Energy efficient networking via dynamic relay node selection in wireless networks

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Abstract

Mobile wireless ad-hoc networks need to maximize their network lifetime (defined as the time until the first node runs out of energy). In the broadcast network lifetime problem, all nodes are sending broadcast traffic, and one asks for an assignment of transmit powers to nodes, and for sets of relay nodes so that the network lifetime is maximized. The selection of a dynamic relay set consisting of a single node (the 'master'), can be regarded as a special case, providing lower bounds to the optimal lifetime in the general setting. This paper provides a preliminary analysis of such a 'dynamic master selection' algorithm, comparing relaying to direct routing.

Keywords: network lifetime, ad hoc networks, average case analysis, random graphs

1. Introduction

The finite amount of battery energy in sensor- or ad-hoc networks gives rise to a number of issues concerning energy saving at the physical, MAC and network layers. For such networks it is important to maximize the network lifetime. Here, the network lifetime is defined as the time until the first node runs out of energy. The broadcast network lifetime problem asks for settings

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of transmit powers and (node-dependent) sets of relay nodes, that maximize the network lifetime, while other nodes originate broadcast traffic. If we do not consider the node dependencies, we can ask for a fixed *set* of relay nodes to maximize the network lifetime, while allowing transmissions from multiple sources. This leads to lower bounds for the general network lifetime problem. This paper presents a preliminary analysis of a special case in this setting, where we ask for a *single* relay node (the 'master'), which is allowed to change over time. We compare this dynamic master selection to a direct routing approach (without multihop communication). This is of interest for ad hoc networks and sensor networks. Here one could envisage a distinction between very simple devices (clients), and more powerful devices (eligible masters). Implementing a dynamic master selection algorithm imposes little memory requirements while enhancing the relaying capabilities, and increasing network lifetime. Motivated by the fact that communication is a much more energy expensive tasks than data processing (see e.g. [1]), we focus on the efficiency of the distance-dominated communication related power consumption.

Section 2 of this paper presents an overview of related literature. Section 3 introduces the model and notation. In Section 4 of this paper we provide an approximate analysis of this problem, in a non-geometric setting, where the transmit power thresholds are randomly chosen in the unit interval. In Section 5 we address a geometric setting, where nodes are randomly distributed in the plane. There we distinguish two cases: the case where transmit power levels can be adjusted continuously, and the case where only a discrete set of transmit power levels is supported. Section 6 presents the conclusions.

2. Literature overview

Considerable research efforts in the literature attempts to reduce energy consumption and maximize the network lifetime for ad-hoc and wireless sensor networks. In [2] a general overview of strategies to alleviate power consumption in wireless networks is presented. It is natural to use power control to reduce the

transmission power and thus minimize the energy consumption at the physical layer [3]. In the context of mobile ad-hoc networks (MANETs), the complexity is reduced by assuming transmissions originate from a single source ([4], [5] and [6]). The related problem of minimizing the total energy consumption for broadcast traffic has also been widely studied, because it provides a crude upper bound to the lifetime of the network. In [7] and [8] it is shown that minimizing the total transmit power is NP-hard. A general approximation framework for fault tolerant topology control problems is developed in [9]. However, this problem does not address the residual energy of the nodes.

There is also work focusing on selection of multihop routes in order to maximize the network lifetime. In [10] a new routing algorithm is proposed in terms of maximizing the system lifetime, which can also be interpreted as maximizing the amount of information transfer between origin and destination given the limited energy. In [3] this approach is further extended to take into account Shannon capacity of each link. Addressing the heterogeneous case, where nodes run on batteries or are connected to the mains is [11], where a new energy-aware routing algorithm is developed. These approaches typically address unicast traffic over a multihop network, whereas we address broadcast traffic in a single hop situation. When the locations of part of the nodes are a variable, the problem is to find the (energy-optimal) location of relay nodes, given the location of the sensors within the network, this problem is for example studied in [12] and in [13].

The work in this paper is closest related to the research on hierarchical routing protocols for sensor networks. This involves the partitioning of nodes into a number of small groups called clusters. The member nodes send their data to their immediate cluster heads (corresponding to our master). These perform data aggregation and send the message to the next destination. As discussed in [14], [15], LEACH (Low Energy Adaptive Clustering Hierarchy) is perhaps the first cluster based routing protocol for wireless sensor networks, which uses a stochastic model for cluster head selection. This protocol forms clusters by using a distributed stochastic algorithm. However, LEACH does not take residual

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energy into account. In [16], an energy efficient cluster head (EECHE) selection algorithm is proposed, by adjusting the threshold (determining the likelihood of cluster head selection) based on the residual energy. The algorithm of [17], focuses on minimizing the number of communication messages. The problem of finding the optimal path for data transmission between cluster heads and the base station is addressed in [18].

The main contribution of this paper is that we do not target a *single (unicast) destination* (the base station), but that in our case the master (cluster head) should broadcast the data to *all* nodes. Moreover, the algorithm we propose is not stochastic but deterministic in nature, and takes residual energy explicitly into account. In addition, we complement the simulation results on the algorithm with a formal analysis. We believe the analytical method as presented here could be used to analyze the various clustering algorithms as well.

