

Letter to the Editor

COMMENTS ON "A DEFICIENCY OF NATURAL DEDUCTION" [1]

Dear sir,

I think it is appropriate that a word of comment be made on Wiltink's article [1]. The author of this article aims at pointing out that Gentzen's system of Natural Deduction cannot live up to its claim to provide for efficient formalization of mathematical proofs. Whether Natural Deduction is actually suited for such a formalization may be disputable, but the way the author goes about stating his case is, in my opinion, certainly disputable, and in some aspects even wrong. What makes Wiltink's article hard to understand is the following. Apparently, he wants to offer an alternative proof system providing for shorter and more efficient proofs than possible in Natural Deduction, but he does so without giving any explicit reference to what his proof system actually amounts to: there is no mention of the syntax, of the axioms, nor of the inference rules employed in his logic. It is this complete lack of explicitness that makes it almost impossible to evaluate the merits of his work.

In one of his examples, the author offers a (short) proof making use of properties of (logical) equivalence (such as associativity and reflexivity). Apparently, the kind of logical system the author has in mind is based on this special logical connective. He also claims that it is a defect of Natural Deduction that it does not handle equivalence efficiently, one reason being that, in Natural Deduction, equivalence is a defined construct (for example, a conjunction of implications). For two reasons this is not a fair argument. First of all (as already mentioned above), the author should give a full axiomatization of his version of predicate logic to actually demonstrate that predicate

logic can be suitably based on logical equivalence instead of implication. Without such a comparative formalization, further discussion seems rather senseless. In the second place, Natural Deduction in its primitive axiomatized form offers a framework for manipulating basic constructs from which properties of defined constructs (such as equivalence) can be derived in a sometimes lengthy but straightforward manner. (This is, in fact, the major strong-point of Gentzen's system: it is one of the few known axiomatizations of first-order predicate logic in which *syntactical* proofs can actually be carried out in a more-or-less straightforward and intuitively simple manner.) The basic Natural Deduction system enhanced with such derived properties (*theorems* in Natural Deduction) can then provide for a powerful framework (by means of a *meta*-theorem known as 'substitution for equivalences'), in which finding proofs of new properties can be immensely shortened.

In another example, the author asks for a short proof of the following problem: from $A \equiv B$ conclude that $A \vee C \equiv B \vee C$. In the Natural Deduction system enhanced with the above-mentioned substitution rule, the result follows immediately, so no real problem here. Wiltink, in his article, proposes a completely different approach: he makes an appeal to the so-called rule of Leibniz! But this rule, employing quantification over function symbols, is a construct from *second-order* predicate logic, which is a completely different game! Gentzen's system of Natural Deduction is aimed at axiomatizing *first-order* predicate logic, hence an appeal to Leibniz' rule is quite out of place.

I agree with Wiltink that proofs in the basic Natural Deduction system are sometimes rather

lengthy but, in an enhanced system as described above, the lengths of these proofs can usually be cut down substantially. I do not agree with Wiltink that he has offered insight in constructing actual shorter proofs, since he fails in providing anything near an alternative formalization of first-order predicate logic.

H. BALSTERS

*Department of Computer Science
Twente University of Technology
P.O. Box 717
7500 AE Enschede
The Netherlands*

Reference

- [1] J.G. Wiltink. A deficiency of natural deduction, *Inform. Process. Lett.* 25 (4) (1987): 233-234.

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