

Stengel RF: *Stochastic Optimal Control*. John Wiley, New York 1986.

The title of this book is misleading. It is not a book specifically devoted to stochastic control, but covers all standard topics in modern control theory – deterministic optimal control, optimal state estimation, stochastic control theory and some aspects of linear multivariable control. To include all these topics, along with the necessary mathematical preliminaries, in one manageable single volume is a daunting assignment. The author succeeded in that, although occasionally at the cost of clarity.

After a short introduction on the objectives of optimal (stochastic) control, the author assembles the necessary mathematical background material in Chapter 2. They include introduction to vectors and matrices, detailed properties of and operations on matrices, dynamical system models, elements of random variables and stochastic processes, system theoretic properties of dynamic models and frequency domain modeling and analysis. This last topic, which forms the central feature of classical control theory, is treated rather sketchily. Both continuous- and discrete-time systems have been studied, although the treatment of continuous-time stochastic (nonlinear) systems is superficial and may even be misleading for a beginning student.

Chapter 3 deals with deterministic optimal control theory. Both the minimum principle of Pontryagin and the Hamilton-Jacobi-Bellman equations are derived. Equality and inequality constraints on the state and control, and singular control are studied, along with the numerical algorithms for determining the optimal controls. One special feature of the book is a separate section on “neighbouring-optimal solutions”. The idea is to first determine the optimal solution and then add to it a “small variation” linear-quadratic optimal solution to take care of any eventual variation in initial condition, final condition, parameters or deterministic disturbances which may alter the optimal state and control histories. The discussion on numerical schemes is very cryptic and should be looked at as only providing the essential information which must be pursued further if one wants seriously to grasp the material.

Chapter 4 is on optimal state estimation; that is, on Kalman filter. The author starts with (weighted) least squares estimation of an unknown parameter vector, derives its recursive version and then obtains Kalman filter by directly referring to these equations. The derivation of Kalman filter equations is correct and compact, but the role of the “innovation process” remains obscure. Continuous-time case is treated heuristically as the limit of the discrete-time situation. Nonlinear estimation, including extended Kalman-Bucy filter, is also discussed. The chapter ends with a section on adaptive filtering, including a short discussion on multiple-model estimation. Smoothing algorithms are not discussed at all.

Chapter 5 combines the knowledge of the two preceding chapters to study the stochastic optimal control theory. Dynamic programming equation is derived in the continuous-time somewhat heuristically and then applied to obtain the detailed solution of the linear-quadratic stochastic control problem with complete informa-

tion of the system state. The corresponding results in the discrete-time case are merely stated. Neighbouring-optimal control is also derived. The author then formulates the stochastic control problem with incomplete and inaccurate information of the system state. The dynamic programming equation is again heuristically derived and the notion and role of “sufficient statistic” in solving this problem is stressed inadequately. However, a reasonably thorough treatment of discrete-time dual control is given. The LQG (Linear-Quadratic-Gaussian) problem is then solved, both in the continuous and discrete-time cases, along with a discussion on the “separation principle” (certainty equivalence). Finally, the author considers the infinite-horizon problem (steady state case). In this section, the steady state behavior of the LQ-problem in Chapter 3 and the steady state behavior of Kalman filter in Chapter 4 are also treated for the first time and then combined to study the most general infinite horizon LQG-problem.

The last Chapter on linear multivariable control touches only on some aspects of this subject as they relate to the linear-quadratic (stochastic) control theory. It begins with the numerical solution of the algebraic Riccati equation. The rest of the chapter deals with several design issues which may be satisfactorily resolved with proper choice of matrices in the criterion function. This connects the modern time-domain theory with classical frequency-domain one. One, of course, needs to develop multivariable equivalent of Bode plot, Nyquist criterion etc., which are discussed in this chapter. These are used for studying robustness of linear-quadratic regulators. This is relatively recent area of research and the contributions of researchers to various topics are not always clearly mentioned. This is inconvenient for a reader who may want to pursue one particular topic further. In addition, the recent active area of  $H^\infty$ -optimization is not mentioned at all.

As mentioned in the beginning, the author wanted to include too many subjects in one manageable single volume and this meant that many topics are only discussed cursorily. In this sense, this book is not suitable for graduate students in the area of systems and control. On the other hand, this book may be extremely useful for engineering students and researchers in other fields who may need modern control theory intensively in their (future) research. In particular, mechanical engineers interested in robotics and aerospace engineers will find this book valuable. There are a number of interesting problems at the end of each chapter and numerous practical examples are either worked out or stated as problems, many of which arise in aerospace application. This reflects the expertise of the author. Some examples from economics might make this book also a good source for economists interested in economic dynamics and control. The book is written in a lucid manner and too much technical details is avoided. Despite its shortcomings, this book is definitely a useful contribution to the field.