

## HEAVY SUBGRAPH PAIRS FOR TRACEABILITY OF BLOCK-CHAINS

BINLONG LI<sup>a,b1</sup>, HAJO BROERSMA<sup>b</sup>

AND

SHENGGUI ZHANG<sup>a2</sup>

<sup>a</sup> *Department of Applied Mathematics*  
*Northwestern Polytechnical University*  
*Xi'an, Shaanxi 710072, P.R. China*

<sup>b</sup> *Faculty of EEMCS, University of Twente*  
*P.O. Box 217, 7500 AE Enschede, The Netherlands*

**e-mail:** h.j.broersma@utwente.nl

### Abstract

A graph is called traceable if it contains a Hamilton path, i.e., a path containing all its vertices. Let  $G$  be a graph on  $n$  vertices. We say that an induced subgraph of  $G$  is  $o_{-1}$ -heavy if it contains two nonadjacent vertices which satisfy an Ore-type degree condition for traceability, i.e., with degree sum at least  $n-1$  in  $G$ . A block-chain is a graph whose block graph is a path, i.e., it is either a  $P_1$ ,  $P_2$ , or a 2-connected graph, or a graph with at least one cut vertex and exactly two end-blocks. Obviously, every traceable graph is a block-chain, but the reverse does not hold. In this paper we characterize all the pairs of connected  $o_{-1}$ -heavy graphs that guarantee traceability of block-chains. Our main result is a common extension of earlier work on degree sum conditions, forbidden subgraph conditions and heavy subgraph conditions for traceability.

**Keywords:**  $o_{-1}$ -heavy subgraph, block-chain traceable graph, Ore-type condition, forbidden subgraph.

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