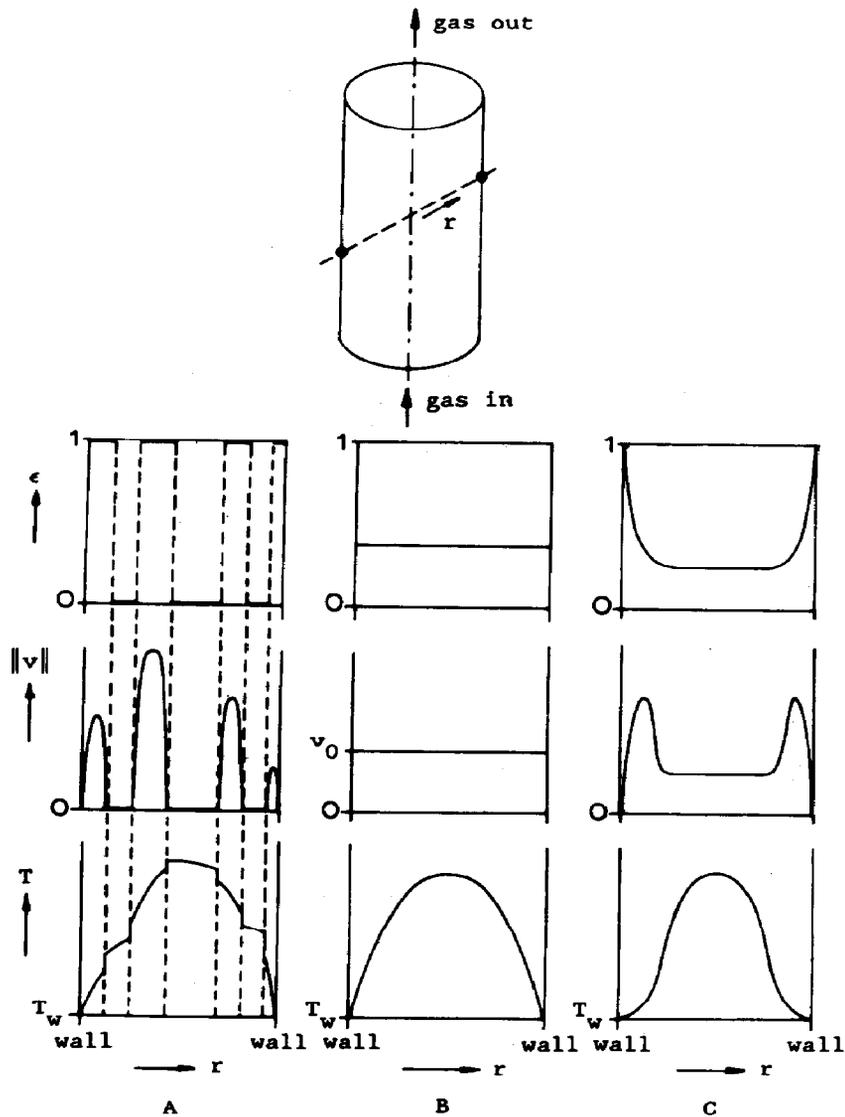


Fig. 1. Porosity, ϵ , velocity norm, $\|v\|$, and temperature, T , vs the distance r in a packed bed: (A) actual profiles, (B) homogeneous model without wall effect, (C) homogeneous model with wall effect.

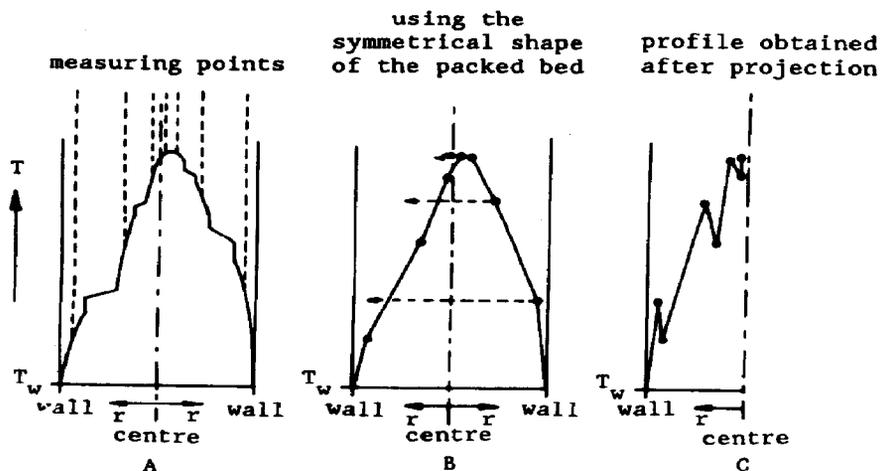


Fig. 2. Illustration of how scatter in a temperature profile is obtained: (A) actual profile and measuring points, (B) measured temperature profile, (C) profile which is obtained if the cylindrical symmetry of the packed bed is included.