3. General model and notation

In order to formally define the problem, we introduce some notation. For a set V (denoting the potential master nodes), a power assignment is a function $p : V \rightarrow \mathbb{R}$. To each ordered pair (u, v) of transceivers we assign a transmit power threshold, denoted by $c(u, v)$, with the following meaning: a signal transmitted by transceiver u can be received by v only when the transmit power is at least $c(u, v)$. We assume that the $c(u, v)$ can be determined, and that these are symmetric. A node can only be chosen a master if it can reach all other nodes when transmitting at maximum power. In our analysis, we only consider those nodes that are eligible as master, i.e., those nodes that can reach all other nodes when transmitting at maximum power. For a node $m \in V$, let p_m denote the power assignment $p_m : V \rightarrow \mathbb{R}$ defined as:

$$p_m(v) = \begin{cases} c(v, m) & \text{for } v \neq m, \\ \max_{v \in V} c(v, m) & \text{for } v = m. \end{cases} \tag{1}$$

$p_m(v)$ can be interpreted as the power assigned to v when m is master.

In [11] and [13] and (and references herein), the power consumption due to transmission of a packet consists of a distance independent part for transmission and reception (due to activation of transmitter and receiver circuits) and a distance related part for transmission. In this paper, we focus on the efficiency of the distance-dominated communication related power consumption, assuming a linear battery model. So, each vertex is equipped with battery supply b_v, which is reduced by amount $\lambda p_m(v)$ for each message transmission by v with transmit power $p_m(v)$. However, the analysis presented above is extendable to case of more complicated power models.

We assume that all nodes $v \in V$ transmit at a constant rate a_v, where a_v denote the number of messages per time unit. We call a series of transmissions were each node $v \in V$ transmits a_v times a *round*. With these assumptions, we obtain for the battery reduction after one round (with master node m):

$$b_v = \begin{cases} b_m - \lambda p_m(v) \sum_{v \in V} a_v & \text{for } v = m, \\ b_v - \lambda a_v p_m(v) & \text{for } v \neq m. \end{cases}$$

In [19] we analyzed the case where a master m is kept constant for the whole lifetime of the network. This paper is concerned with a dynamic version of this problem: given a graph $G = (V, E, c, b, a)$, where $c : E \to \mathbb{R}$ denotes the transmit power thresholds, and $b : V \to \mathbb{R}$ denotes the initial battery levels $b_v, v \in V$, and the relative frequencies $a_1, \ldots, a_n$. We ask for the number of rounds x_v for each node v to be master. Here, the $x_v \geq 0$ have to be chosen in such a way that $\sum_{v \in V} x_v$ is maximized under the condition that the remaining battery capacity of each node is positive during the lifetime of the network. Here, x denotes the vector $(x_1, \ldots, x_n)$. Corresponding to the fact that each node can reach all other nodes when transmitting at maximum power, we assume that E corresponds to a complete graph.

We call $x = (x_1, \ldots, x_n) \in \mathbb{N}_+^n$ *feasible* if for all $m \in \{1, \ldots, n\}$,

$$b_m - \lambda \sum_{v \neq m} a_m x_v p_v(m) - \lambda x_m p_m(m) \sum_{v \in V} a_v \geq 0. \tag{2}$$

The terms $\lambda \sum_{v \neq m} a_m x_v p_v(m)$ and $\lambda x_m p_m(m) \sum_{v \in V} a_v$ in (2) indicate the reduction in battery capacity of node m during the periods when nodes $v \neq m$
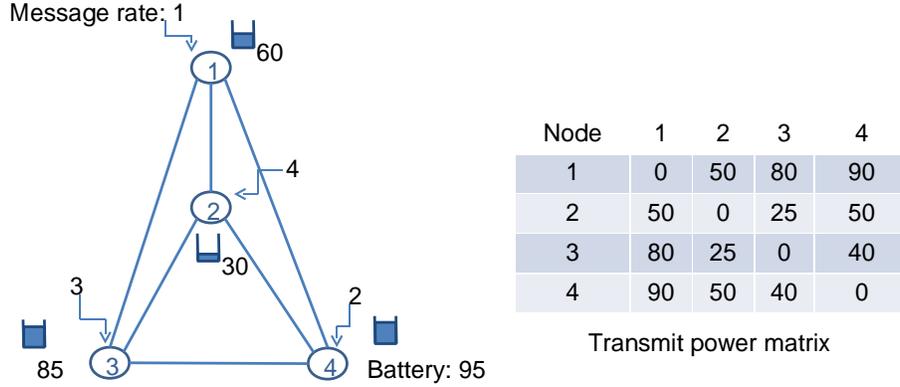
Node	1	2	3	4
1	0	50	80	90
2	50	0	25	50
3	80	25	0	40
4	90	50	40	0

Transmit power matrix

Figure 1: Example of graph $G = (V, E, c, b, a)$: V consists of 4 vertices $\{1, 2, 3, 4\}$, E consists of all pairs of edges, the filled boxes indicate the remaining battery levels b_v in % of the total level, the arrows indicate the rates a_v at which each vertex generates messages. The transmit power thresholds $c(u, v)$ are indicated in the table.

are master, and when m is master, respectively.

