

## WEAR NUMBERS FOR BALL CUP AND JOURNAL BEARINGS

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### Summary

A wear number is defined for ball cup bearings and for journal bearings where the cup and the cylindrical bearing are made of soft material. This dimensionless wear number provides a relation between the following five quantities: the radius of the ball or the length of the journal bearing in millimetres, the wear modulus in newtons per square millimetre, the maximum wear depth rate of the cup or the cylindrical bearing in millimetres per second, the force between the mating surfaces in newtons and the angular speed of the ball or the journal in radians per second. The wear volume of the plastic cup of a hip joint prosthesis was calculated and from the point of view of wear the Charnley prosthesis is probably superior to the Charnley-Muller prosthesis.

### 1. Wear number

The wear modulus in the plastic cup of a hip joint prosthesis has been calculated [1] from the results of wear tests [2]. The initial formula for the wear modulus is given by

$$\frac{F\omega}{4rW(\Delta d_z/\Delta t)} = \frac{1}{2L}$$
$$= I = \int_{\phi=0}^{\phi=\pi/2} \int_{\theta=0}^{\theta=\theta_e} \frac{\cos^2\theta \sin\theta}{(\cos^2\theta + \sin^2\theta \cos^2\phi)^{1/2}} d\theta d\phi \quad (1)$$

where  $F$  is the load on the ball of the prosthesis in newtons,  $W$  is the wear modulus in newtons per square millimetre,  $\omega$  is the angular speed of the ball with respect to the cup in radians per second,  $\Delta d_z/\Delta t$  is the maximum wear depth rate in the cup in millimetres per second,  $L$  is defined as a wear number,  $r$ ,  $\theta$  and  $\phi$  are the spherical coordinates in millimetres and radians and  $\theta_e$  is the contact angle between the ball and the cup (Fig. 1). The following approximate solution was used for the double integral in eqn. (1):

$$I = \int_{\phi=0}^{\phi=\pi/2} \int_{\theta=0}^{\theta=\theta_e} \frac{\cos^2 \theta \sin \theta \, d\theta \, d\phi}{(\cos^2 \theta + \sin^2 \theta \cos^2 \phi)^{1/2}}$$

$$\approx 0.92\theta_e^2 - 0.424\theta_e^3 \quad (2)$$

However, a more accurate approximation can be derived:

$$I = \int_{\theta=0}^{\theta=\theta_e} \int_{\phi=0}^{\phi=\pi/2} \frac{\cos^2 \theta \sin \theta \, d\phi \, d\theta}{(\cos^2 \theta + \sin^2 \theta \cos^2 \phi)^{1/2}}$$

$$= \int_{\theta=0}^{\theta=\theta_e} \int_{\phi=0}^{\phi=\pi/2} \frac{\cos^2 \theta \sin \theta \, d\phi \, d\theta}{(1 - \sin^2 \theta \sin^2 \phi)^{1/2}} \quad (3)$$

where the complete elliptic integral in eqn. (3) can be well approximated by [3]

$$\int_{\phi=0}^{\phi=\pi/2} \frac{d\phi}{(1 - \sin^2 \theta \sin^2 \phi)^{1/2}} = \ln \left\{ \frac{4}{(1 - \sin^2 \theta)^{1/2}} \right\} + 0.167(1 - \sin^2 \theta) \times$$

$$\times \ln \left\{ \frac{3.0176}{(1 - \sin^2 \theta)^{1/2}} \right\} \quad (4)$$

for  $0 \leq \sin^2 \theta \leq 1$  and with an error less than 0.002. Substitution of eqn. (4) into eqn. (3) gives

$$I = \int_{\theta=0}^{\theta=\theta_e} \left\{ \ln \left( \frac{4}{\cos \theta} \right) + 0.1671 \cos^2 \theta \ln \left( \frac{3.0176}{\cos \theta} \right) \right\} \cos^2 \theta \sin \theta \, d\theta \quad (5)$$