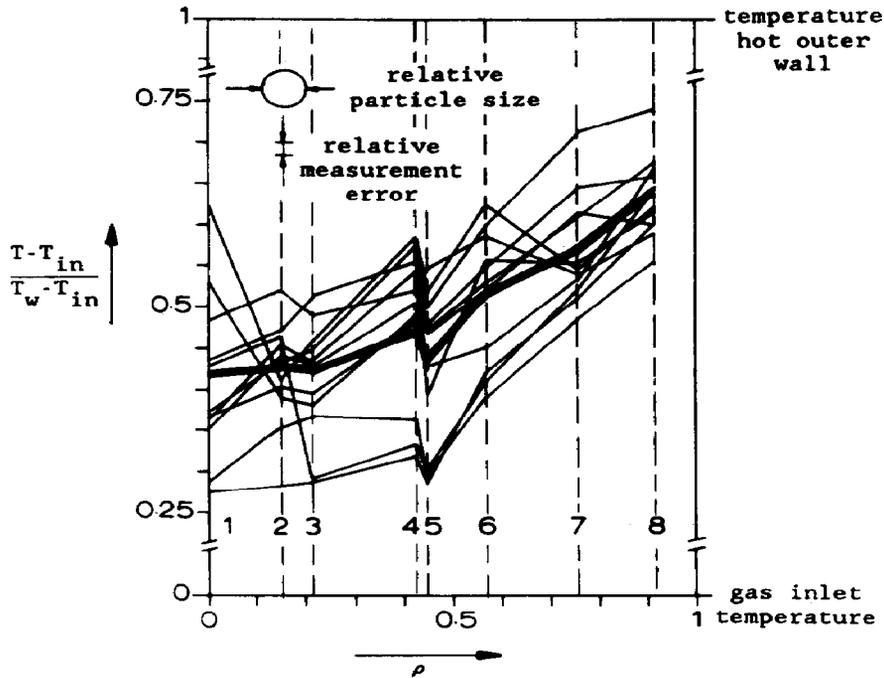


Fig. 3. Measured radial temperature profiles. In between the measurements the bed was fluidized and allowed to settle again. The bold line gives the average profile. Pellet size as indicated.

profile. The profiles were measured with thermocouples stuck into the bed, some 20 particle layers above the bed inlet. The packed bed contained 25 particles on a diameter. Further data: the pellets are Raschig rings with height \times outer diameter \times inner diameter = $8 \times 8 \times 4$ mm; cylindrical bed with height \times diameter = 30×20 cm; the gas is air; the superficial velocity is 21 cm/s.

Notice—that the amount of scattering is considerable. Bed properties, e.g. the effective heat conductivity of the packed bed, λ_{eff} , and the heat transfer coefficient at the wall, α_w , are determined by fitting a model to the measured temperature profiles. The scatter in temperature can induce a scatter in the bed properties like λ_{eff} and α_w or a comparable statistical character of the bed. This will be discussed in the next section.

PACKED-BED HEAT TRANSPORT PROPERTIES

For a cold gas flowing through a packed bed heated at the wall, the temperature as a function of the dimensionless radius, $\rho = r/R_t$, and the dimensionless height, $\omega = z/L_t$, is given by

$$\frac{T_w - T(\rho, \omega)}{T_w - T_{in}} = 2 \sum_{i=1}^{\infty} \left\{ \frac{J_1(\lambda_i) J_0(\lambda_i \rho)}{\lambda_i \{ [J_0(\lambda_i)]^2 + [J_1(\lambda_i)]^2 \}} \times \exp\left(-\frac{\lambda_i^2 \omega}{Pe^{\ominus}}\right) \right\} \quad (1)$$

with λ_i being the i th root of the equation

$$Bi^{\ominus} J_0(\lambda) - \lambda J_1(\lambda) = 0 \quad (2)$$

and

$$Pe^{\ominus} = \frac{\rho_g C_{p,g} v_0 R_t^2}{\lambda_{\text{eff}} L_t} \quad (3)$$

$$Bi^{\ominus} = \frac{\alpha_w R_t}{\lambda_{\text{eff}}} \quad (4)$$

[e.g. Lerou and Froment (1978)]. It is assumed that the temperature of the wall of the packed bed is constant over the entire bed length.

