

INITIAL SLOPE OF THE HYSTERESIS CURVE

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Abstract. - An analytical expression for the initial slope T of the hysteresis curve is derived for a stripe domain structure in a thin magnetic film, giving that T^{-1} is proportional to $t^{-1/2}$ (t = film thickness). This is confirmed by measurements on RF sputtered CoCr films with $20 \text{ nm} \leq t \leq 950 \text{ nm}$.

Theory and comparison with experimental results

One of the numerical quantities used to describe the hysteresis curve is its initial slope T (or initial susceptibility). Wielinga *et al.* [1] used this parameter as a function of the film thickness t for RF sputtered CoCr. In this article an analytical expression is derived that is very convenient for analysing experimental data because we found a linear relationship between T^{-1} and $t^{-1/2}$.

The energy of a stripe domain structure in a thin magnetic film was calculated by Kooy and Enz [2]. The total free energy per unit area F_T of the film is the sum of the energy of the magnetization in the external field, the wall energy and the demagnetization energy and is given by:

$$F_T = -\mu_0 t H M_s \varepsilon + \frac{2t\sigma_w}{P} + \frac{1}{2}\mu_0 t M_s^2 \varepsilon^2 + \frac{2\mu_0 M_s^2 P}{\pi^3} \sum_{k=1}^{\infty} \frac{1}{k^3} \sin^2 \left(\frac{\pi k}{2} (1 + \varepsilon) \right) \times \left[1 - \exp \left(\frac{-2\pi k t}{P} \right) \right] \quad (1)$$

where $\varepsilon = M/M_s$, P the stripe domain period, σ_w the domain wall energy density, M the magnetization and M_s its saturation value. In order to calculate the initial slope an approximation of the Kooy and Enz model for low magnetization is calculated, i.e. $\varepsilon \ll 1$. Furthermore it is assumed that $t/P > 0.5$ in order to obtain a 4 % accuracy in the approximation of the series summation. By expanding the series summation into powers of ε^2 and dividing the energy by $\mu_0 M_s^2$ it can be shown that

$$f_T = -t h \varepsilon + \frac{2t\lambda}{P} + \frac{t\varepsilon^2}{2} + \frac{2P}{\pi^3} \left[\frac{7}{8} \zeta(3) - \left(\frac{\pi\varepsilon}{2} \right)^2 \ln 2 \right] + O(\varepsilon^4) \quad (2)$$

where $f_T = F_T / \mu_0 M_s^2$, $h = H/M_s$ and $\lambda = \sigma_w / \mu_0 M_s^2$ (the characteristic material length). Applying the equilibrium condition $\frac{\partial f_T}{\partial \varepsilon} = 0$ we obtain

$$\varepsilon = h \cdot \left(1 - \frac{\ln 2P}{\pi t} \right)^{-1} \quad (3)$$

Using the equilibrium condition $\frac{\partial f_T}{\partial P} = 0$ we find

$$\frac{\partial f_T}{\partial P} = \frac{2}{\pi^3} \left(\frac{7}{8} \zeta(3) - \left(\frac{\pi\varepsilon}{2} \right)^2 \ln 2 \right) - \frac{2t\lambda}{P^2} = 0 \quad (4)$$

and when ε tends to zero the period P becomes

$$P = \sqrt{\frac{8\pi^3 \lambda t}{\tau \zeta(3)}} \quad (5)$$

Substituting (5) into (3) we obtain the initial slope

$$T = \left(\frac{\partial M}{\partial H} \right)_{M=0} \text{ and hence} \\ T^{-1} = 1 - \ln 2 \left(\frac{8\pi}{7\zeta(3)} \right)^{1/2} \left(\frac{\lambda}{t} \right)^{1/2} = 1 - 1.20 \left(\frac{\lambda}{t} \right)^{1/2} \quad (6)$$

From (6) we can conclude that if T^{-1} is plotted against $t^{-1/2}$ a straight line is found, which has a slope $-1.20\sqrt{\lambda}$.