This integral can readily be solved by making the following substitution:

$$\cos \theta = x$$

Thus

$$\theta = \arccos x$$

$$d\theta = -\frac{1}{(1-x^2)^{1/2}} dx$$

After substitution, integration and some calculations we obtain from eqn. (5)

$$I = 0.03342 \cos^5 \theta_e \ln \cos \theta_e - 0.04359 \cos^5 \theta_e +$$

$$+ 0.33333 \cos^3 \theta_e \ln \cos \theta_e - 0.57320 \cos^3 \theta_e + 0.61679 \quad (6)$$

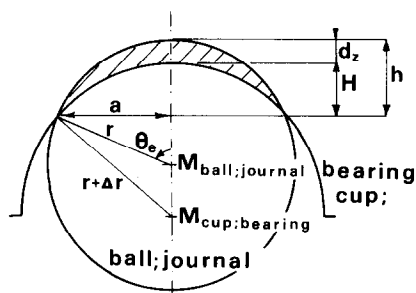


Fig. 1. Ball cup hip joint prosthesis or journal in cylindrical bearing.

From eqn. (1)

$$W = \frac{F\omega}{4r(\Delta d_z/\Delta t)I} \quad (7)$$

and

$$L \equiv \frac{2rW(\Delta d_z/\Delta t)}{F\omega} = \frac{1}{2I} \quad (8)$$

Thus

$$L_{\theta_e \rightarrow 0} = \frac{1}{2 \times 0.7854\theta_e^2} \quad (8a)$$

$$L_{\theta_e \rightarrow 0} = 2090/\theta_e^2$$

where  $\theta_e$  is in radians and degrees respectively.

The wear modulus  $W$  and the wear number  $L$  for a journal bearing can now be derived. Substituting  $\phi = 0$  into eqn. (7) of ref. 1 gives

$$\sigma_n = \frac{W}{\omega r} \cos\theta \frac{\Delta d_z}{\Delta t} \quad (9)$$

From Fig. 2 of ref. 1 and Fig. 1

$$dA = rb \, d\theta \quad (10)$$

and

$$\sigma_z = \cos\theta \sigma_n \quad (11)$$

where the infinitesimal contact area  $dA$  is a function of the radius  $r$  of the journal in millimetres, the length  $b$  of the bearing in millimetres and the angle  $\theta$  in radians. Equations (9), (10) and (11) together with eqn. (8) of ref. 1 give

$$\begin{aligned}
 F &= 2 \int_{\theta=0}^{\theta_e} \cos\theta \frac{W}{\omega r} \cos\theta \frac{\Delta d_z}{\Delta t} r b d\theta \\
 &= \frac{bW}{\omega} \frac{\Delta d_z}{\Delta t} 2 \int_{\theta=0}^{\theta_e} \cos^2\theta d\theta
 \end{aligned}$$

(see ref. 4)

$$F = \frac{bW}{\omega} \frac{\Delta d_z}{\Delta t} 2 \left( \frac{1}{2} \theta_e + \frac{1}{4} \sin 2\theta_e \right) \tag{12}$$

According to eqn. (7)

$$W = \frac{F\omega}{b(\Delta d_z/\Delta t)(\theta_e + \frac{1}{2} \sin 2\theta_e)} \tag{13}$$

According to eqn. (8)

$$L \equiv \frac{bW(\Delta d_z/\Delta t)}{F\omega} = \frac{1}{\theta_e + \frac{1}{2} \sin 2\theta_e} \tag{14}$$

$L$  is defined as a wear number:

$$\begin{aligned}
 L_{\theta_e \rightarrow 0} &= \frac{1}{2\theta_e} \\
 L_{\theta_e \rightarrow 0} &= \frac{28.65}{\theta_e}
 \end{aligned} \tag{14a}$$

where  $\theta_e$  is in radians and degrees respectively. Equations (6), (8) and (14) are given in Table 1 and Fig. 2. The asymptotes (eqn. (8a) and eqn. (14a) are also given in Fig. 2.