By scaling, we may assume that $\lambda = 1$. With $x = (x_1, \ldots, x_n)$, (2) can be rephrased as: $Ax \leq b$, where $b : V \to \mathbb{R}^+$, and where A is an $n \times n$-matrix where the entry corresponding to (v, m) is defined by:

$$A(v, m) = \begin{cases} p_m(m) \sum_{v \in V} a_v & \text{for } v = m, \\ a_m p_v(m) & \text{for } v \neq m. \end{cases} \tag{3}$$

Now, dropping the integrality constraints on the number of rounds, lifetime maximization (for a dynamically chosen single relay node) corresponds to the solution of a simple linear program, a fact which is exploited in [20]. We call this algorithm OPT (Optimal Master Selection). To be explicit, under OPT, we choose $x \geq 0$, so that $\sum_{i=1}^{n} x_i$ is maximized, under condition $Ax \leq b$, with matrix A as defined in (3).

In the next section, we will compare this with the algorithm DIR (Direct Transmission). Under DIR, there is no master: all nodes reach all other nodes via a single hop transmission, from source to destination. A related variant of this algorithm is also analyzed in [14].

As an example, for the graph in Figure 1, the inequality $Ax \leq b$ becomes:

$$
\begin{bmatrix}
900 & 50 & 80 & 90 \\
200 & 500 & 100 & 200 \\
240 & 75 & 800 & 120 \\
180 & 100 & 80 & 900
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4
\end{bmatrix}
\leq
\begin{bmatrix}
60 \\ 90 \\ 85 \\ 30
\end{bmatrix}. \tag{4}
$$

We illustrate the calculation of entries $A(1,1)$ and $A(1,2)$, for the example in Figure 1. We have $\sum_{v \in V} a_v = 10$, moreover $p_1(1) = 90$, because 1 has to reach all other nodes. So $A(1,1) = 10 \cdot 90 = 900$. Moreover, for the message rate: $a_1 = 1$, and $p_2(1)$ (the power assigned to 1 when 2 is master) is 50. So $A(1,2) = 1 \cdot 50 = 50$.

4. Analytical approach to fixed and uniform distributed powers

In order to get some preliminary intuition, we address the constant power case, meaning that all nodes transmit with the same transmit power p. Here, the matrix A as defined in (3) equals $A = (n-1)pI_n + pE_n$, where I_n denotes the identity matrix and E_n the all-one matrix.

Theorem 1. *Let $G = (V, E, c, b, a)$ be given, with E the complete graph, $a_v = 1$ for all $v \in V$, and $n \geq 2$. Then the network lifetime, expressed in rounds, for algorithm DIR is:*

$$
L(G) = \min_{i=1,\ldots,n} \{b_i/p\}, \tag{5}
$$

for the algorithm OPT we obtain the following network lifetime:

$$
L(G) = \min_{v \in V} \{\frac{b_v}{p}, \frac{\sum_{v \in V} b_v}{p(2n-1)}\} \tag{6}
$$

PROOF. Statement (5) is immediate. To see (6), we may assume w.l.o.g. $V = \{1, \ldots, n\}$, $p = 1$ and $b_1 \leq \ldots \leq b_n$. By exploiting the well-known LP-duality (see e.g. [21], page 62), $\max\{1^T x | Ax \leq b, x \geq 0\} = \min\{y^T b, yA \geq 1, y \geq 0\}$, where y^T denotes the transpose of the dual variable vector, and 1 denotes the all-one vector. Considering $y = (2n-1)^{-1}1^T$, it follows that $\sum x_i \leq (2n-1)^{-1} \sum_{v \in V} b_v$. To see the other upper bound, consider $y = [1, 0, \ldots, 0]$, which

implies that $nx_1 + \sum_{i=2}^n x_i \le b_1$, whence also $\sum_{v \in V} x_v \le b_1$. To see that the upper bounds are attainable, first assume $b_1 \ge \sum_{i=1}^n b_i/(2n-1)$. Next consider x as given by $x_i = (b_i - \frac{\sum b_i}{2n-1})/(n-1)$. By assumption x is feasible. Moreover: $\sum x_i = \sum b_i/(2n-1)$ by simple substitution. To see that the lower bound b_1 is attainable, assume (2) does not hold, so $b_1 < \sum_{i=1}^n b_i/(2n-1)$. Choose $x_1 = 0$, and repeat this procedure until we are back in the situation under (a). With the corresponding assignment also the lifetime b_1 is realized $\qquad\square$

Remark 1. Equation (6) can be interpreted as: when the battery capacities are 'equally' distributed over the network (so the smallest battery capacity is not 'too small' compared to the others), the optimal lifetime is determined by fact that the total battery capacity of the network after one round is reduced by $(2n-1)p$. This can be seen as follows: after one round, all $n-1$ 'slave' nodes have transmitted once, reducing the total power with $(n-1)p$, all these transmissions have been relayed by the master which has performed $n-1$ relaying (rebroadcasting) actions, reducing the power further with $(n-1)p$. Furthermore, the master initiated a single broadcast as source. So the total power in the network is reduced by $p(2n-1)$ at each round. So the number of rounds cannot be more than $\sum_{v \in V} b_v/p(2n-1)$. When there are one or more nodes with small battery capacity, the upper bound cannot be reached, due to the fact that even when other nodes are master, they are unable to act as 'slave'. The collowing corollary is immediate.