Figure 1 shows the initial slope of the volume (T_v) and surface (T_s) hysteresis curves of RF sputtered CoCr single layers measured using respectively, a VSM and a Kerr tracer. The films described here those of [3] which were prepared with an argon pressure $P_{Ar} = 0.4 \times 10^{-2}$ mbar, radio frequency voltage $V_{RF} = -1.6$ kV. These films can be classified as having a high coer-

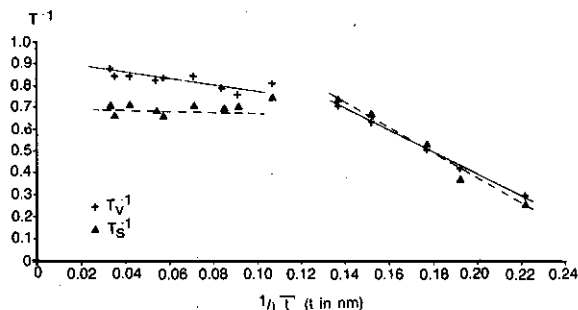


Fig. 1. - Experimental values of the inverse of the initial slope of surface (T_s^{-1}) and volume (T_v^{-1}) hysteresis curves versus the inverse of the square root of the thickness $t^{-1/2}$.

civity [4] since $H_{c\perp}/H_k > 9\%$ ($H_k = 2K_1/\mu_0 M_s$, where K_1 is the first order anisotropy constant) for the samples thicker than 30 nm. From the figure it can be seen that for $t \leq 80$ nm the slope of the volume hysteresis curve is much steeper than for $t \geq 80$ nm. This coincides (accidentally?) with the point where maximum value of the coercivity is measured. For the thinner films T_s and T_v are very similar, while for thicker films this is not the case. In [3, 5, 6] there are indications for the possible presence of small reversed domains at the surface, known as spikes, (proposed by Privorotskii [7]). At a certain depth inside the thin film each domain begins to split. The width of the new domains increases with decreasing distance to the surface until it becomes equal to $P/3$. Then a new splitting occurs. This process continues until the dimensions of the new domains become comparable to the thickness of the domain wall. Therefore spikes would occur especially in thicker films and maybe therefore also the surface and volume coercivity would differ [3]. To allow for spikes which diminish the domain period at the surface a factor r is introduced so that

$$P_n = P/3^n = Pr \quad (7)$$

where P_n is the new period formed after the n^{th} splitting. Equation (2) can be roughly modified to include this effect by multiplying the term within square brackets by r . Using the same procedure as above we obtain the following:

$$P = \{8\pi^3 \lambda t / 7\zeta(3) r\}^{1/2} \quad (8)$$

$$T^{-1} = 1 - \ln 2 \left(\frac{8\pi}{7\zeta(3)} \right)^{1/2} \cdot \left(\frac{\lambda r}{t} \right)^{1/2} = \\ = 1 - 1.20 \left(\frac{\lambda r}{t} \right)^{1/2} \quad (9)$$

If $r = 1$ equations (8) and (9) coincide with (5) and (6) respectively. Applying equation (6) to the volume initial slope for $t \leq 80$ nm we find $\lambda = 17.4$ nm. This corresponds to $\sigma_w = 4.6 \times 10^{-3}$ J/m². If $K_1 = 90$ kJ/m³ [8] then the exchange constant $A = 1.5 \times 10^{-11}$ J/m ($\sigma_w = 4\sqrt{AK_1}$). Using spin wave resonance Cofield *et al.* [9] found $A = 1.0 \times 10^{-11}$ J/m for RF sputtered CoCr with an in plane anisotropy. Wielinga *et al.* [1] determined $\sigma_w = 1.0 \times 10^{-3}$ J/m² for their films. Other values of σ_w for magnetron sputtered CoCr were determined in [10] using measurements of the domain period for low and medium coercivity films where $\sigma_w = 1.0 \times 10^{-3} - 2.0 \times 10^{-3}$ J/m² and in [4] measuring T and P simultaneously for a low coercivity film where $\sigma_w = 4.2 \times 10^{-3}$ J/m². For the volume slope of the thicker films $r = 0.087$ i.e. $r \simeq (1/3)^2$ which as a power of $1/3$ agrees with [7]. This indicates that 2 splittings have occurred. Whether this is indeed the case remains to be seen however, because our results may

also be influenced by the hysteresis of the material, especially in high coercivity films, and the fact that the material properties are not constant as a function of film thickness. From ferromagnetic resonance measurements in combination with etching of magnetron sputtered CoCr films we find an initial layer which has a thickness of about 55 ± 25 nm. Moreover the μ^* effect has not been included in these calculations.

Conclusions

An analytical approximation (for low values of the magnetization) for the energy of a stripe domain structure which was originally calculated by Kooy and Enz is presented. From this we find that the inverse of the initial slope T^{-1} is proportional to $t^{-1/2}$ the inverse square root of the film thickness. When this approximation is applied to CoCr the graph splits up into two parts at $t \simeq 80$ nm. This happens also to be the point where the maximum of the coercivity is measured. The fact that the coercivity increases with t for $t \leq t_c$ and decreases for $t \geq t_c$ could be due to the presence of spikes in the thicker films. From the slope of the hysteresis curve we find that the number of splittings in the sense of Privorotski's model is equal to two.

Acknowledgments

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