

## CRITERIA FOR SUCCESSFUL SEPARATION BY CONTINUOUS ELECTROPHORESIS AND ELECTROCHROMATOGRAPHY IN BLOCKS AND COLUMNS

E. RAVOO\* AND P. J. GELLINGS

*Afdeling Chemische Technologie, Technische Hogeschool Twente, Enschede (The Netherlands)*

TH. VERMEULEN

*Department of Chemical Engineering, University of California, Berkeley, Calif. (U.S.A.)*

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Continuous electrophoresis and electrochromatography have found analytical as well as preparative application in recent years. The principles and development of these techniques have been reviewed by PUČAR<sup>1</sup>. Most of the work has related to specific biochemical and clinical problems and little attention has been paid to the physics of the method. The object of this paper is to report an analysis of some important process variables and to derive criteria for successful separation. In connection with the theory, some experimental results obtained by VERMEULEN and coworkers at the University of California, Berkeley will be presented.

The principle of continuous electrophoresis may be described as the separation of electrically charged species of different mobility from a continuous feedstream by application of direct current perpendicular to the direction of flow. Here only packed-bed systems will be considered. The feed is admitted continuously from a line-source into a column or rectangular block filled with either an inert or an adsorptive packing or support. This material is bathed continuously and uniformly by a suitable carrier electrolyte or eluant. In the bed (see Fig. 1) the ionic components from the feed will move in zones (bands), vectorially determined by (a) convective transport in the direction of bulk flow, possibly retarded by adsorptive action of the packing, (b) electromigration in the direction of the field.

In steady-state operation each feed component can be collected continuously from the respective position of its zone at the downstream end of the bed.

The process is termed continuous electrophoresis in the absence of adsorption or migration in the adsorbed state. If on the other hand adsorption plays a role, the process is known as continuous electrochromatography. Adsorptive action of the packing need not necessarily have a favourable effect on the separation. However, CAPLAN<sup>2</sup> has demonstrated that the use of ion-exchange resins, particularly in combination with complexing agents, will generally improve the separation.

The major fundamental difficulties in packed-bed continuous electrophoresis and electrochromatography fall into four categories.

(1) Prediction of electrophoretic mobilities in moderately concentrated, multi-

\* 1961-1963, Department of Chemical Engineering, University of California, Berkeley.

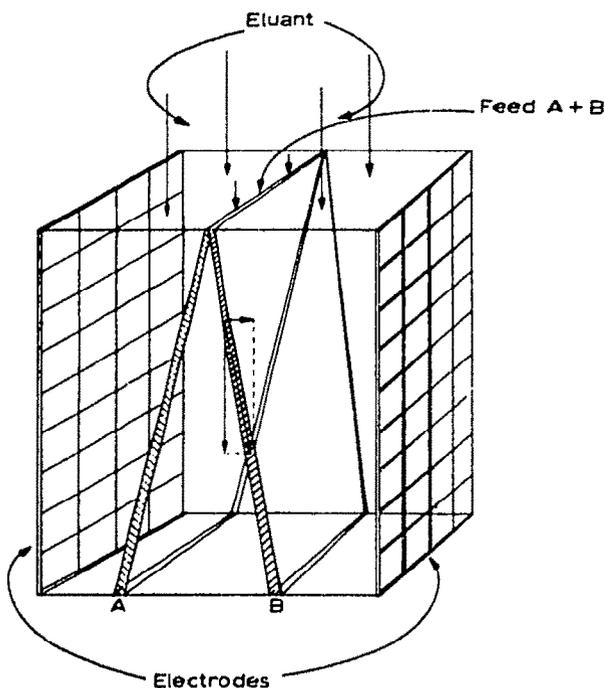


Fig. 1. Schematic representation of continuous electrophoretic separation.

component, temperature-dependent systems. In spite of considerable theoretical progress by various authors, *e.g.* OVERBEEK<sup>3</sup> and more recently WIERSEMA<sup>4</sup>, the practical worker is still confronted with a general lack of data or at best finds a poor collection of semi-empirical correlations at his disposal.

(2) Problems associated with the nature of the supporting medium. Intended to minimize convection due to local density gradients, the supporting medium may introduce serious complications. The packing will generally affect mobilities and other transport properties, and may in some cases promote electroosmotic flow. The complexity of the transport phenomena will be aggravated by the heterogeneity of the bed. An interesting discussion on the role of supporting media in electrophoresis has been written by KUNKEL AND TRAUTMAN<sup>5</sup>.

(3) The rise of temperature due to the passage of current represents a serious limitation in continuous electrophoresis. This is especially important for the separation of heat-labile substances, with high voltages, in apparatus with a high volume to cooling surface ratio. Part of this paper will be devoted to a general theory correlating power input, residence time and temperature rise for rectangular and cylindrical geometries.

(4) The occurrence of transverse dispersion by molecular and eddy-diffusion may impose another limiting factor. One of the following sections will describe how and to what extent the separating capacity can be maximized without undue overlap of the bands.

