Matrix Factorization Methods in the Theory of Queues

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We discuss several queueing models in which explicit solutions can be obtained by matrix factorization methods. These models fall into three classes:

1. *The single server semi-Markov queue.* The general model was discussed in [1]. It gives the possibility to describe dependencies among interarrival times and service times which in the classical GI/G/1 queue are ignored. Of particular interest is the special case of the Markov-modulated M/G/1 queue.

2. *The multi-server queue with phase-type service times.* The queue GI/PH/s gives rise to a system of Wiener–Hopf type equations which can be solved whenever the symbol of this system can be factorized. Explicit solutions have been found for the case of hyper-exponential service times. The form of waiting time and queue length distributions is found (see [2]) and a numerical algorithm is derived.

3. *Other models.* Several other models in queueing theory can be solved by factorization methods (see [3]), in particular
   (i) a buffer storage process in a Markovian environment;
   (ii) an interleaved memory system, modeled as a queueing system with Markov chain driven input and constant services.

References

[1] J.H.A. de Smit, The single server semi-Markov queue, Stoch. Proc. Appl. 22 (1986) 37–50.

[2] J.H.A. de Smit, The GI/M/s with customers of different types or the queue GI/H_m/s, Adv. Appl. Probab. 15 (1983) 392–419.

[3] G.J.K. Regterschot, Wiener–Hopf factorization techniques in queueing models, Ph.D. Thesis (University of Twente, 1987).

Central-Limit-Theorem Versions of $L = \lambda W$

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Underlying the fundamental queueing formula $L = \lambda W$ (Little's Law) is a relation between cumulative processes in continuous time (the integral of the queue length process) and in discrete time (the sum of the waiting times of successive customers). In addition to the familiar relation between the w.p.1 limits of the averages, there are corresponding relations among the central-limit-theorems (CLTs) [2, 4]. Roughly speaking, the sequence of customer waiting times and interarrival times obey a joint CLT if and only if the continuous-time queue length and arrival counting process obey a joint CLT, in which case all four processes obey a joint CLT and the marginal limits are simply related. Similar results hold for extensions of $L = \lambda W$ such as $H = \lambda G$ [5].