Use of a genetic algorithm to improve predictions of alternate bar dynamics

M. A. F. Knaapen and S. J. M. H. Hulscher

Water Engineering and Management, Department of Engineering Technology, University of Twente, Enschede, Netherlands

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Alternate bars may form in sandy beds of straight rivers and channels. These bars are characterized by the alternation of crests, all moving downstream at a speed of several meters per day. The aim of this paper is to predict the dynamics of alternate bars. To that end, we tested predictions of measured alternate bars in flume experiments, as derived from an amplitude evolution model. Weakly nonlinear stability analysis underlies this amplitude evolution model, so that it applies to situations in which the width-to-depth ratio is close to the critical ratio, above which alternate bars occur. The experiments have a width-to-depth ratio far above the critical value, well outside the range of formal validity of the model. While wavelengths and heights of the alternate bars are still well predicted, we found that the migration rate is not: the amplitude evolution model produces an underestimation of close to a factor of two. Therefore we took a slightly different approach. We tested the predictive capability for this amplitude evolution model by using a genetic algorithm to tune the model to bathymetric data. After tuning, the model is indeed able to predict the migration rate of the bars over periods that exceed the tuning period by far. Limits to the prediction time, i.e., failure of this method, could not be derived for the data sets used in this work.

INDEX TERMS: 1815 Hydrology: Erosion and sedimentation; 1824 Hydrology: Geomorphology (1625); 3210 Mathematical Geophysics: Modeling;

KEYWORDS: morphology, alternate bars, data assimilation, genetic algorithm, optimization, modeling


1. Introduction

[2] The interaction between a noncohesive bed and water motion results in interesting phenomena. Several types of wavy patterns can develop on sandy beds. Alternate bars are the largest free bed forms observed in fixed-bank rivers and channels. This bar pattern consists of an alternation of crests and troughs between the channel banks (see Figure 1). The wavelength of the bars is several times the width of the river, while the height is up to several meters. Alternate bars have been observed in rivers worldwide: for example the Tokachi in Japan (Figure 1), the Jamuna [Ashworth et al., 2000] in Bangladesh, the Rhine in Switzerland, and in North Boulder Creek, [Furbish et al., 1998] in the USA.

[3] The existence of alternate bars is thought to reflect an inherent instability of the bed and depth-averaged-flow system. This system has a straight forward equilibrium state, a spatially uniform river bed with a constant longitudinal slope and a constant flow over it. However, under certain conditions this state is unstable. Under these conditions, the interaction of the flow with the sand results in rhythmic bed patterns like ripples, dunes, or alternate bars.

[4] Such instabilities can be modeled using a stability analysis. The equilibrium state is disturbed by infinitesimal perturbations with a wide range of wavelengths. In the linear stability analysis, the growth rates of these perturbations are examined based on the linearization of the flow and sediment transport equations. For most conditions, all perturbations have negative growth rates and the bed will stay flat. For certain conditions, some perturbations have positive growth rates. If the model is correct, the wavelength of these waves should correspond to the observed bed pattern, alternate bars in our case.

[5] As the linear approximation is invalid once the amplitude of the perturbations are no longer infinitesimal, this approach is unable to describe the actual dynamics of the waves. This purpose requires a weakly nonlinear analysis. Next to the linear terms in the equations, the 2nd and 3rd order approximation of the system equations are evaluated. Consequently, the system is valid for larger perturbations as well. Both Colombini et al. [1987] and Schielen et al. [1993] assessed such a nonlinear stability analysis for alternate bars in rivers, resulting in an amplitude evolution model describing the slow temporal variations of the bar in time. The latter model is an extension of the former as it also accounts for large-scale spatial variations in the amplitude.

[6] The advantage of a stability approach over a straightforward numerical analysis is twofold. Firstly, the stability analysis is based on simplified equations, that are solved analytically. The equations contain the bare elements necessary to represent the solution. Consequently, this approach gives good insight on the relevant physical processes, whereas numerical model results are more difficult to analyze. Second, the analytical approach is inherently
stated, while in every numerical model it is difficult to find the right balance between allowing for perturbation growth and noise reduction. So far, no numerical model has been published that is able to describe the long term evolution of free rhythmic bed features.