TABLE 1

Wear number  $L$  as a function of the contact angle  $\theta_e$  for a hip joint prosthesis (ball cup bearing) and for a journal bearing

Contact angle $\theta_e$ (°)		0	15	30	45	60	75	90
Wear number $L$	Hip joint prosthesis	$\infty$	9.58	2.634	1.375	0.972	0.835	0.811
	Journal bearing	$\infty$	1.953	1.045	0.778	0.676	0.641	0.637

## 2. Wear volume in the cup

The wear volume in the cup of the prosthesis was approximated (eqn. (22) of ref. 1) by the difference of two cones. However, this was not

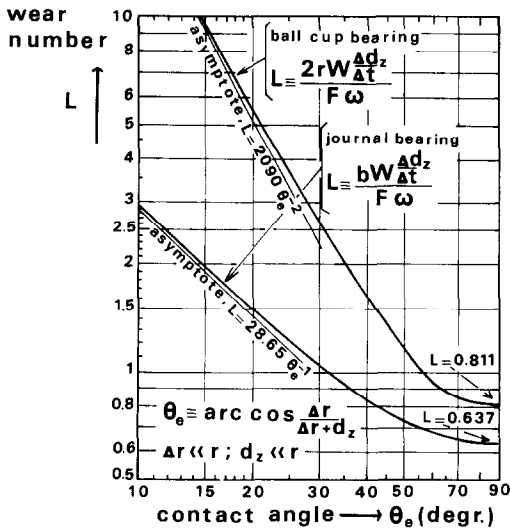


Fig. 2. Wear number  $L$  as a function of the contact angle  $\theta_e$  for a hip joint prosthesis (a ball cup bearing) and for a journal bearing. The ball and journal are hard, and the cup and cylindrical bearing are soft.

satisfactory since it is too rough an approximation. It can be more accurately calculated by using (Fig. 1) [4]

$$\begin{aligned}
 V_w &= \frac{1}{6} \pi h(3a^2 + h^2) - \frac{1}{6} \pi H(3a^2 + H^2) \\
 &= \frac{1}{6} \pi (r - r \cos \theta_e) \{3r^2 \sin^2 \theta_e + r^2 (1 - \cos \theta_e)^2\} - \\
 &\quad - \frac{1}{6} \pi (r - r \cos \theta_e - d_z) \{3r^2 \sin^2 \theta_e + (r - r \cos \theta_e - d_z)^2\} \\
 &= \frac{1}{6} \pi (6r^2 d_z - 6r^2 d_z \cos \theta_e - 3rd_z^2 + 3rd_z^2 \cos \theta_e + d_z^3) \quad (15)
 \end{aligned}$$

The conditions for eqn. (15) are

$$6r^2 d_z \gg 3rd_z^2$$

$$2r \gg d_z$$

The last three terms in eqn. (15) can be neglected giving

$$V_w \approx \pi r^2 d_z (1 - \cos \theta_e) \quad (16)$$

From eqn. (18) of ref. 1

$$\cos \theta_e \approx \frac{\Delta r}{d_z + \Delta r} \quad (17)$$

where  $\Delta r$  is the radial clearance in millimetres. Substitution of eqn. (17) into eqn. (16) gives

$$V_w \approx \pi r^2 \frac{d_z^2}{d_z + \Delta r} \quad (18)$$

The wear volume of the Charnley prosthesis is greater than that of the Charnley–Muller prosthesis for the first 500 h of testing. However, in the second 500 h the wear volume in the Charnley prosthesis is 7.17 mm<sup>3</sup> compared with 7.78 mm<sup>3</sup>, *i.e.* 10% more, in the Charnley–Muller prosthesis (Table 2). Therefore, assuming that this trend continues, the Charnley prosthesis is superior to the Charnley–Muller prosthesis from the point of view of wear provided that it is run-in prior to implantation.

TABLE 2

Wear volume obtained in the plastic cup of hip joint prostheses

	<i>Ball radius</i> <i>r</i> [2] (mm)	<i>Maximum</i> <i>wear depth</i> <i>d<sub>z</sub></i> [2] (mm)	<i>Radial</i> <i>clearance</i> $\Delta r$ (mm)	<i>Wear</i> <i>volume</i> <i>V<sub>w</sub></i> (mm <sup>3</sup> )
Charnley–Muller prosthesis after 500 h	16	0.057	0.140	13.26
Charnley prosthesis after 500 h	11	0.120	0.210	16.59
Charnley–Muller prosthesis after 1000 h	16	0.075	0.140	21.04
Charnley prosthesis after 1000 h	11	0.150	0.210	23.76

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