Corollary 1. *Let $G = (V, c, b)$ be given, with $n \ge 2$ and $b_v \in [1/2, 1]$, for $v \in V$. Then network lifetime for algorithm OPT equals $(p(2n-1))^{-1} \sum_{v \in V} b_v$.*

In many practical situations, the free-space conditions are not satisfied and there is no simple power law possible (see e.g. [22], [23]). For that reason we first consider the uniform model, which - in its purest form - assigns a transmit power threshold $c(u, v)$ to each of the $\binom{n}{2}$ pairs $\{u, v\}$, with $c(u, v)$ uniformly distributed over $[0, 1]$, and with $c(v, u) = c(u, v)$. We also write B_v for the random battery capacities and assume $B_v \cong U[\alpha, 1]$, for some $\alpha \ge 0$, where U

denotes the uniform distribution. As above, for the transmission frequencies we assume $a_v = 1$ for all $v \in V$. As an approximation to this 'pure' model, where a single edge determines the transmit power threshold for two vertices, we analyze an 'independent model' in which, for each node v, the transmit power thresholds $U_1, \ldots, U_{n-1}$ for its $n-1$ neighbors are randomly generated, independent from the other nodes. We analyze the OPT and DIR algorithms, starting with the latter:

Theorem 2. *Under the independence assumption, with all-one battery capacities $B = 1$, the expected lifetime L for a network using algorithm DIR is:*

$$E[L] = \frac{n(n-1)}{n(n-1) - 1} \tag{7}$$

PROOF. Under the independence assumption, with $B_v = 1$, for all $v \in V$, the power X_v with which node v transmits the broadcast messages is determined by $X_v = \max_i \{U_i\}$ with $U_i \cong U[0, 1]$ $(i = 1, \ldots, n-1)$ (all nodes must receive the message via a direct transmission). The lifetime of a node $v \in V$, L_v is then determined by: $L_v = 1/X_v$, and the lifetime of the network, L, is determined by: $L = \min_{v \in V}\{L_v\}$. Again by the independence assumption, we obtain the distribution function of L: $F_L(y) = 1 - P(L_1 \geq y)^n$, where the 1 indicates vertex 1 in the graph (arbitrary but fixed). We obtain: $P(L_1 \geq y) = P(X_1 \leq 1/y)$. This yields the following expressions for $P(L_1 \geq y)$ for the various regimes:

$$\mathrm{P}(L_1 \geq y) = \begin{cases} 1 & \text{for } y \leq 1, \\ \frac{1}{y^{n-1}} & \text{for } y \geq 1 \end{cases}$$

As $F_L(y) = 1 - P(L_1 \geq y)^n$, we have $F_L(y) = 1 - y^{-n(n-1)}$, for $y \geq 1$, and $F_L(y) = 0$, for $y \leq 1$. So we can calculate $f_L(y) = n(n-1)y^{-n(n-1)-1}$, and $E[L] = \int_1^\infty y f_L(y) dy$, which yields (7). $\square$

Note that $E[L] = \frac{n(n-1)}{n(n-1)-1}$, quickly tends to 1 for $n \to \infty$. This is intuitively clear, as for a large number of nodes the maximum weight is tending to 1 (from below) and we are back in the constant power case (see (5). The next theorem provides a more general view on the expected network lifetime, addressing the

case where the battery levels are uniformly selected from the interval $[\alpha, 1]$ with $0 \leq \alpha < 1$.

Theorem 3. *Let $0 \leq \alpha < 1$, under the independence assumption, with $B \cong U[\alpha, 1]$, the expected lifetime L for a network with algorithm DIR is:*

$$E[L] = f_1(n, \alpha) - f_2(n, \alpha) \tag{8}$$

with

$$f_1(n, \alpha) = \frac{n(n-1)(\frac{1-\alpha^n}{(1-\alpha)n})^n}{n(n-1)-1} = \frac{n(n-1)}{n(n-1)-1} \left(\frac{\sum_{j=0}^{n-1} \alpha^j}{n} \right)^n \tag{9}$$

and

$$f_2(n, \alpha) = \frac{n \int_\alpha^1 y \left(\frac{n-(n-1)y-a^n y^{1-n}}{n-\alpha-(n-1)\alpha} \right)^{n-1} (1 - n + \alpha^n (n-1) y^{-n}) \, dy}{n - \alpha - \alpha(n-1)}. \tag{10}$$

Remark 2. Note that for $\alpha = 0$ the expression (8) reduces to

$$E[L] = \frac{n(n^{-n} + n^2 - n - 1)}{(n^2 - 1)(n^2 - n - 1)},$$

which quickly tends to 0, corresponding to the intuition that in such networks, it is highly likely that a node starts with an almost empty battery, determining the network lifetime.