[7] The amplitude evolution models, resulting from the stability analysis, describe the dynamics of alternate bars in rivers. So far, however, there has been no evidence that the amplitude evolution models can also predict these dynamics of alternate bars. The predictability, in a deterministic sense, of the small-scale morphodynamic processes is restricted in time [De Vriend, 1998, 2001]. Because of the recursive interactions, even smallest errors in the sediment transport models eventually become unacceptably large. However, the large-scale changes in themselves may still have fairly regular, and to some extent predictable, properties. We think that the amplitude evolution models resulting from a stability analysis can be used to predict these large-scale changes. In this paper we test this idea for alternate bar dynamics using the flume experiments of Lanzoni [2000b, 2000a].

[8] To utilize the amplitude evolution model for predictions of alternate bar dynamics, accurate estimates of both the initial state and the model parameters are required. Theoretical values of the parameters have been found to result in good estimates of both the wavelength and the height of alternate bars [Knaapen et al., 2001]. However, those studies did not consider bar dynamics. As this paper will show, the model tuned with theoretical parameters does not predict accurate migration rates.

[9] Therefore a data assimilation procedure, called genetic algorithm, is used to find those parameter values that do result in accurate migration rates. Data assimilation is the science in which available measurements are used to optimize predictive models. This approach is now often used in oceanography [Canizares et al., 1998; Voorrips et al., 1999; Van Leeuwen, 1999; Feddersen and Guza, 2000; Heemink et al., 2002; Knaapen and Hulscher, 2002; Morelissen et al., 2003].

[10] The genetic algorithm finds those parameter values that give the best fit to part of the available data. Starting from a random initial set of model parameter combinations, the genetic algorithm combines and modifies the parameters. This algorithm, that resembles genetic reproduction in biology, results in a model that predicts the remaining measurements accurately.

[11] This paper is organized as follows. First the genetic algorithm is explained in section 2. Section 3 describes the flume experiments that are used to test the amplitude evolution model. The model predictions based on theoretical parameters are evaluated in section 5, while in section 6, the model is tuned using the genetic algorithm. Section 8 describes possible applications of the amplitude evolution model combined with a genetic algorithm. Our findings are summarized in section 9.

2. Using a Genetic Algorithm for Data Assimilation

[12] Data assimilation is the mathematical science to incorporate measurements into models.

[13] With online data assimilation, the model with an initial set of parameters is used to make predictions. During the predictions, the model variables and parameters are changed every time new data show discrepancies between the model and the data. This changes of the parameters depend on the difference between the predicted and the measured values. The main advantage of online data assimilation is that it allows parameters to vary in time. The main disadvantage is that online techniques only work if the model and the initial parameters already are fairly accurate. In our problem the initial accuracy of the model is limited as will be shown in section 5. For a thorough introduction, the interested reader is referred to Anderson [1979] or Lewis [1986]. Canizares et al. [1998], Voorrips et al. [1999], and Van Leeuwen [1999] all give applications of online data assimilation in physical oceanographic problems.

[14] Off-line data assimilation, in general referred to as optimization algorithms [see Fletcher, 1987], aim to find the best values for both the parameters and the initial state of a model relative to measured data. In general, these algorithms are based on a gradient analysis. The dependency of the model results to the parameters is analyzed using the derivatives of the model. On the basis of the derivatives the best parameter change is chosen. A state-of-the-art example of the gradient search is the adjoint approach (see Heemink et al. [2002], Leredde et al. [2002], and Moore et al. [2002] for applications in oceanography).

