A Novel Nonlinear Programming Model for Distribution Protection Optimization

Eduardo Zambon, Débora Z. Bossois, Berilhes B. Garcia, and Elias F. Azeredo

***Abstract*—This paper presents a novel nonlinear binary programming model designed to improve the reliability indices of a distribution network. This model identifies the type and location of protection devices that should be installed in a distribution feeder and is a generalization of the classical optimization models given in the literature. This new approach is more flexible and leads to better placement solutions. Numerical results of the tests performed on real feeders are presented for analysis. Furthermore, we make the reliability data of the 36 feeders that were used in our work available to the research community.**

***Index Terms*—Nonlinear integer programming, power distribution protection, power distribution reliability.**

I. INTRODUCTION

RELIABILITY of distribution networks is a permanent concern for electric utilities and is usually evaluated and regulated by an utility board or similar commission. To quantify the quality of the utilities' services, reliability indices, such as the ones presented in [1], can be used. In this paper, we focus on two of the most commonly used customer oriented indices: system average interruption frequency index (SAIFI) and system average interruption duration index (SAIDI).

To reduce the effects of failures in power distribution, protective devices, such as reclosers, fuses, and isolating switches can be installed, increasing the reliability of a network and, therefore reducing its SAIFI and SAIDI indices. This paper presents a method to minimize the SAIFI or SAIDI index of a distribution feeder. It does so by identifying where the protective devices should be installed and which types should be used, while ensuring that operational and cost constraints are satisfied.

A significant amount of research has been developed about the topic presented here. For brevity's sake, we limit our literature review to works that focus on mathematical programming models. To our knowledge, Soudi and Tomsovic were the first to present a mathematical programming model to minimize the SAIFI index of a feeder [2]. Initially, the authors propose a binary programming model with a nonlinear objective function subject to linear constraints. From now on, we shall refer to a problem with those characteristics as a nonlinear binary programming (NLBP) model. To employ a commercial software package that can solve linear integer programming problems, they use a simple technique to transform the nonlinear objective function to a linear one, thus ending with a linear model of binary decision variables. Here, we call that a linear binary programming (LBP) model. To limit the amount of complexity of the LBP problems to be solved, Soudi and Tomsovic perform a heuristic division of a distribution feeder in one main line and zero or more lateral lines, each classified in one of three categories. A shortcoming of the model by Soudi and Tomsovic is that it does not take into account the interrelated effects of failures between the main line and the laterals, which can lead to suboptimal solutions. Additionally, Soudi and Tomsovic briefly discuss some "utility practices" [2] that were used to define the heuristic division employed. However, it should be noted that each utility has its own set of practices that may be different from the ones employed in the model by Soudi and Tomsovic, hence hindering or even preventing the use of the model given in [2] by some utilities.

In [3], Soudi and Tomsovic present a similar variant model that can be used to minimize the SAIFI or an average system interruption frequency index (ASIFI) index of a feeder, or minimize the cost of protective device acquisition while ensuring some desired indices levels. Later, in [4], these same authors discuss the worst-case analysis for the computational effort needed to solve their model and show how the feeder division can bound the problem complexity. Finally, in [5], Soudi and Tomsovic propose the use of goal programming, together with their original model, to search a balance in the optimization of more than one reliability index simultaneously.

Following the work of Soudi and Tomsovic, Bupasiri *et al.* [6] developed an NLBP model to minimize the total monetary cost of distribution reliability. That total cost is the sum of outage costs, life-cycle costs of equipment, and investment costs on protective devices. Bupasiri's formulation is more complete than the original one by Soudi and Tomsovic [2] since it explicitly handles the effect of isolating switches on the SAIDI index.

Also based in the original model by Soudi and Tomsovic [2], Silva *et al.* [7] present another NLBP model to optimize the SAIFI index of a feeder. Silva's model considers the interrelated effects of failures between the main line and the laterals. Silva uses the same form of feeder division proposed by Soudi and Tomsovic, originally devised to limit the interrelated effects of failures between each part, only to arrive at a final model that seeks to exactly take these interrelated effects into account.

Again, based in [2], Sohn *et al.* [8] propose an NLBP model to minimize the total cost of reliability. Sohn's model is similar to that of Bupasiri's [6] and handles the effects of isolating

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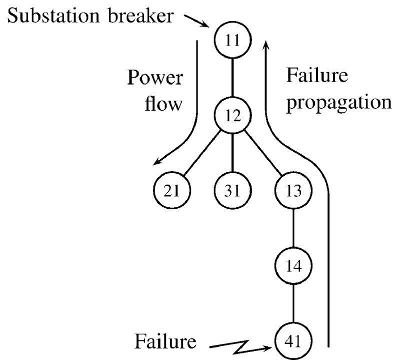

Fig. 1. Simple seven-section feeder [2].

switches in the objective function, since it has terms to represent maneuvers, such as upstream and downstream restoration. As in [7], they wish to take the interrelated effects between the main line and laterals into account. The authors also employ the same form of division of Soudi and Tomsovic.