For $\alpha \uparrow 1$, expression (8) tends to (7). To see this: note first that $f_1(n, \alpha)$ tends to $\frac{n(n-1)}{n(n-1)-1}$ for $\alpha \uparrow 1$ (note that $\left(\frac{\sum_{j=0}^{n-1} \alpha^j}{n} \right)$ tends to 1 for $\alpha \uparrow 1$). Second, we show that $f_2(n, \alpha)$ tends to 0 when $\alpha \uparrow 1$. To see this, define $\beta = 1 - \alpha$, and let $\beta \uparrow 0$. The denominator then reduces to β. The numerator then becomes $\int_{1-b}^1 g(y) \mathrm{d}y$ where $g(y) = y \left(\frac{n+y-ny-(1-\beta)^n y^{1-n}}{\beta n} \right)^{n-1} (n-1)(1 - (1-\beta)^n y^{-n})$. Series expansion learns that the latter part $(n-1)(1 - (1-\beta)^n y^{-n})$ behaves as $n(n-1)\beta$ for small β. The fraction $\left(\frac{n+y-ny-(1-\beta)^n y^{1-n}}{\beta n} \right)^{n-1}$ tends to 1 for $y \uparrow 1$ and $\beta \downarrow 0$. So, for $\beta \downarrow 0$ the expression $\int_{1-\beta}^1 g(y) \mathrm{d}y / \beta$ behaves as $n(n-1) \int_{1-b}^1 y \mathrm{d}y$ which clearly tends to 0.

PROOF (OF THEOREM 3). Under the independence assumption, the power X_v with which node v transmits the broadcast messages is determined by $X_v = \max_i \{U_i\}$ with $U_i \cong U[0, 1]$ $(i = 1, \ldots, n-1)$. The lifetime of a node $v \in V$,

L_v is then determined by: $L_v = B_v/X_v$, and the lifetime of the network, L, is determined by: $L = \min_{v \in V}\{L_v\}$. So, assuming independent nodes, we obtain the distribution function of L: $F_L(y) = 1 - P(L_1 \geq y)^n$, where the 1 indicates vertex 1 in the graph (arbitrary but fixed). Conditioning for $B = b \in [\alpha, 1]$ we obtain:

$$P(L_1 \geq y | B = b) = P(X_1 \leq b/y) = \begin{cases} 1 & \text{for } y \leq b, \\ \frac{b^{n-1}}{y^{n-1}} & \text{for } y \leq b \end{cases}$$

Deconditioning on B yields expressions for $P(L_1 \geq y)$ for the various regimes:

$$P(L_1 \geq y) = \begin{cases} 1 & \text{for } y \leq \alpha, \\ \int_\alpha^y \frac{b^{n-1}}{y^{n-1}} \frac{1}{1-\alpha} db + \int_y^1 \frac{1}{1-\alpha} db = \frac{y - y^{1-n}\alpha^n}{(1-\alpha)n} + \frac{1-y}{1-\alpha} & \text{for } \alpha < y < 1, \\ \int_\alpha^1 \frac{b^{n-1}}{y^{n-1}} \frac{1}{1-\alpha} db = \frac{y^{1-n}(1-\alpha^n)}{(1-\alpha)n} & \text{for } y \geq 1 \end{cases}$$

As $F_L(y) = 1 - P(L_1 \geq y)^n$, we have $F_L(y)$ and we can calculate $f_L(y)$, and $E[L] = \int_1^\infty y f_L(y) dy$. This leads to (8) $\qquad \square$

Theorem 4. *Let $0 \leq \alpha < 1$, under the independence assumption, with $B \cong [\alpha, 1]$, an upper bound for the lifetime L' for the network lifetime L using the algorithm OPT is:*

$$L' = \frac{B}{nZ + W}, \tag{11}$$

where Z, W, and B are random variables $Z = \max\{U_1, \ldots, U_{n-1}\}$ ($n \geq 3$), with $U_i \cong [0, 1]$, $W = U_1 + \cdots + U_{n-1}$, $B = \sum_{v \in V} B_v$ with $B_v \cong U[\alpha, 1]$. In addition,

$$\frac{B/n}{nZ/(n-1) + W/(n-1)} \xrightarrow{\mu} \frac{\alpha + 1}{3}, \tag{12}$$

*where $\xrightarrow{\mu}$ denotes convergence in mean.*

PROOF. Assume $B_v \cong U[\alpha, 1]$, $v \in V$. Equation (3) determines the matrix A where the diagonal entries are determined by $n \max\{U_1, \ldots, U_{n-1}\}$, and all remaining entries are uniformly $[0, 1]$ distributed. Adding all rows then leads to the upper bound for the lifetime $L = \sum_{v \in V} x_v$ as indicated by (11). The second part follows from the fact (see [24] Theorem 1.3.6, page 11), that if V_n and W_n are sequences of r.v.'s, where $V_n \xrightarrow{p} c$ and $W_n \xrightarrow{p} d$, and $|W_n| \leq |Y|$

with probability 1 (all n), and $E|Y| < \infty$, then the r.v. $V_n/W_n \xrightarrow{\mu} c/d$. In our case $B_n/n \xrightarrow{p} (\alpha+1)/2$ and $nZ/(n-1) + W/(n-1) \xrightarrow{p} 1 + 1/2$. Moreover, $W_n \leq W_2$ with probability 1 (so the role of Y is played by W_2, and $E|W_2|$ is finite). $\hfill\square$