[15] Because of the nonlinearity of our model and the amount of noise on the measurement data, the problem has many local minima. Consequently, gradient search algorithms are not always effective, because they are very sensitive to the occurrence of such local minima. Therefore we use a global optimization routine. Global optimizers aim to find the minimum of any function, regardless of its properties. Although efforts are made to construct deterministic global optimizers, most global optimizers are based on a stochastic search. A genetic algorithm is one type of global optimizer.
[16] In a genetic algorithm, all unknown parameters are gathered in a vector. A vector with allowed parameter values is referred to as “individual”, while a collection of vectors is named “population”. For each individual, a model run is executed using the parameter values of the individual. Each individual receives a fitness parameter, which is the root-mean-square difference between the measured bathymetry and the bathymetry predicted by the model using the parameter values of that individual (RSME).

[17] At the start of the algorithm, a population of individuals is generated with random parameter values, with uniform parameter distributions within imposed boundaries. From this initial population, new individuals are produced, analogously to genetic reproduction. A new individual can be created through two types of operations. With crossover, parameter values from two individuals are exchanged. Mutation is the operation in which one or more parameter values of a single individual are changed. Because of the reproduction, the population grows.

[18] However, some individuals die, thereby reducing the population size. Each individual has a chance on survival, depending on its fitness. The fittest individuals, with a low RSME, have a better chance on survival than individuals with high RMSEs. Thus new populations are generated iteratively. According to this approach, the genetic algorithm explores the whole parameter domain. The combination of the genetic reproduction and the survival of the fittest is able to identify the areas with low RMSEs. These promising areas are explored more extensively than other areas. The only thing left to do is to identify which individual has the lowest RSME. This individual contains an optimal parameter combination.

[19] In principle, this approach is not effected by local minima. However, since the approach partly depends on chance, one might end up in a local minimum, especially in the case of a model with numerous local minima. Therefore it is preferred to repeat the algorithm a number of times. The case of a model with numerous local minima. Therefore one could start a final run with the local minima as initial guess.


3. Large-Scale Flume Experiments on Alternate Bars

[21] In 1995, Lanzoni [2000b, 2000a] generated alternate bars under steady flow conditions in the large, straight sand flume of WL/Delft Hydraulics. The flume was 1.5 m wide, 1 m deep and 50 m long. The bathymetry was measured over a length of 43.8 m.

[22] During the experiments, all flow characteristics were controlled. Both the water level and the water flux were kept constant at the values chosen for the experiments. Sand leaving the flume was weighed and subsequently fed back at the upstream end of the flume, evenly distributed over the flume’s width.

[23] From the series of experiments [Lanzoni, 2000b, 2000a], two were selected to test the amplitude evolution model. Experiment P1801 is a very long test (816 h) with relatively deep water (7.3 cm) and low flow velocities (0.27 m/s). This run was part of the series with uniform size sediment ($d_{30} = 481 \mu m$, $d_{90} = 710 \mu m$, $\rho_s = 2650 \text{ kg/m}^3$).

[24] Experiment P0109 is much shorter (51 hours), and had shallower water (4.7 cm) and higher flow velocities (0.57 m/s). This run was part of the series with graded sediment ($d_{30} = 262 \mu m$, $d_{90} = 3210 \mu m$, $\rho_s = 2650 \text{ kg/m}^3$).

[25] A water level indicator and a profile indicator measured the water level and the bed profile respectively. In brief periods of 4 to 6 min, measurements were taken along three longitudinal sections, one along the central line and the other two at 0.20 m from each wall. At the end of an experiment, bed profiles were taken at 0.40 and 0.60 m from each wall. The interval between consecutive measurements depends on the rate of the morphological change.

[26] Figure 2 shows an example of the three resulting longitudinal bed profiles. Each crest on one side of the flume clearly corresponds with a trough on the opposing side. Ripples, which can be up to 2 cm high, appear as noise in the bathymetry.

[27] The measurements of the water level and the bed level can be used to determine the water depth: $h_s = \zeta - z_b$, in which $z_b$ and $\zeta$ are the bed level and the water level, respectively. For each section, the average longitudinal bed slope ($i_{b1}$, $i_{b2}$, $i_{b3}$) is calculated using linear regression.

[28] The Chezy coefficient can be estimated from the measured water depth, by using:

$$C_z = \frac{u}{\sqrt{h_s i_b}}$$

in which $h_s$ is the water depth, averaged over time and space, $i_b = (i_{b1}, i_{b2}, i_{b3})/3$ the mean bed slope, and $u$ the flow velocity averaged over time and space.