Along with these fundamental problems many constructional and operational difficulties arise. Since most of these aspects are well covered, *e.g.* by BLOEMENDAL<sup>6</sup>, BIER<sup>7</sup> and FINN<sup>8</sup> (the latter with emphasis on large-scale techniques), further discus-

sion is unnecessary. This paper is restricted to an analysis of heat dissipation and transverse spreading, and a discussion of the implications for the limitations and optimization of continuous electrophoresis.

## ANALYSIS OF HEAT DISSIPATION

The first analysis of temperature distributions in electrophoresis columns was published by PORATH<sup>9</sup>.

His work, however, centered on batch processes in cylindrical beds with the electrodes at the ends of the column. Here a more general theory will be presented for continuous separation in rectangular as well as cylindrical (annular) beds. The symbols used are defined in Table I. In all cases the following assumptions are made:

(a) The bulk process-stream (feed + eluant) moves with uniform velocity  $v_x = L/\tau$  in the longitudinal direction of the bed.

TABLE I

## SYMBOLS AND DEFINITIONS USED

|              |   |
|--------------|---|
| A, B         | Typical feed components   |
| C            | Concentration of feed component (moles/cm <sup>3</sup> )  |
| $c_p$        | Specific heat of process stream (J/g °C)  |
| $D_y$        | Total transverse diffusion coefficient (cm <sup>2</sup> /sec)   |
| $D_m$        | Molecular diffusion coefficient (cm <sup>2</sup> /sec)  |
| $D_e$        | Transverse eddy diffusion coefficient (cm <sup>2</sup> /sec)  |
| $d_p$        | Packing diameter (cm)   |
| E            | Voltage across the bed; $E_y$ , voltage across rectangular bed; $E_{nb}$ , voltage across cylindrical bed (V) |
| L            | Length of the bed (cm)  |
| Pe           | Péclet-number for transverse eddy diffusion   |
| q            | Strength of feed line-source (moles/cm sec)   |
| r            | Radial coordinate; $r_a$ , outer radius of annular bed; $r_b$ , inner radius of annular bed (cm)              |
| $r_F$        | Feed position in annular bed; $r_L$ , take-off position in annular bed (cm)                                   |
| R            | Ratio of lateral dimensions for rectangular bed   |
| Re           | Reynolds number for flow in packed bed  |
| S            | Bandwidth (cm)  |
| T            | Local bed temperature; $T_{max}$ , maximum bed temperature (°C)   |
| $T_0$        | Temperature of coolant and entering process stream (°C)   |
| $\tau_{c,c}$ | Dimensionless residence time in cylindrical bed   |
| $\tau_{c,r}$ | Dimensionless residence time in rectangular bed   |
| u            | Effective electrophoretic mobility of feed component in the bed (cm <sup>2</sup> /Vsec)                       |
| $v_x$        | Plug flow velocity in longitudinal bed direction (cm/sec)   |
| x            | Lateral coordinate in rectangular bed (perpendicular to electric field) (cm)                                  |
| X            | Lateral bedwidth in x-direction (cm)  |
| y            | Lateral coordinate in rectangular bed (in direction of electric field) (cm)                                   |
| $y_b$        | Transverse coordinate measured from the centre of the band (cm)   |
| $y_F$        | Feed position in rectangular bed; $y_L$ , take-off position in rectangular bed (cm)                           |
| Y            | Lateral bedwidth in y-direction (cm)  |
| z            | Longitudinal coordinate (cm)  |
| $\epsilon$   | Porosity of the bed   |
| $\Theta$     | Dimensionless temperature rise; $\Theta_{max}$ , maximum dimensionless temperature rise                       |
| $\kappa$     | Effective electrical conductivity of the bed ( $\Omega^{-1} \text{cm}^{-1}$ )                                 |
| $\lambda$    | Effective thermal conductivity of the bed (W/cm °C)   |
| $\rho$       | Density of process stream (g/cm <sup>3</sup> )  |
| $\tau$       | Residence time (sec)  |
| $\Phi_{m,k}$ | Algebraic function, see definitions to eqn. (2)   |

- (b) A constant voltage drop  $E$  in transverse direction is maintained over the entire length of the bed.
- (c) The bed is cooled by keeping two or more of its lateral faces at a constant temperature equal to the inlet temperature of the process-stream  $T_0$ .
- (d) Longitudinal convection of heat is large compared to longitudinal conduction.
- (e) Transverse convection (*e.g.* by electro-osmosis) is negligible.
- (f) Physical properties of the fluid and the packing are constant throughout the bed.
- (g) End effects are neglected.
- (h) The separation proceeds at steady state.