This paper presents a new NLBP model to minimize the SAIFI or SAIDI index of a distribution feeder. As in [7] and [8], we seek a model that can fully represent the interrelation of protective devices. We, however, depart from the established notation and feeder division of [2], and end up with a generalized model that can handle the one from Soudi and Tomsovic as a particular case. From now on, for brevity, we shall use the term ST model to refer to the original model of Soudi and Tomsovic proposed in [2], and the term ZB model to refer to our model presented here.

II. Problem Definition

In this section, we present some concepts required to state the problem definition. Some of these concepts are defined in [1]. We briefly give them here for completeness.

A. Network Structure

Regarding the distribution task within itself, we can assume that a substation is the source of electrical power. The distribution network served by the substation is subdivided in one or more distribution feeders. Each feeder, in turn, is split into one or more sections. A section encompasses a geographic region and is formed by the primary and secondary distribution cabling, transformers, capacitors, voltage regulators, etc. In our allocation problem, we can safely abstract from most of this physical configuration: only some general information is required for each section, as detailed in Section III.

Since this is the most common form of network configuration, we assume that each feeder of the substation is radially operated (i.e., we consider that the substation is the only source of electric power in the network and that each feeder is isolated from each other). For simplicity, we do not take switching between two feeders connected through normally open switches into account, such as downstream restoration.

From the aforementioned considerations, we can represent a feeder using a tree structure, as shown in Fig. 1, where each node represents a numbered section of the feeder, and each edge shows a dependency on the power flow between the two nodes. This feeder in Fig. 1 is the same one presented by Soudi and Tomsovic in [2]. Section 11 is marked as the root R of the tree and is considered to be part of the substation. Usually, a breaker is installed in R. Since the feeder is radially operated, the feeder tree root can be regarded as the source of the electric power, which flows toward the tree leaves. From now on, we shall use the terms section and node, and distribution feeder and feeder tree interchangeably.

B. Failures and Interruptions

An interruption is defined [1] as a complete loss of electrical power to one or more customers. Each interruption can be classified by its duration as either momentary or sustained. As defined in the IEEE Standard 1366 [1], to be considered momentary, an interruption should not exceed 5 min; otherwise, it is a sustained one. The failures that occur in the distribution feeder are also divided in two types: 1) temporary and 2) permanent. Temporary failures are events that clear themselves away after a short period of time. As an example of this type of failure, we can consider a brief short circuit between two phases of an overhead line. A temporary failure can cause either a momentary or a sustained interruption, determined by the type of the protection device it activates. Permanent failures are the ones caused by more serious events, such as the fall of a pole. A permanent failure always causes a sustained interruption to occur, and requires the dispatch of a repair crew to fix it.

A failure always propagates upstream, toward the root of the feeder tree, until it reaches a node where a protection device is installed. Taking Fig. 1 as an example: assuming that no protective device is installed except at node 11, a failure in section 41 would propagate through nodes 14, 13, and 12, until reaching the breaker at 11.

C. Protection Devices

An electric utility must follow two lines of action regarding failures in the distribution network: prevention and mitigation. Prevention covers tasks that should be performed to diminish the occurrence of faults (e.g., trimming of trees close to overhead lines). On the other hand, should a failure occur, its effects must be mitigated to a minimum acceptable level. A single failure should not cause all of the customers of a feeder to be interrupted, for example. To handle the effects of failures, utilities install protection (or protective) devices along the distribution network. Here, we use the term recloser to represent the three-phase automatic sectionalizing devices that are treated equally by the model: line reclosers, breakers, sectionalizers, and interrupters. Similarly, the term "fuse" is used to represent low-cost sectionalizing devices that lack reclosing capability. As noted in [2], after finding a solution to the NLBP problem, the decision-maker can select which type of three-phase device to use, based on the type of circuit and coordination concerns.

The relation between failures and protective devices considered in this work is as follows.

- A permanent failure always causes a sustained interruption regardless of the type of protective device that it activates.
- A temporary failure can be cleared by a recloser, causing only a momentary interruption, but will cause a sustained interruption if it reaches a fuse.

We also assume that all protective devices operate properly and that they are installed at the beginning of the sections. Thus, an interruption in section i, where a protective device is located, affects all customers from that section and all others in the sections downstream i. From now on, every time we say "all sections downstream i," we want to include section i.