4. Amplitude Evolution Model

[29] Stability analysis approaches have been used for a variety of morphodynamic problems in rivers [Ikeda et al., 1981; Colombini et al., 1987; Johannesson and Parker,
In this equation, $U$ is the flow velocity vector and $z_b$ is the deviation of the bed as a traveling wave:

$$z_b(x, y, t) = h_c \varepsilon A(\xi, \tau)e^{ixk + \omega t} \cos(\gamma y) + NLT + c.c.$$ (3)

in which $(x, y)$ determine the position and $t$ denotes time. Furthermore, $\varepsilon$ is a small dimensionless parameter, while $k$ and $\omega$ are the wave number and frequency, respectively. The cosine in equation (3) models the cross-stream variation of the crest, $c.c.$ denotes the complex conjugates and $NLT$ indicates the nonlinear terms (see Appendix A for the exact formulations).

[32] The amplitude $A(\xi, \tau)$ follows from:

$$\frac{\partial A}{\partial \tau} = \alpha_0 A + \alpha_1 \frac{\partial^2 A}{\partial \xi^2} + \alpha_2 |A|^2 A$$ (4)

which is known as the Ginzburg-Landau equation. The coefficients $\alpha_i$ $(i = 0, 1, 2)$ in equation (4) are complex functions of the drag coefficient $C_d$ and the transport parameters $b$ and $\gamma$ (see Appendix A for the exact formulations of Schielen et al. [1993]). Furthermore, $\tau$ is the morphological time and $\xi$ is the spatial morphological coordinate in a frame moving with the group speed $v_g$ of the bars:

$$\tau = \frac{\xi}{\nu_\xi}, \quad \xi = \xi(x + \nu_\xi t)$$ (5)

The group speed follows from the weakly nonlinear model as explained in Appendix A.

5. Predictions With Theoretical Parameters

[33] To use this amplitude evolution model for predicting bed level changes with time, we need values for the coefficients $\alpha_i$ $(i = 0, 1, 2)$ of equation (4), all parameters in equation (3) as well as an initial state. According to Schielen et al. [1993], however, all coefficients and parameters are functions of the known width-to-depth ratio $R$, the drag-coefficient $C_d$ and the transport parameters $b$, $\gamma$ and $\sigma$.

[34] According to Knaapen et al. [2001], the parameters $C_d$, $b$, $\gamma$ are accurately represented by:

$$C_d = \frac{g}{C_z} = \frac{gh_c}{u_c^2}$$ (6)

$$b \approx \frac{3}{1 - \frac{\theta_1}{\theta_2}}$$ (7)

$$\gamma \approx 0.75 \left( \frac{\theta_1}{\theta_2} \right)^{\frac{3}{2}}$$ (8)

where $C_z$ is the Chezy coefficient and $\theta$ and $\theta_2$ are the Shields parameter and the critical Shields parameter, respectively. Furthermore, $\mu$ is the bed form or efficiency factor.

[35] In their analysis, Knaapen et al. [2001] used the results of Sekine and Parker [1992] to find the value for $\gamma$. Equation (7) is derived from the Meyer-Peter and Müller [1948] formula for bed load transport. The proportionality parameter $\sigma$ can also be estimated using this formula. After some calculations and using equation (6) one finds:

$$\sigma \approx 13.3 \sqrt{g \Delta d_0 \left[ \theta_2 - \theta_1 \right]^3 \left( \frac{C_d}{g \Delta d_0 \omega} \right)}$$ (9)

in which $\Delta$ is the relative submerged weight of the sediment and $d_0$ the median grain size.