### Rectangular geometry

Three cases will be considered (see Fig. 2): (1) cooling at the electrode-faces

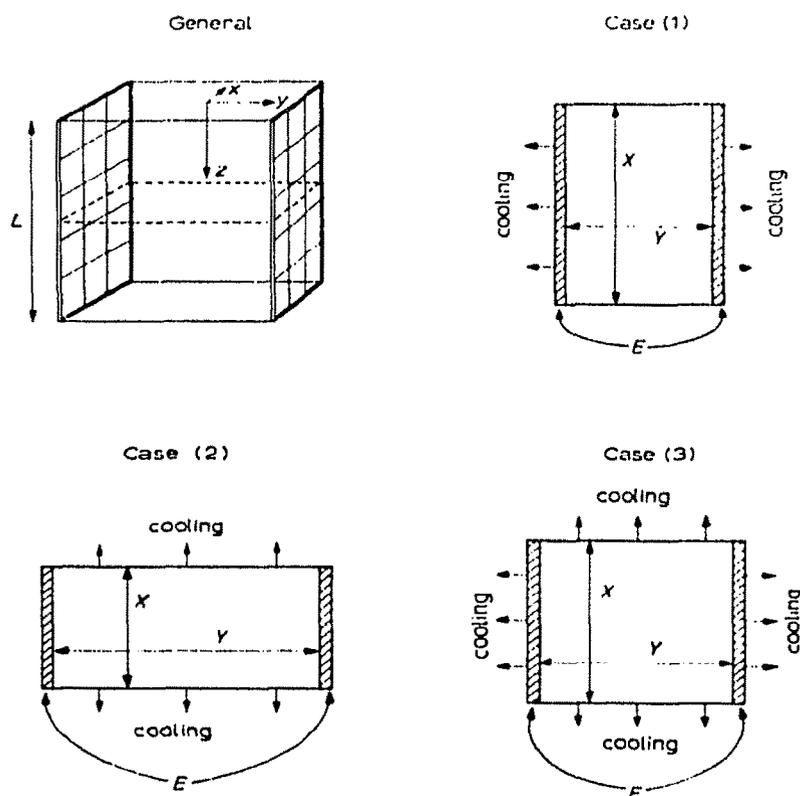


Fig. 2. Rectangular models of electrophoresis units with lateral cooling.

of the bed only; (2) cooling at the two lateral faces perpendicular to the electrode-faces; (3) cooling at all four faces.

The analysis of model (3), although considerably more tedious, is certainly most rewarding. Not only does it generate special solutions for the first two cases as a by-product, it also allows an evaluation of their usefulness for approximation purposes.

Under the assumptions mentioned above, the heat balance for an infinitesimal volume element  $dx \cdot dy \cdot dz$  in the bed takes the form:

$$\lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \rho c_p \frac{\partial T}{\partial \tau} + \kappa \left( \frac{E}{Y} \right)^2 = 0 \quad (1)$$

with the boundary and initial conditions:  $T = T_0$  at  $x = \pm X/2$ , at  $y = \pm Y/2$ , and at  $\tau = 0$ . The respective terms in equ. (1) represent thermal conduction in the two lateral directions, longitudinal convection and ohmic heat generation.

A full analytical solution of the problem is available, from which the maximum temperature—which is of course of most concern—can be derived. In dimensionless form, the following expression for the maximum temperature rise (located at the center of the bed) is obtained:

$$\Theta_{\max} = \frac{16}{\pi^4} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^{k+m}}{(2k+1)(2m+1)} \cdot \frac{1 - \exp(-\pi^2 \Phi_{m,k} \tau_r)}{\Phi_{m,k}} \quad (2)$$

where  $\Theta = \{\lambda(T - T_0)\} / \kappa E^2$ ;  $\tau_r = \lambda \tau / \rho c_p Y^2$ ;  $\Phi_{m,k} = \{(2m+1)/R\}^2 + (2k+1)^2$ ;  $R = X/Y$ .

Calculated values of the dimensionless maximum temperature rise versus dimensionless residence time with varying ratio of the lateral bed dimensions are summarized in Fig. 3.

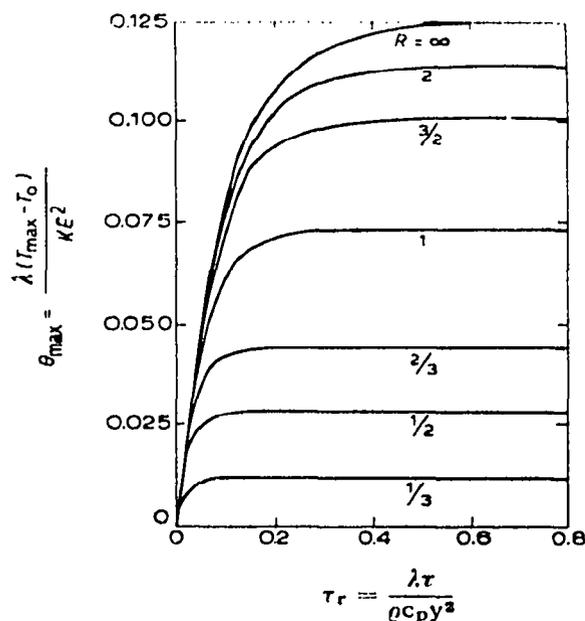


Fig. 3. Maximum temperature in rectangular beds, in terms of (lateral) dimension ratio and residence time (in dimensionless units).

As can be seen from the curves,  $\Theta_{\max}$  rapidly approaches an asymptotic value\*, depending upon the dimension ratio  $R$ . Since in most practical separations the dimensionless residence time  $\tau_r$  will be of the order 0.3 to 30, the asymptotic maxima provide a safe and sufficiently exact guide for successful operation.