D. Reliability Indices

The most common reliability indices are defined in [1]. The SAIDI index of a distribution network can be calculated with the following definition:

$$\text{SAIDI} = \frac{\sum \text{Customer interruption durations}}{\text{Total number of customers served}} \quad (1)$$

and the SAIFI index by

$$\text{SAIFI} = \frac{\text{Total number of customer interruptions}}{\text{Total number of customers served}}. \quad (2)$$

Equations (1) and (2) can be used to calculate the SAIDI and SAIFI indices of a feeder, given the history of interruptions within a period of time (usually a year.) However, we want to estimate the value of an index that a given configuration of protection devices will yield. Thus, we have to define new formulae for the estimation of a feeder's SAIDI and SAIFI indices. The SAIDI index estimation can be calculated by

$$\text{SAIDI} = \frac{\sum_{i \in B} H_i N_i}{N_T} \quad [\text{hours/year}] \quad (3)$$

and the SAIFI index estimation by

$$\text{SAIFI} = \frac{\sum_{i \in B} I_i N_i}{N_T} \quad [\text{interruptions/year}] \quad (4)$$

where

- B set formed by all sections of a feeder;
- N_i number of customers in section i;
- N_T total number of customers on the feeder;
- H_i estimation on the number of hours per year that a section i is without service; and
- I_i estimation of the number of interruptions per year suffered by section i.

To estimate H_i and I_i, we need to know the permanent (λ) and temporary (γ) failure rates of each section. These rates are usually obtained from the feeder history of interruptions.

III. Proposed Model

In this section, we present the developed ZB model. An example that shows how to apply this model to a distribution feeder is given in the Appendix.

A. Motivation

To apply the ST model, we must divide a feeder in the main line and lateral lines, classified in categories one, two, or three. A category one lateral is short and its effects in the reliability indices are subsumed in the main line. The model requires a fuse to be installed in the tap of a category two lateral and a protective device (recloser or fuse) to be present in the tap of a category three lateral. As noted by Sohn *et al.* [8], due to these restrictions, the ST model cannot deal with the effects between sections in the main line and other laterals.

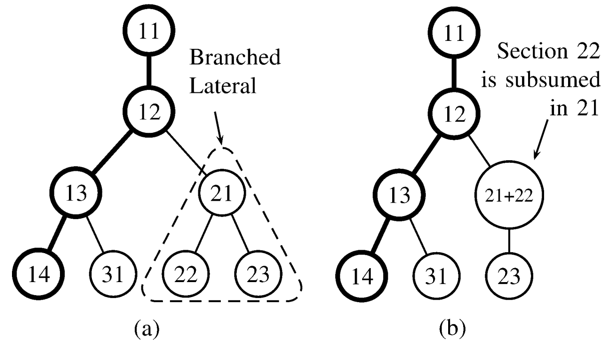

Fig. 2. Example of the difficulties encountered when the ST model is applied. (a) After choosing the main line (shown in bold,), the lateral rooted at node 21 has two branches. (b) Section 22 is subsumed in node 21.

Another limitation of the ST model is that the laterals cannot have branches. But sometimes this situation can occur in a feeder and in order to apply the ST model, the effects of the sections in the lateral branches must be subsumed in other sections of the lateral, similar to what has been done with category one laterals. An example where this situation occurs is shown in Fig. 2. Taking the feeder tree in Fig. 2(a), we define the main line (formed by sections 11, 12, 13, and 14—shown in bold) and we obtain two laterals, one beginning at node 21 and the other at node 31. The latter poses no problem since it is formed only by one section, but the former has two branches (nodes 22 and 23.) In order to use the ST model in this case, node 22 is subsumed in section 21 (i.e., the failures rates and the customers of node 22 are added to the ones of section 21), forming a new section, labeled $21 + 22$ in Fig. 2(b). If the lateral branches are long, this limitation of the ST model is rather troublesome since it burdens the decision-maker. After obtaining a solution, he or she must ponder whether the devices' placement is indeed adequate or if some equipment should be moved to subsumed sections. Returning to the example, if the programming problem obtained when the ST model is applied to the feeder of Fig. 2(b) yields a solution that indicates that a recloser should be installed in node $21+22$, the decision-maker has to decide whether the recloser is to be placed either in node 21 or in node 22.

The aforementioned problems presented led to the development of the ZB model, designed to handle feeders with a generic tree structure. In a more general approach, our proposed model can also handle a feeder with arbitrary divisions, thereby increasing the freedom for device allocation.