[36] The migration rate $c_b$ can be calculated from the model frequency $\omega$ and wave number $k$ using:

$$c_b = -\frac{\omega}{k} = \frac{\omega_c - \omega_0}{k_c + k_d} = \frac{\omega_c - \omega_0 K - \frac{e^2 W}{k_c}}{k_c + \frac{e^2 W}{k_c}}$$ (10)

In this equation, $\omega_0$ and wave number $k_c$ are the critical values of $\omega$ and $k$, respectively. The critical values are the
wavelength and frequency of the fastest growing mode resulting from the linear stability analysis, which can be computed directly (see Appendix A). According to the weakly nonlinear theory, the actual wavelength and frequency are close to these critical values. The real-valued nonlinear terms \((W, K)\) are caused by the amplitude variations \(G\) described by the Ginzburg-Landau equation and are determined by:

\[
Re(\alpha_0) + Re(\alpha_1)K^2 + Re(\alpha_2)G^2 = 0
\]

\[
Im(\alpha_0) + Im(\alpha_1)K^2 + Im(\alpha_2)G^2 = W
\]

This system is under-determined (2 equations, 3 variables: \(K, G, W\)). However, the range of possible wave number variations can be estimated using:

\[
K^2 < Re(\alpha_0)\bar{f}_c
\]

with \(\bar{f}_c\) a known function of \(\alpha_i, i = 1, 2\) [see Schielen et al., 1993, equation (5.11)]. With this range estimate we can calculate the ranges of the nonlinear terms \((W, K)\). Table 1 gives the values of the parameters \(\alpha_i, i = 0,1,2\) and the frequency components for the conditions of two experiments by Lanzoni [2000b, 2000a].

<table>
<thead>
<tr>
<th>Parameters and Coefficients for the Conditions of the Experiments of Lanzoni [2000b, 2000a]</th>
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<tr>
<td>(\alpha_0)</td>
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<tr>
<td>P1801</td>
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<td>P0109</td>
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\[Q = S_{xy} \left[ \left( \frac{m}{\pi} \right) \right],\] however, are in good agreement with the measured discharges (Figure 5).

The difference between the predicted migration rates and measured values can be explained from the assumption of weak nonlinearity, on which the amplitude evolution model of Schielen et al. [1993] is based. On the basis of this assumption, they derived the complex frequency by using an expansion around the critical values \((\kappa_c, \lambda_c)\). However, the real width-to-depth ratios \(R\) in the experiments are far above the critical values \(R_c\) (see Figure 6). Therefore the linear estimates are invalid, and we need a different approach for estimating the frequency and thus indirectly the migration rates.

6. Predictions With the Model and a Genetic Algorithm

In practice, we are often interested in (low order) general information on alternate bars. For navigation purposes, for example, it is sufficient to know where the bars are located. From section 5 we know that the model is unable to predict such information using theoretical parameters; it under-estimates the migration rates considerably. Therefore in this section a genetic algorithm tunes a simplified version of the amplitude model to predict the average shape and the migration of bars in Lanzoni’s experiments.

In addition we will neglect all temporal variations of the amplitude. On the basis of the observed pattern in the experiments (Figure 2), we assume that the spatial variation in the amplitude of the alternate bars is a boundary effect. This boundary effect is included,
through an exponential growth curve $\eta$ in the bed form equation (3):

$$z_b = \varepsilon h_z \eta(x) \cos(k_x x + i\omega t + \phi_0) \cos(\pi y) + NLT + c.c.$$  \hspace{1cm} (14)

[41] In this data assimilation model, a simplified version of the amplitude evolution model, the average depth $h_z$, position $x$ and time $t$ are known. Frequency $\omega$, phase $\phi_0$ and boundary effect parameters, $c_1$, $c_2$, should be estimated using the genetic algorithm. The higher order terms $NLT$ are similar to the terms in the original model (see Appendix A), whereas $\varepsilon$ and wave number $k_x$ can be calculated directly from theory. Knaapen et al. [2001] show that this gives accurate estimates for the wavelength and amplitude. To increase the accuracy, the genetic algorithm is allowed to correct all parameters within a 25% margin of their theoretical values.

[42] Note that we use the wave frequency $\omega$ instead of the migration rate $c_b = \frac{\omega}{k_x}$. Also note that the initial value problem is reduced to one unknown $\phi_0$. The boundary relaxation is probably not optimal, but it does allow for both initial exponential growth and exponential decay. Since we are not interested in relaxation effects, this choice suffices.