By fitting the temperature profiles to the above equation, we determined the effective radial heat conductivity, λ_{eff} , and the heat transfer coefficient at the wall, α_w , as a function of the gas velocity for the packed bed with the temperature profiles as given in Fig. 3. Values of λ_{eff} and α_w are obtained by fitting average radial temperature profiles.

In Fig. 4 the results of the fitting procedure are presented for λ_{eff} in the form of $\Gamma = \lambda_{\text{eff}}/\lambda_g$ with $\lambda_g = 0.037$ W/m K vs $Re = v_0 d_p/v_g$, with $d_p = 6V_p/A_p = 6.1$ mm and $v_g = 3.3 \times 10^{-5}$ m²/s. For α_w , the results are presented in Fig. 5 in the form of $Bi = \alpha_w d_p/\lambda_{\text{eff}}$ vs Re .

Two sets of data have been obtained. The packed bed was fluidized and allowed to settle again and the radial temperature profile was measured again. Usually, this yielded temperature profiles full of scatter as illustrated in Fig. 3. However, we repeated fluidizing and settling the packed bed over again, until by chance a smooth temperature profile was obtained. For this specific packing we determined Γ and Bi for several values of Re , yielding the full dots. For Γ it can be seen from Fig. 4 that the straight-line relation

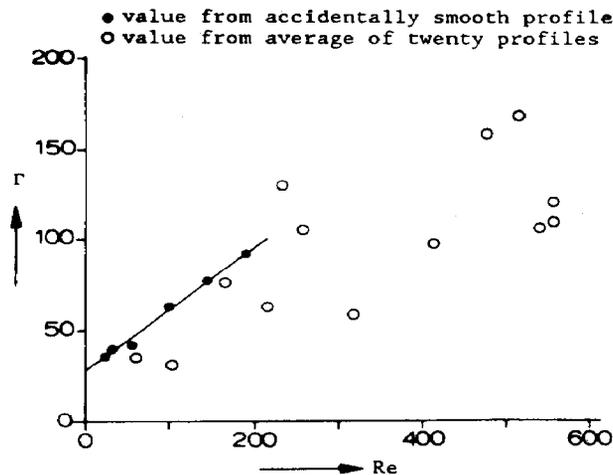


Fig. 4. Dimensionless effective heat conductivity, $\Gamma = \lambda_{\text{eff}}/\lambda_g$, vs the particle Reynolds number.

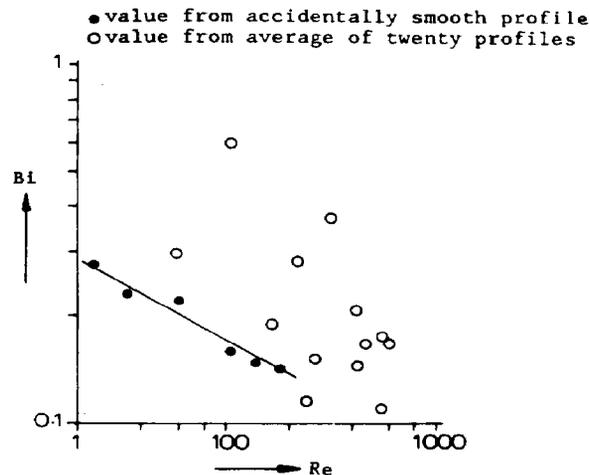


Fig. 5. Biot number, $Bi = \alpha_w d_p / \lambda_{\text{eff}}$, particle Reynolds number.

commonly found in the literature is obtained: the intersection with the Γ -axis gives us the static contribution to λ_{eff} and the slope, the dynamic contribution. Notice that in Fig. 5, for this selected case, the amount of scatter in Bi is small. The deviation between the measured and fitted temperature profiles never exceeded 1°C .

The second set of data are represented by the open dots. For the open dots we fitted the average temperature profile obtained from 20 individual temperature profiles full of scatter. All profiles were measured for one specific gas velocity. For each determination of an individual temperature profile, we fluidized the packed bed and let it resettle. Then we adjusted the gas flow so that the same specific velocity was obtained, and measured the radial temperature profile. Regardless of the amount of scatter, this temperature profile was added to the 19 obtained in a similar way: this gave us one average temperature profile for this specific velocity. This average profile was then fitted to the model. The deviation between the average measured and fitted profiles never exceeded 1°C . Notice that, if an average profile is fitted, a tremendous amount of scatter is found in the values of Γ and Bi . This scatter is induced by the scatter in the temperature profiles.