B. Arbitrary Feeder Division

Before applying the ZB model, we must specify a parameter d_i for each section i of the given feeder. Each d_i is defined as the first section upstream from i on which we can guarantee that a protective device will be installed. This parameter d_i allows us to divide the feeder in an arbitrary fashion. In an extreme case, we can analyze the feeder without divisions. To do so, it suffices to define $d_i = R$, for all $i \in B$, since we assume that a breaker is installed at the root (R) of the feeder tree.

It is important to note that the ZB model can give us an NLBP problem that is equivalent to the one obtained with the ST model. In this case, the parameters d_i should be defined in the following way:

- for all i on the main line, $d_i = R$;
- for each lateral l and for all i in l, d_i is defined as the tap of l.

We call an "equivalent problem," an NLBP problem that differs from the one obtained with the ST model but has the same optimal solution.

C. Definition

This section presents the definition for the model's objective function (OF). Initially, we define three auxiliary variables X, Y, and Z. These variables simplify the OF formulation and ease the understanding of the model. Later, these auxiliary variables are replaced by the decision variables and the final form of the OF can be found with only some simple algebraic manipulations. Binary variables X, Y, and Z are defined as

$$X_j = \begin{cases} 1, & \text{if a recloser is installed on section } j \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$Y_j = \begin{cases} 1, & \text{if a fuse is installed on section } j \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

$$Z_j = X_j + Y_j. \tag{7}$$

The main role of (7) is to establish an equality of symbols (i.e., instead of writing $X_j + Y_j$, we can just write Z_j). From (7), we note that variable Z_j assumes a nonzero value when any protection device is installed on j, and is 0 otherwise. The situation where Z_j has a value greater than 1 is avoided by a proper constraint, as shown in Section III-D.

For these three variables, we also define three opposite value complements $\bar{X}, \bar{Y}$, and $\bar{Z}$ as

$$\bar{X}_j = \begin{cases} 1, & \text{if no recloser is installed on section } j \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

$$\bar{Y}_j = \begin{cases} 1, & \text{if no fuse is installed on section } j \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

$$\bar{Z}_j = \bar{X}_j \bar{Y}_j. \tag{10}$$

The model's objective function is derived from the SAIFI index estimation equation. The numerator of (4) can be rewritten as

$$\sum_{i \in B} \lambda_i \left[\sum_{j \in U_i} T_j Z_j \left(\prod_{k \in C_i^j} \bar{Z}_k \right) \right] + \sum_{i \in B} \gamma_i \left[\sum_{j \in U_i} T_j Y_j \left(\prod_{k \in C_i^j} \bar{Z}_k \right) \right] \tag{11}$$

where

B	set formed by all sections of a feeder;
λ_i	permanent failure rate of section i;

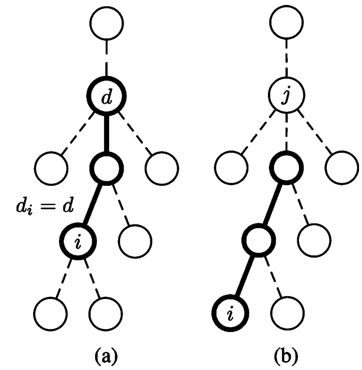

Fig. 3. Example of each set U_i and C_i^j. (a) The set U_i is formed by the nodes along the path between i and d_i. Both extreme nodes are included in the set. (b) When $i \neq j$, the set C_i^j is formed by the nodes along the path between i and j. Among the extreme nodes, only node i is included in the set. When $i = j$, the set C_i^j is defined to be empty.

γ_i	temporary failure rate of section i;
U_i	set formed by the nodes along the path between i and d_i, including both extreme nodes;
C_i^j	set formed by the nodes along the path between i and j, including i and excluding j, when $i \neq j$ and is an empty set, otherwise;
T_j	total number of customers downstream section j.

Since we know the number of customers in each section, the values of T_j can be calculated *a priori* for all sections of a feeder. An example of the composition of sets U_i and C_i^j is shown in Fig. 3. The set U_i can be enumerated by starting at node i and following the feeder tree upstream through each parent until the d_i node is reached. Both extreme nodes are included in the set, as can be seen in Fig. 3(a). When $i \neq j$, that same procedure can be used to enumerate the set C_i^j, except that node j must be excluded, as shown in Fig. 3(b). When $i = j$, the set C_i^j is defined to be empty. Since 1 is the neutral element of multiplication, when $C_i^j = \emptyset$, the term $\prod_{k \in C_i^j} \bar{Z}_k$ is considered to be equal to 1.