[43] Lanzoni was interested in the initial development of alternate bars and his experiments were stopped when the amplitudes of the bars reached an equilibrium height. However, the data assimilation model assumes a constant amplitude. This restriction reduces the available data. We used the data assimilation model in equation (14) to predict the dynamics of alternate bars in experiments P1801 with uniform sand [Lanzoni, 2000b] and P0109 with graded sand [Lanzoni, 2000a].

[44] Of Lanzoni’s experiment P1801, starting on 16 February 1995, at 16:08 hours, a number of measurements are used to tune the model. It turns out that even with only two surveys, taken just 9 min after each other, accurate predictions can be made. After the tuning, the model predicted the bed topography for over 100 hours. We compare the predicted bathymetry to the measurements and conclude that the model predicts the measured bathymetry well (Figures 7, 8, and 9). At all times, the model predicts the peaks and the troughs at the correct locations and reproduces the shape bars accurate as well. Almost everywhere, the prediction errors are smaller than the ripples that are observed in the measurements.

[45] Furthermore, the differences between the predicted bathymetry and the measured values do not seem to change with time, which implies that the prediction error does not increase in time. This observation is confirmed by Figure 10. The dimensionless error (scaled by the measured amplitude) increases only marginally and, across the entire time domain, is about 20% larger than the standard deviation of

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**Figure 6.** Prediction versus measurements: the real width-to-depth ratio against the critical width-to-depth ratio. All true ratios are about twice the critical value (dashed line).

**Figure 7.** The last hindcast result compared to the measured bathymetry at the end of the tuning period ($t = 0$) in test P1801. The header gives the date and time of the measurement.

**Figure 8.** The model results compared to the measured bathymetry in test P1801. The header gives the date and time of the measurement.
the measurement noise. Small-scale bed perturbations, like ripples or dunes, cause most of the noise.

Of Lanzoni’s experiment P0109, two measurements are used to tune the data assimilation model. For this experiment the minimal required tuning period is 15 min, starting on September 1, 1995, at 11:00. After tuning, the model predicted the bed changes for the following 1:45 hours. Figures 11, 12, and 13 show the results. The only available shorter tuning period, 7.5 min results in a significant 20% increase of the error after 1 hour of predictions (Figure 14). This additional error is caused by an under-prediction of the propagation speed of the bars, which becomes significant within one period already.

The alternate bars are less regular than those of experiment p1801. The model predicts the positions of the bars and the troughs correctly, but there appear to be some amplitude modulations. However, there is not enough spatial variation to link these variations to the amplitude evolution model. Still, the differences between the predicted and the measured bathymetry are small in the downstream part of the flume. The statistics of the results of P0109 (Figure 14) are similar to those of test P1801. The errors are about 50% larger than the measurement noise, but now the difference between the errors and the noise slowly increases with time.

7. Discussion of the Results

For both tests, the differences between the predicted and the measured bathymetry are mainly related to the longitudinal asymmetry of the alternate bars. It is unlikely that higher order terms make up for this difference. It is probably another effect of the weakly nonlinear approach. Under the assumptions of near criticality, the nonlinearity of

Figure 9. The model results compared to the measured bathymetry in test P1801. The header gives the date and time of the measurement.

Figure 10. The root mean square error (Erms, solid line) compared to an estimate of the standard deviation of the measurement noise (dashed line) for test P1801. Both quantities are scaled by the measured amplitude of the alternate bars.

Figure 11. The last hindcast results compared to the measured bathymetry at the last hindcast resulting from the tuning (t = 0) in test P0109.

Figure 12. The model prediction compared to the measured bathymetry in test P0109 at the time indicated.
the system is insufficiently included in the model to describe the very steep fronts of the bars. In addition, a large part of the errors results from the boundary effect on the wavelength of the bars. This boundary effect was not imbedded in the amplitude relaxation in equation (14), but is clearly present in the graded-sediment experiment (P0109).