Figure 4 depicts the asymptotic temperature rise as a function of  $R$ . It is

\* Physically this means that all ohmic heat is dissipated by conduction in a lateral direction.

interesting to note that the behaviour of beds with cooling at the electrode faces only is reflected by the upper end of the curve, and the case of cooling perpendicular to the direction of current by the lower part. Apparently, the cross-section of the bed need not be very oblong in order to approximate quasi-two-sided cooling.

In that event the following approximate criteria can easily be derived from eqn. (2)

$$R > 4 \quad \Theta_{\max} = \frac{1}{8} \quad \text{or} \quad T_{\max} - T_0 = \frac{\kappa E^2}{\lambda} \frac{1}{8}$$

$$R < \frac{1}{4} \quad \Theta_{\max} = \frac{R^2}{8} \quad \text{or} \quad T_{\max} - T_0 = \frac{\kappa E^2 X^2}{\lambda} \frac{1}{8Y^2}$$

For  $\frac{1}{4} < R < 4$  the use of Fig. 4 is recommended. From these considerations it may be concluded that the best dissipation of heat during continuous separation in rectangular systems can be attained in relatively thin slabs, with narrow electrode

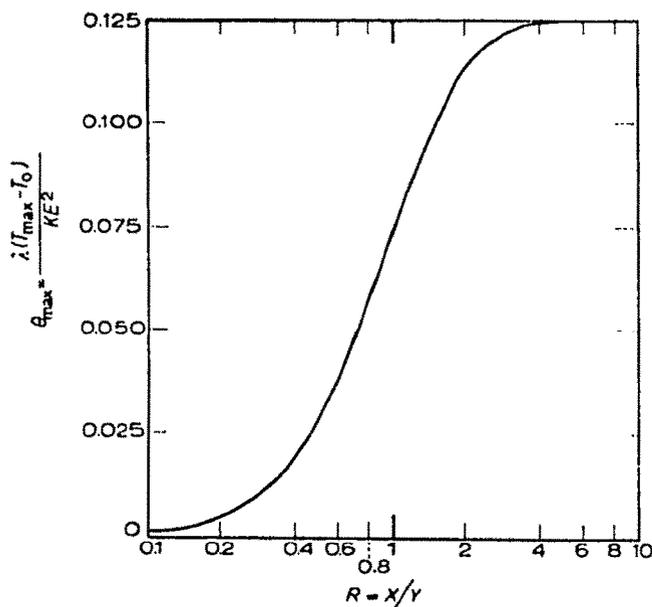


Fig. 4. Asymptotic behaviour of the maximum bed temperature as a function of dimension ratio.

faces and sidewise cooling, perpendicular to the passage of current. A high ratio of thermal to electrical conductivity in the bed is desirable, and the voltage should be kept as low as possible. Even in less favourable geometries  $\Theta_{\max}$  will never exceed  $1/8$ .

#### Cylindrical geometry

A diagram of a cylindrical electrophoresis unit, with electrodes situated at the center and at the periphery of the column, and cooled via the electrode compartments, is shown in Fig. 5.

With the same assumptions as before, the balance between radial conduction, axial convection and ohmic production of heat in an annular element  $2\pi r \cdot dr \cdot dz$  can be formulated:

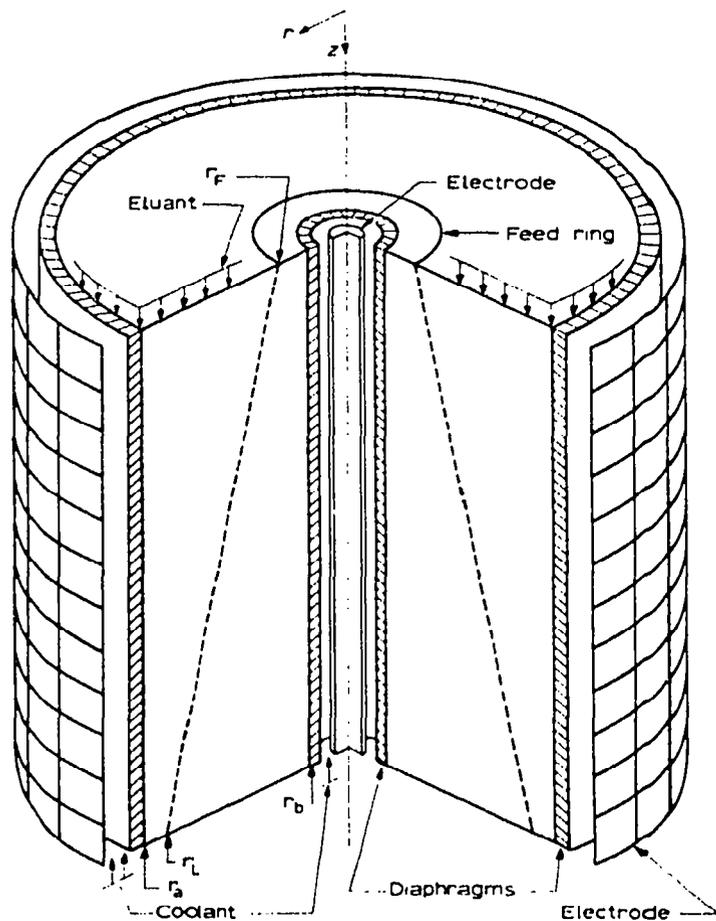


Fig. 5. Exploded view of cylindrical electrophoresis column with annular bed.