Let us further analyze the upper bound $E[L']$, with $L' = B/(nZ+W)$. This upper bound is due to the fact that the total power in the network determines the network lifetime. An exact value of L' can, in principle, be calculated by conditioning on $B = b$ and $W = w$, and then calculating $P(L \leq x | B = b, W = w)$ for the regimes $w \geq b/x$, $w \leq b/x - n$ and otherwise. Then we would have to decondition first on W and then on B. This leads to a complicated expression which yields no additional insight. From Theorem 4, we obtain: $E[L'] \approx n(\alpha + 1)/3(n-1)$. With MATLAB [25], we numerically evaluated the actual network lifetimes, obtained by using OPT and DIR and the approximations of Theorem 3 and 4 according to the independent model for $\alpha \in \{0, 1/3, 2/3, 1\}$. In Figure 2 we numerically evaluate the quality of this approximation, as well as the accuracy of the independent model, by comparing the results of simulations of the uniform model with the theoretical results. The figures show that the independent model for DIR provides a very good approximation of the network lifetime. Clearly OPT converges to $(\alpha+1)/3$, as explained in Theorem 4. Figure 2 (a) shows that for $b \in [\alpha, 1]$, with $\alpha = 0$, the linear approximation of OPT yields an overestimation. This is conform the remark under Theorem 1: when there is one node with small battery capacity, it determines the lifetime instead of the total power in the network. Clearly, with $\alpha = 0$ OPT yields a better lifetime than DIR: under OPT nodes with small remaining battery capacity can transmit to a 'nearby' master instead of having to transmit to all nodes. (b) For $\alpha = 1/3$ the simulated (dependent) model and the independent approximation for OPT are surprisingly well in line. The difference for small values of n can be explained by the independence assumption for the calculation, whereas in reality there is a dependency as a single edge determines the weight for two nodes. For such a small value of α, still OPT yields a better lifetime than DIR.
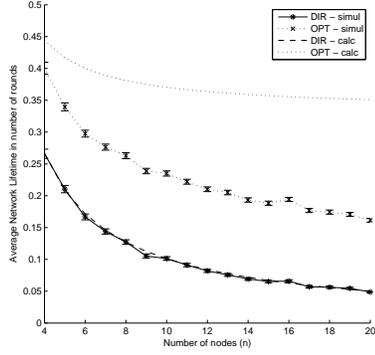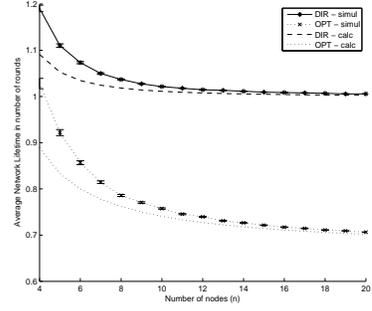
(a) $\alpha = 0$
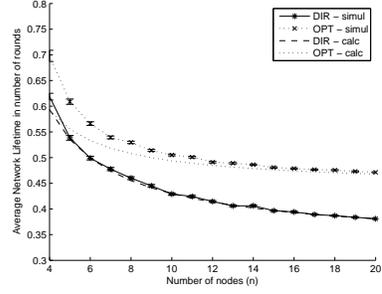
(b) $\alpha = 1/3$
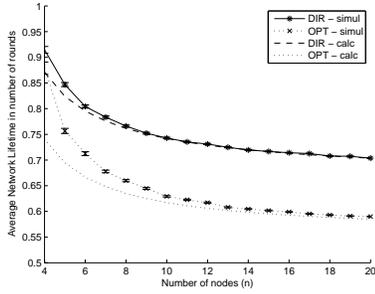
(c) $\alpha = 2/3$

(d) $\alpha = 1$

Figure 2: Comparing DIR and OPT for graphs with $\binom{n}{2}$ uniformly $[0, 1]$ distributed transmit power thresholds with uniform $[\alpha, 1]$ battery capacities, to the independent model approximations (8) and (12) for various values of α. We evaluate the network lifetime in number of rounds for n, ranging from 4 to 20. For each algorithm, the average network lifetime was calculated over 1000 simulations. Confidence intervals are calculated as one standard deviation.

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(c) For $\alpha = 2/3$ this changes and DIR yields a better lifetime than OPT. Under (d) the lifetime for DIR clearly tends to 1. This shows that for (approximately) equal battery capacities direct routing leads to longer lifetimes than dynamic master selection.