The first term of (11) represents the impact of permanent failures of a given section i when a protective device on section j is activated. We can read this term as follows: compute a sustained interruption to all customers downstream section $j\ (T_j)$, weighted by the permanent failure rate of the section where the failure occurred (λ_i), if a protective device is installed in $j\ (Z_j)$ and there is no other protective device installed between sections i and $j\ (\prod_{k \in C_i^j} \bar{Z}_k)$. The second term of (11) represents the impact of temporary failures of a given section i when a fuse on section j is activated, causing a sustained interruption. We can read this term as follows: compute a sustained interruption to all customers downstream section $j\ (T_j)$, weighted by the temporary failure rate of the section where the failure occurred (γ_i), if a fuse is installed in $j\ (Y_j)$ and there is no other protective device installed between sections i and $j\ (\prod_{k \in C_i^j} \bar{Z}_k)$.

Grouping the terms of (11), we have

$$\sum_{i \in B} \left[\sum_{j \in U_i} T_j (\lambda_i Z_j + \gamma_i Y_j) \left(\prod_{k \in C_i^j} \bar{Z}_k \right) \right]. \tag{12}$$

Now, the auxiliary variables X, Y, and Z can be replaced by the decision variables. We define variables x_j and y_j as

$$x_j = \begin{cases} 0, & \text{if a recloser should be installed on } j \\ 1, & \text{otherwise} \end{cases} \tag{13}$$

$$y_j = \begin{cases} 0, & \text{if a fuse should be installed on } j \\ 1, & \text{otherwise.} \end{cases} \tag{14}$$

Using (13) and (14), we can rewrite variables X_j and Y_j, and their complements as

$$X_j = 1 - x_j, \quad \bar{X}_j = x_j \tag{15}$$

$$Y_j = 1 - y_j, \quad \bar{Y}_j = y_j. \tag{16}$$

For variable Z_j and its complement, we replace (15) and (16) in (7) and (10), yielding

$$Z_j = 2 - x_j - y_j, \quad \bar{Z}_j = x_j y_j. \tag{17}$$

Finally, we use (16) and (17) to rewrite (12). Rearranging the coefficients of x_j and y_j, we arrive at

$$\sum_{i \in B} \left\{ \sum_{j \in U_i} T_j [2\lambda_i + \gamma_i - \lambda_i x_j - (\lambda_i + \gamma_i) y_j] \left(\prod_{k \in C_i^j} x_k y_k \right) \right\}. \tag{18}$$

We could have used the normally more intuitive inverse definition for the decision variables x_j and y_j. However, in that case, the Z_j complement would be $\bar{Z}_j = 2 - x_j - y_j$, and since $\bar{Z}_j$ is part of a product operator, the number of product terms in the objective function final equation would have been greater. Hence, this is our choice for the counterintuitive definition of x_j and y_j.

To calculate the SAIDI index of a feeder, (18) should be replaced by

$$\sum_{i \in B} \left\{ \sum_{j \in U_i} T_j t_i [2\lambda_i + \gamma_i - \lambda_i x_j - (\lambda_i + \gamma_i) y_j] \left(\prod_{k \in C_i^j} x_k y_k \right) \right\} \tag{19}$$

where t_i is the mean time to repair the failures of section i.

D. Constraints

This section presents only the minimal mandatory set of constraints for the model. However, some operational constraints presented in [2], similar to the ones handling coordination issues, can also be employed.

For every section where we know that a protective device will be installed, we must consider

$$x_j + y_j = 1, \quad j \in \{d_i\}, \quad i \in B. \tag{20}$$

With the remaining sections, we know that it is not possible to install more than one protective device in each section. Hence, the following constraint must hold:

$$x_i + y_i \geq 1, \quad i \in B. \tag{21}$$

Since a breaker is always installed at the feeder's root, we have

$$x_R = 0. \tag{22}$$

Regarding cost limitations, the following constraint is required:

$$\sum_{i \in B} x_i \geq |B| - (r + 1), \tag{23}$$

where r is the number of available reclosers for allocation on the feeder, excluding the substation breaker, and $|B|$ is the number of sections of the feeder.

IV. Tests and Results

A. Test Data

To evaluate the ZB model, we applied it to 36 test feeders selected from the distribution network of the Central Utility of the State of Espírito Santo, Brazil (ESCELSA). These feeders were selected because they cover urban and rural counties and are deemed as good representatives of the characteristics of the utility's distribution network. The test feeders are divided among six substations, marked from A to F. The smallest feeder is C7, with only eight sections; and the largest one is D1, with 246 sections.