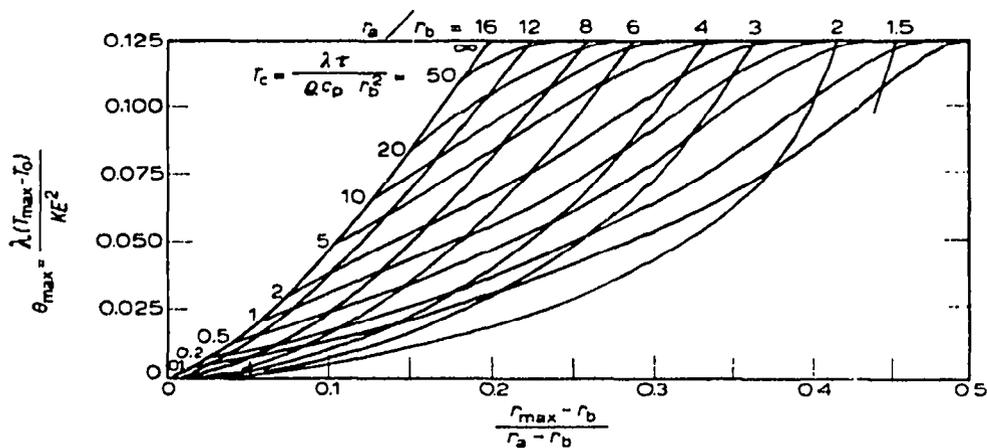


Fig. 6. Maximum temperature and its radial location in annular beds, in terms of radius ratio and residence time (in dimensionless units)

$$\frac{\lambda}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - \rho c_p \frac{\partial T}{\partial \tau} + \kappa \left[ \frac{E}{r \ln(r_a/r_b)} \right]^2 = 0 \quad (3)$$

The boundary and initial conditions are:  $T = T_0$  at  $r = r_a$ , at  $r = r_b$  and at  $\tau = 0$ .

The analytical solution of eqn. (3) has been reported by HYBARGER *et al.*<sup>10</sup> and results in an expression for the temperature rise as a function of voltage, residence

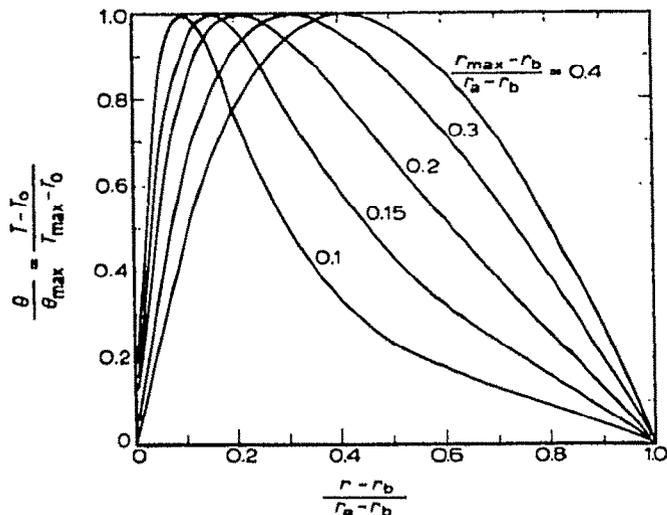


Fig. 7. Temperature distribution in annular beds as a function of maximum temperature location.

time and geometry, in the form of a series of Bessel-functions. The maxima in the temperature profiles, though not accessible analytically, were determined by a combination of graphical and computer techniques by RAVOO *et al.*<sup>11,12</sup>. The results for a wide range of radius ratio  $r_a/r_b$  and dimensionless residence time  $\tau_c = \lambda\tau/\rho c_p r_b^2$ , are summarized in Fig. 6. The graphs show that  $\theta_{\max}$  will approach  $1/8$  for sufficiently long residence times, particularly in thin annuli. Naturally this behaviour is similar

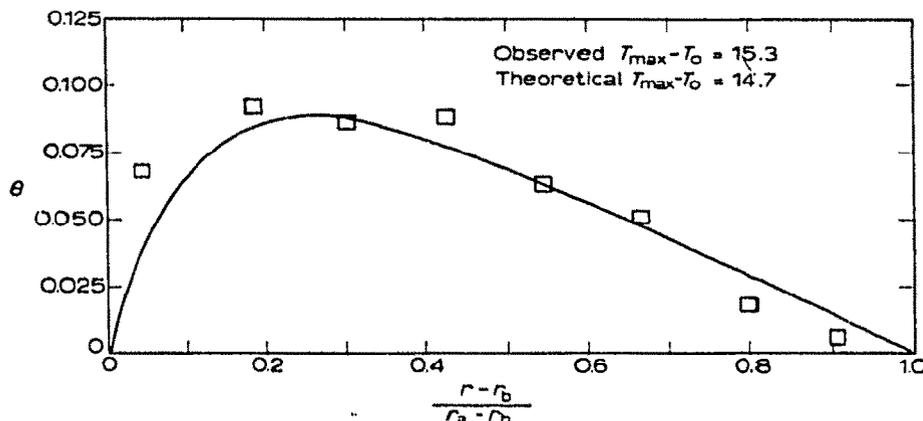


Fig. 8. Comparison of theoretical (solid line) and measured temperature profiles (based on data by NADY *et al.*<sup>13</sup>). Configuration as in Fig. 5;  $r_a = 9.7$  cm,  $r_b = 1.73$  cm,  $r_a/r_b = 5.6$ ; voltage drop  $E_{ab} = 13.7$  V; residence time 2 h; eluant composition 0.05 M  $\text{NaNO}_3$ ; packing 80–150 mesh polystyrene beads.