5. The geometrical case: continuous power and discrete power levels

We are interested in the impact of the geometrical setting for the network lifetime problem with dynamic relay node selection. Following Section 2, we assume the graph $G = (V, E, c, b, a)$, with $a_v = 1$ for all $v \in V$ and E a complete graph embedded in $\mathbb{R}^2$ with for every pair $c(u, v) = \|u - v\|^2$. In addition to OPT and DIR we also consider *Central Master Selection (CEN)* and *Maximum Battery Master Selection (BAT)*, all providing feasible solutions $x = (x_1, \ldots, x_n) \in \mathbb{R}^n_+$, while being computationally much simpler, as will be clear from the following description, hence more easily implementable in practice.

The algorithm BAT is easiest to explain. Under BAT we select a master node m in such a way that it has maximum battery capacity: $b_m = \max_{v \in V}\{b_v\}$. We choose this node m to be master for the next Δt rounds. After this, we re-evaluate the master choice, that is, we choose a new master $b_{m'} = \max_{v \in V}\{b_{v'}\}$, where the accent denotes the battery capacities at the time of re-evaluation. We repeat this process periodically at each Δt rounds. In the simulation we choose $\Delta t = 0.1$ (we deal with non integral number of rounds by pro rata reducing the battery capacity of each node, compared to one round).

The algorithm CEN follows a similar pattern. In order to describe CEN, we refer back to (2). From this expression, for each of the nodes $m \in V$, we can derive the network lifetime L_m when $m \in V$ would be chosen as a fixed master *during the whole lifetime of the network*:

$$L_m = \min_{v \in V}\{\frac{b_m}{\lambda p_m(m) \sum_{v \in V} a_v}, \frac{b_v}{\lambda a_v p_v(m)}\} \tag{13}$$

Under the condition of a fixed master, the optimal master choice is then the m^*

that maximizes L_m and the network lifetime becomes

$$L = L_{m^*} = \max_{m \in V}\{L_m\}. \tag{14}$$

Similar to BAT, in the algorithm CEN we repeat the process of selecting a master m^* according to (14) periodically at each Δt rounds. In the simulation we choose $\Delta t = 0.1$ (we deal with non integral number of rounds by pro rata reducing the battery capacity of each node, compared to one round).

In this geometric case an analysis as in the previous section turns out to be highly involved. Therefore, we used MATLAB [25] simulations to compare the performance of various algorithms. The network lifetime in number of rounds was evaluated for n, ranging from 4 to 20. In order to avoid corner effects, the nodes were uniformly distributed in a two dimensional disk of unit diameter (circle centered at 0, with radius 1/2). The maximum transmit power assigned to each node is one, enough to cover the complete circle, but the actual transmit powers assigned to nodes were just enough to reach the desired neighbor (e.g. master) according to $p(u,v) = c(u,v) = \|u - v\|^2$. For each algorithm, the average network lifetime was evaluated over 1000 simulations (so 1000 different topologies). Confidence intervals were calculated as one standard deviation.

To investigate the improvement of dynamic master selection as opposed to static master selection, we compare the ratio of lifetime for the algorithm to the lifetime of the optimal static algorithm (as in [19]), see Figure 3(a). Two cases are displayed: all-one battery capacities: $b_v = 1$ for all $v \in V$, and $b_v \cong U[0,1], v \in V$. In all cases $a_v = 1$ for all $v \in V$. From the simulations, we infer the following: (1) dynamic master selection extends the lifetime significantly compared to static master selection; (2) in order of decreasing lifetime the algorithms are : OPT, CEN, BAT and DIR. OPT and CEN are close, and we expect that CEN and OPT are equal when considering infinitesimal values of Δt; (3), as already discussed in the previous section, the performance of OPT vs DIR depends strongly on the initial battery capacities. For uniform [0,1] battery capacities, OPT is about 3 times better than DIR (for 15 nodes or more). For the all-one battery capacities -where the total amount of energy in the network
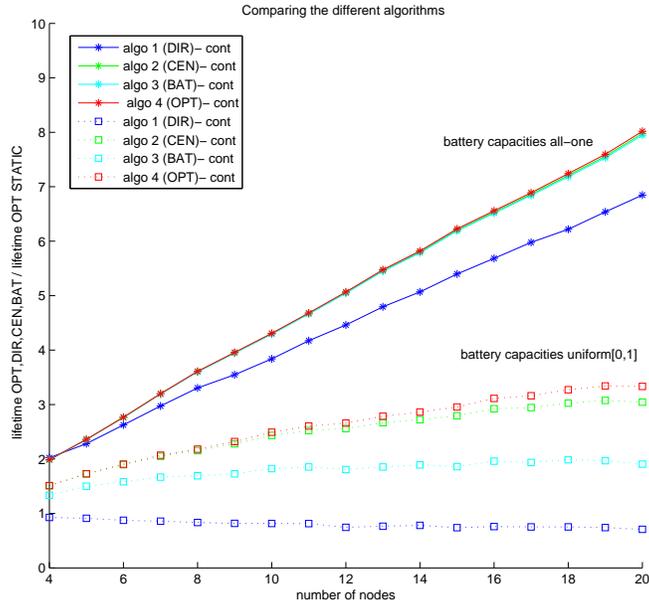
(a) Simulation results for the continuous power case with battery capacities all-one and uniformly distributed.
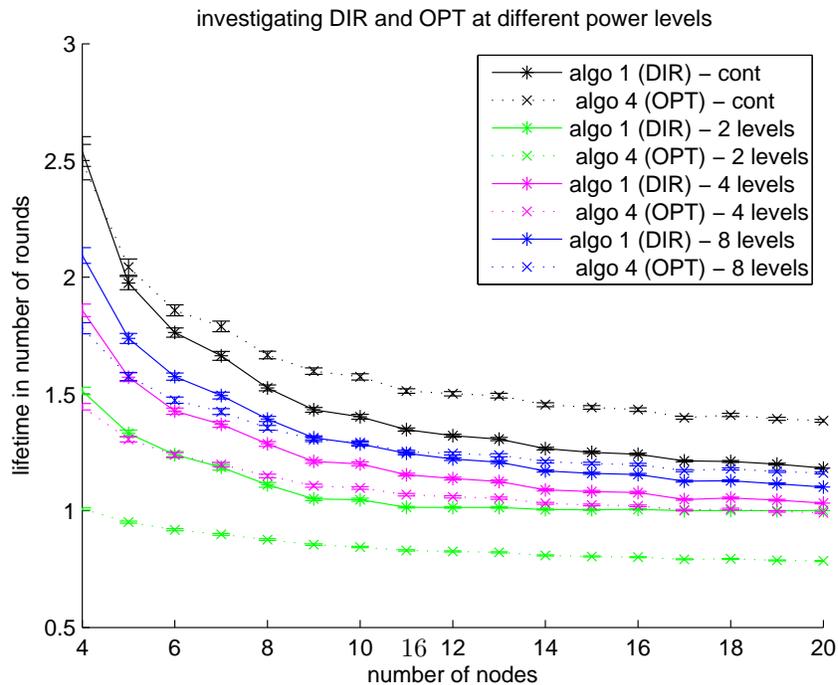
(b) Comparing DIR and OPT for continuous and 2 and 8 discrete power case with all-one battery capacities.