B. Solving NLBP Problems

Due to its characteristics, the ZB model lends itself easily to mechanization. We developed an automatic model application program in the Python programming language that receives as input the required information of a feeder and delivers as output the NLBP problem using the GAMS [9] representation. GAMS is a very popular notational language for the representation of mathematical programming problems and serves as a common interface for different solvers, such as the BARON solver [10]. BARON is a solver based in the branch-and-cut enumeration method, suitable for solving NLBP problems. The BARON solver is freely available through the NEOS optimization server [11], [12].

Since it allows the greatest level of freedom in the placement of protective devices, all of the tests were performed without dividing the feeders. The optimal solutions of these problems define a lower bound for the indices of each feeder and can be used in comparison to results obtained with other forms of divisions on the feeders.

C. Results

As a special case of integer programming, NLBP problems are NP-hard [13] (i.e., they belong to a class of computational problems that usually take a long time to solve). However, as

TABLE I
MODELS RESULTS COMPARISON FOR $r = 1$

Feeder	SAIDI			SAIFI		
	ST	ZB	ST/ZB	ST	ZB	ST/ZB
A1	10.03	9.27	1.082	4.51	4.15	1.087
A2	11.73	8.85	1.325	5.21	3.64	1.546
A3	44.20	44.20	1.000	13.44	12.89	1.042
A4	48.35	44.86	1.078	12.89	11.99	1.075
B1	25.98	25.59	1.015	7.70	8.95	1.014
B2	10.25	9.62	1.066	3.27	3.12	1.049
B3	12.27	11.77	1.042	4.01	3.81	1.054
B4	22.22	19.59	1.135	6.66	5.34	1.248
C1	86.88	86.31[b]	1.007	18.10	18.02[b]	1.004
C2	81.28	73.78	1.102	15.45	13.71	1.127
C3	53.62	46.18	1.161	10.67	9.62	1.109
C4	27.12	23.96	1.132	5.72	5.16	1.109
C5	40.11	38.69	1.037	11.23	11.31	1.010
C6	5.77	4.23[a]	1.366	1.86	1.36[a]	1.369
C7	3.53	1.08[a]	3.257	1.21	0.41[a]	2.928
C8	35.39	27.73	1.276	7.05	5.79	1.218
D1	30.23	30.23	1.000	9.66	13.4	1.344
D2	58.11	44.45	1.307	15.60	11.64	1.340
D3	4.71	2.70	1.743	2.80	1.61	1.736
D4	10.77	8.54	1.262	3.46	2.73	1.264
E1	8.50	7.28	1.168	2.98	2.94	1.015
E2	28.21	27.24	1.035	14.02	13.24	1.059
E3	8.10	6.12	1.325	4.11	3.06	1.345
E4	6.04	5.80	1.041	3.24	3.08	1.052
E5	9.18	9.18	1.000	5.75	5.12	1.124
E6	6.63	6.06	1.093	2.94	2.65	1.109
E7	18.79	13.35	1.408	8.08	6.66	1.301
E8	16.89	16.89	1.000	5.83	5.51	1.111
E9	5.19	5.09	1.020	1.86	1.77	1.052
F1	25.52	24.62	1.037	6.69	6.34	1.056
F2	18.11	17.38	1.042	7.34	7.02	1.046
F3	18.79	17.84	1.053	9.41	8.58	1.097
F4	8.68	8.34	1.042	3.51	3.35	1.047
F5	5.75	5.62	1.024	2.62	2.69	1.005
F6	17.74	17.23	1.030	5.87	5.58	1.052
F7	31.79	–[c]	–	11.76	–[c]	–
	Average		1.192	Average		1.204
	Std. Deviation		0.393	Std. Deviation		0.342
	Minimum		1.000	Minimum		1.004
	Maximum		3.257	Maximum		2.928

[a] Optimal solution.
[b] Solver failure after preprocessing.
[c] Solver failure before preprocessing. No feasible solution found.

noted in [4], a careful analysis of the problem characteristics can point to situations where the required computational time is greatly reduced. When using the BARON solver in our tests, we made considerations as follows.

- We give the possibly suboptimal solution of the ST model as an initial solution for the BARON solver since this reduces the required enumeration of the search space.
- As noted in [14], nearly optimal solutions are usually found within the first minutes of solver execution. In all of the tests presented here, we limited the BARON execution time to 5 min.