to that in a rectangular bed cooled at its electrode faces. The asymptotic value of  $\theta_{\max}$  for  $\mathcal{T}_c \rightarrow \infty$  is located at a radius  $r_{\max,as} = \sqrt{r_a r_b}$ . In most cases, however,  $\theta_{\max}$  will remain well below 1/8 at realistic residence times. Starting from known geometry and conditions of separation, the maximum temperature and its location can rapidly be estimated with Fig. 6. If desired, complete temperature profiles can then be constructed using Fig. 7.

A typical profile, based on actual conditions in a cylindrical column in operation at Berkeley, is shown in Fig. 8. The experimental points coincide fairly well with the theoretical curve. The differences may be accounted for by errors in the measurements and deviations from the assumptions made earlier.

#### TRANSVERSE DISPERSION

Spreading of the bands of different components by transverse dispersion represents a problem, since the actual distance over which separation can take place is limited in a practical unit. To a large extent undue overlap of the bands can be prevented by using a suitable combination of packing and separation length, as will be shown in a simplified analysis for rectangular geometry. Since the zones are assumed to be narrow compared to the electrode distance—which is the desirable situation in practice—the same reasoning may be applied to cylindrical columns with few modifications.

If longitudinal mixing is ignored and the packing is homogeneous, with diameters small compared to the bed dimensions, the diffusion of the bands obeys the differential equation

$$D_y \frac{\partial^2 C}{\partial y_b^2} - \frac{v_z}{\varepsilon} \frac{\partial C}{\partial z} = 0 \quad (4)$$

For a line-source of infinitesimal width and strength  $q$  the solution has the Gaussian form

$$C(y_b, z) = \frac{q}{2\sqrt{\pi\varepsilon v_z D_y z}} \exp\left(-\frac{v_z y_b^2}{4\varepsilon D_y z}\right) \quad (5)$$

With the band-width  $S$  defined as the horizontal range containing a specified percentage of the total solute at any depth, it follows that after passage through the bed

$$S^2 = 16 \frac{\varepsilon D_y L}{v_z} \ln \left[ \frac{C_{\max}}{C(S/2, L)} \right] \quad (6)$$

with  $C_{\max} = q/2\sqrt{\pi\varepsilon v_z D_y L}$ .

The relation between total solute recovery and local concentration (Fig. 9) then allows the prediction of band spreading for given conditions at any arbitrarily chosen recovery value, e.g. 98% for  $C_{\max} = 10 C(S/2, L)$ .

Transverse mixing is the result of true molecular diffusion and microscopic fluctuations of the flow around the particles in the bed, "eddy" diffusion\*. Since the eddy diffusivity is related to the Péclet-number for mass transfer,  $D_y$  can be expressed

$$D_y = D_m + D_c = D_m + v_z d_p / \varepsilon \text{ Pé} \quad (7)$$

\* In fact a misnomer, since there may not be any eddies in laminar flow.

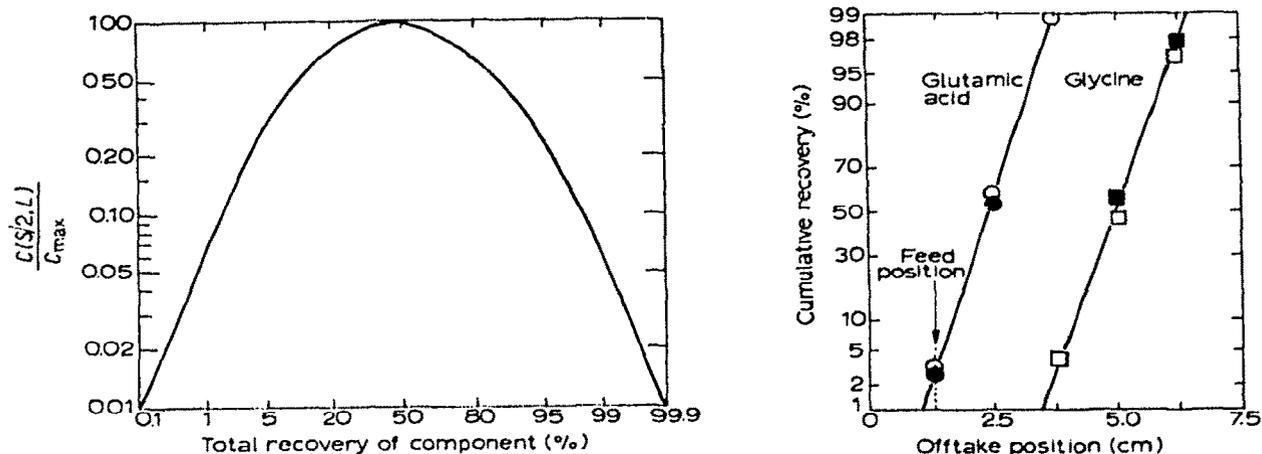


Fig. 9. Gaussian concentration profile in terms of total solute recovery.