Figure 3: Simulation results for dynamic master selection.

is, on average, doubled- this factor amounts to at least 6. Surprisingly, this differs from the result Section 3, where OPT only outperformed DIR in case of uniform [0,1] distributed battery capacities. This is caused by the fact that all nodes are in a disk of unit diameter. With increasing n the set of nodes that is master for a 'long' time, is closer to the center. This way, a master needs lower transmit power than an arbitrary node (under direct routing). This is clearly different from the uniform distribution where the master needs the same power as an arbitrary node (which tends to 1 for $n \to \infty$). The difference is further strengthened by the quadratic power law. For the case of uniform [0,1] battery capacities even *static* master selection is better for the network lifetime than direct routing (shown by the blue squared dotted line dropping below one for increasing number of nodes). In this case OPT,CEN and BAT are very close.

As the dynamic master selection is a highly specific case of ad-hoc multihop routing, this indicates that multihop routing functionality is beneficial for the network lifetime, provided the transmit power levels are continuously adjustable.

In practice, often only a discrete set of transmit power levels is supported in hardware and software. Theorem 1 can be interpreted as an extreme case, where only power level is supported. In this case DIR outperforms OPT, due to the fact that OPT reduces the battery by a constant at each transmission for (at least) 2 nodes. In Figure 3(b) we investigate how many power levels need to be supported before OPT outperforms DIR, as in the continuous power case. Simulations with $U[0,1]$-distributed battery capacities (not displayed) show OPT outperforms DIR already for 2 power levels. For each algorithm, the average network lifetime was evaluated over 100 different topologies. Again confidence intervals of 1 standard deviation were calculated. However, Figure 3(b) shows that, with all-one battery capacities, 2 power levels is not enough. For 8 power levels OPT outperforms DIR for 10 nodes or more. However, with 4 or less power levels, DIR outperforms OPT.

6. Conclusions and future work

We conclude that the case with uniformly distributed power levels and the geometrically case behave fundamentally differently. Both cases agree in the effect that the network lifetime of OPT relative to DIR increases when the initial battery levels become more unequally distributed. In that case OPT outperforms DIR. However, with equal battery capacities DIR outperforms OPT in the uniform case, whereas OPT still outperforms DIR in the geometrical setting. Hence, in most cases, dynamic master selection algorithms extend the network lifetime significantly compared to static master selection. In order of decreasing lifetime the algorithms are : OPT, CEN, BAT and DIR, where CEN and BAT are computationally simpler algorithms. For *discrete power levels*, dynamic master selection can only improve upon direct routing, when there are at least two power levels. Our results suggest that 8 power levels are sufficient for multihop routing to have longer network lifetime than direct transmission, except for small networks. The (linear programming) technique and model of the uniformly distributed power level case can be re-used to analyze more complicated forms of multihop routing, e.g. involving the optimal selection of sets of relay nodes. These ideas can also be applied in the analysis of clustering algorithms. We leave this for future work.

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