The results for the tests performed for the allocation of one recloser $(r = 1)$ in each of the 36 feeders are presented in Table I. We did not limit the number of fuses available. The columns identified as ST and ZB show the feeders indices obtained with the ST model and the ZB model, respectively. In Table I, an^a indicates that the index is the optimal solution, found within the time limit of 5 min of execution time. The BARON solver failed with two feeders: C1 (marked with ^{b}), after the preprocessing routine had produced a solution; and F7 (marked with^c), before

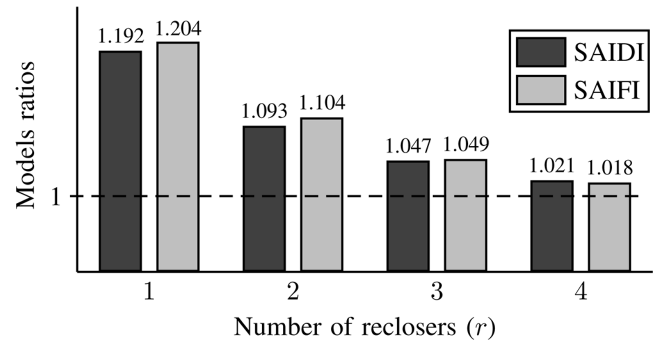

Fig. 4. SAIDI and SAIFI indices variation.

a feasible solution could be found. Both cases were due to the limitation of 32 MB of memory available for the BARON solver in the NEOS server.

To find the significance level of the results obtained in our experiments, we used the Wilcoxon signed-rank test [15], a statistical test that is nonparametric (i.e., that can be employed when we do not know the probability distribution of the data). We performed two Wilcoxon tests, one for each index in Table I, taking as the null hypothesis that the values from columns ST and ZB have an average difference of 0. In both tests, the p-values obtained were smaller than 10^{-5} (i.e., we can reject the null hypothesis with a degree of confidence of greater than 99%). Analyzing the results in Table I, we can see that the average improvement at the SAIDI and SAIFI indices is approximately 20%. Thus, from the ST/ZB ratio and the Wilcoxon tests results, we can conclude that the greater liberty on the protection devices allocation provided by the ZB model indeed leads to better configurations.

We also performed tests for the allocation of two, three, and four reclosers in each feeder. The average improvement ratios for the ZB model over the ST model can be seen in Fig. 4. It should be noted that when the number of reclosers available for allocation increases, the ST/ZB ratio diminishes. This occurs because when the number of available reclosers grows, the average difference between the optimal solution and any other solution in the state space decreases. In fact, in the asymptotic case where the number of reclosers to be installed is equal to the number of sections of the feeder, the ratio is 1.0. Since the ZB model is more general and less restrictive than the ST model, the former produces better placements solutions, especially under tight resources constraints $(r \leq 3)$. Additionally, the ZB model allows the electrical utility to deliver better service with lower cost: it maximizes the effects of each recloser employed and reduces the required amount of such equipment in order to achieve a target value of a reliability index.

To allow the reproduction of the tests performed and the validation of the results here presented, we make the data available for the 36 test feeders. Additionally, this information can be useful for other researchers in the same field. The interested reader is referred to the following Internet address: http://www.inf.ufes.br/~zambon/drd, where besides the feeders data, the GAMS models and the BARON log files can be downloaded. These files can also be requested either by e-mail or traditional mail sent to the address of the first author.

TABLE II
DATA FOR THE FEEDER OF FIG. 1[2]

Section	Permanent (λ) [failures/year]	Temporary (γ) [failures/year]	N [customers]
11	1.00	2.00	300
12	0.75	1.75	180
13	2.25	5.50	125
14	2.00	4.75	190
21	0.25	0.75	20
31	1.00	2.00	50
41	0.50	2.50	10

TABLE III
DOWNSTREAM CUSTOMERS FOR EACH j SECTION

j	11	12	13	14	21	31	41
T_j	875	575	325	200	20	50	10

V. CONCLUSION

This paper presented an NBP model that is more general than the one proposed by Soudi and Tomsovic [2] and can find better solutions for the problem of protective device allocation in a distribution feeder. Moreover, our model can handle arbitrary feeder divisions and, thus, treat the original model of Soudi and Tomsovic as a particular case. Numerical results for the tests performed with 36 distribution feeders show the improvements in the SAIDI and SAIFI indices obtained with our new model. Furthermore, we make the relevant data for all of our 36 test feeders available to the research community.

As a future work, it would be interesting to extend the proposed model here to explicitly handle the effects of isolating switches, following the models presented in [6] and [8].

APPENDIX
EXAMPLE OF APPLICATION OF THE ZB MODEL

To apply the ZB model to a feeder, the following information is needed:

- a structural representation of the feeder as a tree of nodes;
- the permanent (λ) and temporary (γ) failure rates for each feeder section;
- the number of customers (N) located within the boundaries of each section.

To illustrate the model application, we take, as an example, the feeder from Fig. 1, originally presented in [2]. The data required are summarized in Table II, also taken from [2].

First, for each section j of the feeder, we need to calculate the number of customers downstream (T_j), presented in Table III.