Fig. 10. Behaviour of two amino acids in rectangular column, with acids fed separately and combined (data from ref. 13). ( $L = 50$  cm;  $Y = 7.5$  cm;  $\tau = 0.5$ ;  $E_y = 25$  V; packing  $60\text{--}90\ \mu$  glass-beads; eluant  $0.003\ M\ Na_2HPO_4\text{--}0.003\ M\ NaH_2PO_4$ ). Filled in points indicate combined feed.

In the range of interest ( $Re < 10$ ) the Péclet-number is found to be a constant, depending on the nature of the packing, as shown in Table II. From these data it can easily be seen that, even at low flow-rates and with small particle dimensions, the eddy diffusion will usually be predominant. Hence, from eqns. (6) and (7), neglecting  $D_m$ ,

$$S^2 = 16 \frac{d_p L}{Pe} \ln \left[ \frac{C_{\max}}{C(S/2, L)} \right] \quad (8)$$

Thus a simple relationship exists between bandwidth, particle size, bed length and percentage of recovery, which, with Fig. 9 and Table II, allows rapid estimation of suitable separation conditions.

TABLE II

PÉCLET NUMBERS FOR TRANSVERSE EDDY DIFFUSION IN LAMINAR FLOW ( $Re < 10$ )

| $Pe = \frac{v_z d_p}{\epsilon D_0}$ | Packing used                      | Reference |
|-------------------------------------|-----------------------------------|-----------|
| 12                                  | polystyrene pellets               | 14        |
| 12                                  | random packing of mixed spheres   | 15        |
| 43                                  | random packing of uniform spheres | 16        |
| 17                                  | Ottawa sand                       | 14        |

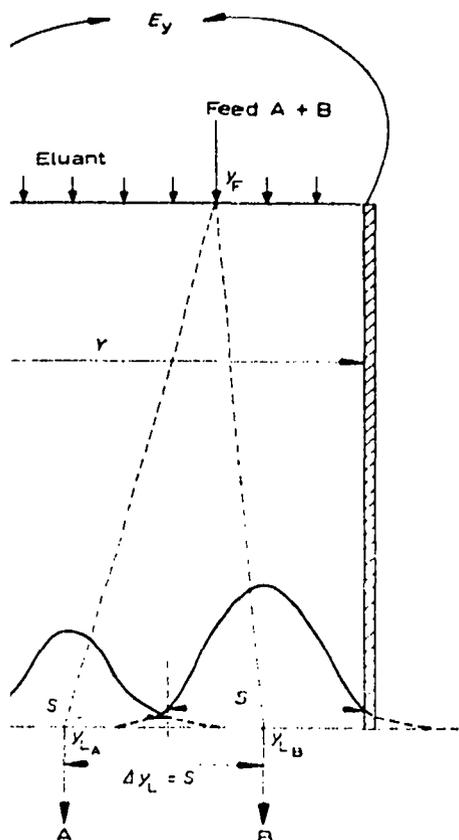
The Gaussian solution of the band-profile will correspond to a straight line if the cumulative sums of the recovered fractions are plotted on a probability scale, against linear offtake position. As illustrated by Fig. 10, a reproducible Gaussian distribution is indeed observed during actual separation, so that, even with the simplifying assumptions made, the developed model appears to be quite satisfactory.

ION OF TWO-COMPONENT SEPARATION

an example of the applicability of the combined operational criteria, the ion of a two-component separation will be discussed. Since the resolution of component systems is determined by the smallest difference in mobility between the components, the example is representative for more involved separations. All conditions and dimensions used are typical for the large-scale rectangular cylindrical electrophoresis units developed by VERMEULEN and coworkers. Designed to study scaling-up of preparative electrophoresis, the apparatus suitable for demonstration of the separation criteria. Details on construction and necessary equipment have been described elsewhere<sup>13,17,18</sup>. For a full understanding of the separation it is necessary to consider the equations of electrophoretic motion of the two components A and B in the bed. For rectangular bed electrophoresis in Fig. 11, the separation distance  $\Delta y_L$  of the two components is found to be

$$L = \frac{(u_A - u_B)\tau E_y}{Y} \tag{9}$$

use of the bedwidth requires that A and B each occupy half of it at the downflow. (All physical properties given, this can always be realized by using a



Schematic representation of optimal resolution of a binary mixture in a rectangular bed.

suitable injection point  $y_F$ .) Therefore, with the same bandwidth for the two components (molecular diffusion ignored), it follows that

$$S = \Delta y_L = Y/2 \quad (10)$$

Since  $S$  is related to packing size, bedlength and degree of recovery by eqn. (8) and Fig. 9, any of these quantities can precisely be adjusted to specified values of the other. Combining eqns. (9) and (10) results in:

$$E_y \tau = \frac{Y^2}{2(u_A - u_B)} = \frac{2 S^2}{u_A - u_B} \quad (11)$$

This implies that for the optimal *resolution* of a given system in a given apparatus, it is immaterial what combination of residence time and voltage is used, as long as the total power input (V h/mole separated) is adequate.