DISCUSSION

The experiments mentioned above lead to the following observations: if we measure λ_{eff} for one single random packed bed, we obtain a very consistent correlation and the results will be highly reproducible for this specific packing. If we alter the packed bed, we will find a new correlation which is just as consistent and reproducible as the first one; nevertheless, it will usually differ from the first one in the parameter values of its correlation equation. This is also consistent with the results reported in the literature. Many authors have reported correlations for λ_{eff} and

α_w [e.g. Hennecke and Schlünder (1973), Zehner (1973), Zehner and Schlünder (1973), Lerou and Froment (1977), Bauer (1977), Schlünder (1966, 1978) and Dixon and Cresswell (1979)]. The prediction of the properties λ_{eff} and α_w by all these correlations differ by a factor of two to four, despite the fact that the authors are known to be accurate, serious and experienced experimentalists. These large discrepancies could be caused in part by the statistical character demonstrated here. Because of the bed-scale statistical behaviour—which, with the same pellets, tubes and gases will only be observed when the experiments are repeated after the bed has been replaced, e.g. by fluidizing and resettling—the probability that different authors find the same correlation is negligible, because the probability to obtain exactly the same random packed bed is negligible. This phenomenon was observed by Gunn and Pryce (1969) for dispersion of mass.

This bed-scale statistical behaviour becomes unimportant if very large packed beds are used with a sufficient number of pellets on a diameter and on a bed height. Regrettably, almost all correlations reported in the literature were obtained from measurements in laboratory set-ups with roughly 10–40 particles on a diameter: the statistical spread must be considerable, and consequently the literature correlations become less universal.

Even if there would exist a reliable correlation obtained by averaging the results of many randomly poured packed beds, for industrial reactors, the statistical spread could still remain important, e.g. for a cooled tubular reactor with a few particles on a diameter, the value of λ_{eff} would exhibit quite some variation along the tube length. Also, a statistical spread must be expected in the behaviour of different tubes. This would demand correlations not only for λ_{eff} and α_w , but also for the distributions of both properties.

CONCLUSIONS

We observed much scatter in the measured temperature profiles. If λ_{eff} and α_w are determined by fitting experimental temperature profiles, the scatter in the profiles will induce a statistical spread in λ_{eff} and α_w . This spread is considerable and provides an explanation for the large discrepancies between the correlations for λ_{eff} and α_w in the literature. Up till now no correlations have been suggested to describe the distributions.

More experimental data, taking into account the statistical character of lumped-bed scale parameters, such as λ_{eff} and α_w and pellet-scale parameters, such as α_p and k_g , should be obtained in order to determine general correlations for the distribution of these parameters. In this way the influence of the statistical character on packed-bed operation in industry can be evaluated.

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NOTATION

A_p	pellet external surface area, m^2
Bi	Biot number based on particle diameter ($= \alpha_w d_p / \lambda_{\text{eff}}$), dimensionless
$C_{p,g}$	specific heat of the gas, J/kg K
d_p	particle diameter equivalent to a sphere ($= 6V_p/A_p$), m
f	pulsation and rotation frequency of the vortices near the particle contact points, $1/\text{s}$
J_0, J_1	Bessel functions of the zeroth and first order
k_g	mass transfer coefficient between the particle and the gas, m/s
L_t	tube length, length of the packed bed, m
Pe^\ominus	modified Péclet number based on tube [$= \rho_g C_{p,g} v_0 R_t^2 / (\lambda_{\text{eff}} L_t)$], dimensionless
r	radial coordinate, m
Re	particle Reynolds number ($= v_0 d_p / \nu_g$), dimensionless
R_t	tube radius, m
Sr	particle Strouhal number ($= \epsilon f d_p / v_0$), dimensionless
T	temperature, K
T_{in}	gas inlet temperature, K
T_w	temperature of the wall of a packed bed, K
v_0	superficial gas velocity based on empty tube cross-section, m/s

V_p	particle volume, m^3
z	axial coordinate, m

Greek letters

α_p	heat transfer coefficient between the particle and the gas, $\text{W/m}^2 \text{K}$
α_w	heat transfer coefficient between the packed bed and the wall, $\text{W/m}^2 \text{K}$
Γ	dimensionless effective radial heat conductivity ($= \lambda_{\text{eff}} / \lambda_g$)
ϵ	porosity at a certain point, being either zero or one, also average bed porosity, dimensionless
λ_{eff}	effective radial heat conductivity of the packed bed, W/m K
λ_g	heat conductivity of the gas, W/m K
ν_g	kinematic viscosity of the gas, m^2/s
ρ	dimensionless radial coordinate ($= r/R_t$)
ρ_g	density of the gas, kg/m^3
ω	dimensionless axial coordinate ($= z/L_t$)

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