Next, we must decide how to divide the feeder. For this example, we want to analyze the feeder without divisions. So we make $d_i = 11$ for all sections $(i \in \{11, 12, 13, 14, 21, 31, 41\})$. To find the objective function for the minimization of the SAIFI index, we replace the numerator of (4) by (18) and use the data in Tables II and III, obtaining the objective function given in (24). For ease of understanding, we included two columns in (24) that show the i and j indices, and the corresponding terms in the equation

$$\begin{aligned}
&i=11 \quad j=11 \quad [(3500-875x_{11}-2625y_{11})+\\
&i=12 \quad j=12 \quad (1868.75-431.25x_{12}-1437.5y_{12})+\\
&\qquad\qquad j=11\ (2843.75-656.25x_{11}-2187.5y_{11})x_{12}y_{12}+\\
&i=13 \quad j=13 \quad (3250-731.75x_{13}-2518.25y_{13})+\\
&\qquad j=12 \quad (5750-1293.75x_{12}-4456.25y_{12})\\
&\qquad\qquad \cdot x_{13}y_{13}+\\
&\qquad j=11 \quad (8750-1968.75x_{11}-6781.25y_{11})\\
&\qquad\qquad \cdot x_{13}y_{13}x_{12}y_{12}+\\
&i=14 \quad j=14 \quad (1750-400x_{14}-1350y_{14})+\\
&\qquad j=13 \quad (2843.75-650x_{13}-2193.75y_{13})x_{14}y_{14}+\\
&\qquad j=12 \quad (5031.25-1150x_{12}-3881.25y_{12})\\
&\qquad\qquad \cdot x_{14}y_{14}x_{13}y_{13}+\\
&\qquad j=11 \quad (7656.25-1750x_{11}-5906.25y_{11})\\
&\qquad\qquad \cdot x_{14}y_{14}x_{13}y_{13}x_{12}y_{12}+\\
&i=21 \quad j=21 \quad (25-5x_{21}-20y_{21})+\\
&\qquad j=12 \quad (718.75-143.75x_{12}-575y_{12})x_{21}y_{21}+\\
&\qquad j=11 \quad (1093.75-218.75x_{11}-875y_{11})\\
&\qquad\qquad \cdot x_{21}y_{21}x_{12}y_{12}+\\
&i=31 \quad j=31 \quad (200-50x_{31}-150y_{31})+\\
&\qquad j=12 \quad (2300-575x_{12}-1725y_{12})x_{31}y_{31}+\\
&\qquad j=11 \quad (3500-875x_{11}-2625y_{11})x_{31}y_{31}x_{12}y_{12}+\\
&i=41 \quad j=41 \quad (35-5x_{41}-30y_{41})+\\
&\qquad j=14 \quad (700-100x_{14}-600y_{14})x_{41}y_{41}+\\
&\qquad j=13 \quad (1137.5-162.5x_{13}-975y_{13})x_{41}y_{41}x_{14}y_{14}+\\
&\qquad j=12 \quad (2012.5-287.5x_{12}-1725y_{12})\\
&\qquad\qquad \cdot x_{41}y_{41}x_{14}y_{14}x_{13}y_{13}+\\
&\qquad j=11 \quad (3062.5-437.5x_{11}-2625y_{11})\\
&\qquad\qquad \cdot x_{41}y_{41}x_{14}y_{14}x_{13}y_{13}x_{12}y_{12}]/875.
\end{aligned} \tag{24}$$

If we have two reclosers available for allocation $(r = 2)$, the following constraints must be satisfied:

$$\begin{aligned}
&x_{11}+x_{12}+x_{13}+x_{14}+x_{21}+x_{31}+x_{41} \geq 4,\\
&x_{11}=0, \qquad x_{11}+y_{11}=1,\\
&x_{12}+y_{12}\geq 1, \quad x_{13}+y_{13}\geq 1, \quad x_{14}+y_{14}\geq 1,\\
&x_{21}+y_{21}\geq 1, \quad x_{31}+y_{31}\geq 1, \quad x_{41}+y_{41}\geq 1.
\end{aligned}$$

The optimal solution for the NLBP problem defined by (24) is an SAIFI of 3.27 interruptions per year.

In [2], Soudi and Tomsovic divide this sample feeder as follows.

- The main line is formed by the sections 11, 12, 13, and 14.
- Sections 21, 31, and 41 each form a category three lateral.

Should we want to apply that division using the ZB model, we must make $d_{21} = 21, d_{31} = 31, d_{41} = 41$, and $d_i = 11$ for $i \in \{11, 12, 13, 14\}$.

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