For the annular configuration similar conclusions apply. The equations corresponding to eqns. (10) and (11) are

$$S = \Delta r_L = \frac{r_a - r_b}{2} \quad (12)$$

and

$$E_{ab} \tau = \frac{(r_a - r_b)^2 \ln(r_a/r_b)}{(u_A - u_B) 4} = \frac{S^2 \ln(r_a/r_b)}{u_A - u_B} \quad (13)$$

In order to find the optimal combination of  $E$  and  $\tau$  for the *separation* as a whole, the other process limitations need to be considered. The combination of short residence time and high voltage ( $\sim 1/\tau$ ) is attractive because it allows a high throughput ( $\sim 1/\tau$ ) with a low energy consumption ( $\sim E\tau^2$ ). However, in practice serious limitations are encountered, like pressure loss in the bed, expensive high-voltage equipment, and above all the temperature lability of the feed components (temperature rise proportional with  $E^2$  and less than proportional with  $\tau$ ).

Thus the allowable maximum voltage and minimum residence time depend on the maximum permissible temperature rise and can be predicted by means of eqn. (11) or (13) and Fig. 4 or 6.

For the rectangular unit at Berkeley, with cooling at the electrode faces and the other lateral faces insulated, the approximate equation,  $T_{\max} - T_0 = \kappa E^2 / 8\lambda$ , applies, so that

$$E_{y \max} = \sqrt{8\lambda(T_{\max} - T_0)/\kappa} \quad (14)$$

and with eqn. (11),

$$\tau_{\min} = \frac{Y^2}{4(u_A - u_B)} \sqrt{\frac{\kappa}{2\lambda(T_{\max} - T_0)}} = \frac{S^2}{u_A - u_B} \sqrt{\frac{\kappa}{2\lambda(T_{\max} - T_0)}} \quad (15)$$

The optimization procedure is illustrated in Table III. For a known system, geometry and bed conditions, the optimal recovery, voltage and residence time are calculated. From the results it is obvious that the rectangular column would allow a slow but very sharp separation, whereas in the cylindrical column a rapid separation in relatively impure fractions would take place.

In sum, the results indicate that the derived criteria may be helpful to the analytical chemist, and contribute to a better insight into optimization and limitations

TABLE III

## OPTIMIZATION OF TWO COMPONENT SEPARATION

Rectangular bed,  $L = 50$  cm,  $Y = 7.5$  cm. Cylindrical column,  $L = 120$  cm,  $r_a = 9.7$  cm,  $r_b = 1.73$  cm. General data:  $u_A = +2 \cdot 10^{-4}$  cm<sup>2</sup>/Vsec,  $u_B = -1 \cdot 10^{-4}$  cm<sup>2</sup>/Vsec,  $0.04 < d_p < 0.06$  cm,  $c_p = 4.2$  J/g °C,  $\rho = 1$  g/cm<sup>3</sup>,  $\lambda = 5 \cdot 10^{-3}$  W/cm °C,  $\kappa = 1 \cdot 10^{-3}$   $\Omega^{-1}$ cm<sup>-1</sup>,  $T_{\max} = 50^\circ$ ,  $T_0 = 25^\circ$ .

| Bed                     |            | Rectangular |           | Cylindrical    |
|-------------------------|------------|-------------|-----------|----------------|
| $S$                     | Eqn. (10)  | 3.75 cm     | Eqn. (12) | 1.98 cm        |
| $P\acute{e}$            | (Table II) | 12          | (Table I) | 12             |
| $C_{(s/2, L)}/C_{\max}$ | Eqn. (8)   | 0.015       | Eqn. (8)  | 0.61           |
| Recovery                | (Fig. 9)   | 99.7%       | (Fig. 9)  | 82%            |
| $E\tau$                 | Eqn. (11)  | 26.0 V h    | Eqn. (13) | 6.3 V h        |
| $E_{\max}$              | Eqn. (14)  | 31.6 V      | (Fig. 6)  | $\sim 110$ V   |
| $\tau_{\min}$           |            | 50 min      |           | $\sim 3.5$ min |

of continuous electrophoresis and electrochromatography. The theory may also be applied for the scaling-up and design of preparative apparatus. Deviations from the theory are to be expected when the assumptions made are not realistic. In particular, electroosmotic flow and variation of the physical properties with temperature might lead to errors. However, further work at Berkeley<sup>13</sup> indicates that electroosmosis can be suppressed by using suitable materials for bed packing and diaphragms. Computer calculations revealed<sup>19,20</sup> that temperature effects can adequately be accounted for by taking all properties at an average bed temperature  $(T_{\max} - T_0)/2$ .

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## SUMMARY

By analysis of some important process variables, criteria for successful separation by continuous electrophoresis and electrochromatography in packed beds are derived. A general theory correlating power input, residence time and temperature rise in cylindrical and rectangular geometries is presented. The limitation of the separating capacity by transverse diffusion effects is shown to be predictable in terms of other operational conditions. These separation criteria appear to be in agreement with experimental evidence, and may find analytical as well as preparative application.

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