For experiment P1801, the data assimilation model predicts a migration rate of about 2.9 m/day. This is slightly higher than reported by [Lanzoni, 2000b]: 2.6 m/day. For experiment P0109, the model optimizer led to a migration rate of about 8.8 m/hour, which again is a bit higher than the value of 8.4 m/hour, found by [Lanzoni, 2000b]. These differences appear to be well within the error margins of the model. Note that the estimates of the Lanzoni experiments were based on the displacement of the crests only. Our estimate is based on the complete profiles of the bars.

The data assimilation model gives accurate predictions for the uniform-sand experiment P1801. The prediction error is slightly larger than the measurement noise for at least 1 wave period, after the model was tuned during only a marginally period (only 0.15% of the wave period). In experiment P0109 with graded sediment, the tuning time was 18% of the wave period. The predictions turn out to be still acceptable after 1.25 wave periods. In this experiment, reducing the tuning period to 9% of the wave period gave unreliable results. Probably the relatively large amplitude modulations increase the required tuning time.

However, it can be concluded that only a short tuning time is required to get a long prediction window. In both tests, the standard deviation of the error increases only marginally with time. This suggests that the predictions stay accurate for a while. The actual prediction window will be much larger than the values found here. Unfortunately, it is impossible to test the accuracy of the predictions over a longer period, as no additional measurements are available.

8. Applicability of Data Assimilation Models in Morphodynamics

In general, straight river channels are convenient for the purpose of navigation. However, when alternate bars occur in the channel, the river authorities have to indicate the navigation channel by using marks. Since the bars migrate downstream at a speed of several meters per day, the channel markings have to be updated frequently. Without a model, every update has to be based on time-consuming bathymetric surveys. With the data assimilation model presented in this paper, the number of measurements can be reduced, without increasing the risk to navigation.

The combination of amplitude evolution models and data assimilation is promising. Stability analyses are performed on a wide range of rhythmic morphodynamic phenomena [Dodd et al., 2003]. In some cases it is necessary to predict the dynamics of these changes. Knaapen and Hulscher [2002] for example, model the regeneration of dredged sand waves in a navigation channel. Data assimilation is an ideal tool for combining amplitude evolution models with available (remote sensing) data. The combination results in a fast and simple predictive model, even if full knowledge about the dynamical processes is incomplete.

9. Conclusions

The derivation of Schielen et al. [1993] assumes a width-to-depth ratio that is near its critical value. However, in the experiments of Lanzoni [2000b] the width-to-depth ratio is at least twice its critical value. Consequently, the approximation is formally invalid for the conditions of the experiments. Nevertheless, Knaapen et al. [2001] show that the model is able to estimate the wavelength and height. The current paper shows that this approach leads to largely underestimated migration rates. Apparently, The size of the alternate bars is not affected under strongly nonlinear conditions, whereas the dynamics of the bars do change significantly when the conditions are no longer weakly nonlinear.

Figure 13. The model prediction compared to the measured bathymetry in test P0109 at the time indicated.

Figure 14. The root mean square error ($E_{rms}$) for the predictions after 15 min of tuning (solid line) and 7.5 min of tuning (dashed line) compared to an estimate of the standard deviation of the measurement noise (dotted line) for test P0109. Both quantities are scaled by the measured amplitude of the alternate bars. The vertical lines denote the end of the tuning period.
Appendix A: Model of Schielen et al. [1993]

\[ z_0 = \varepsilon h_s A e^{i(k_x x + \omega t)} \cos \left( \frac{\pi y}{h_s} \right) \]
\[ + \varepsilon^2 A^2 e^{i(k_x x + \omega t)} \left( z_{222} \sin^2 \left( \frac{\pi y}{h_s} \right) + z_{222} \cos^2 \left( \frac{\pi y}{h_s} \right) \right) \]
\[ + \varepsilon^2 |A|^2 z_{202} \cos(2\pi y) + h.o.t. + c.c. \]  

(A1)

\[ \frac{\partial A}{\partial \tau} = \alpha_0 A + \alpha_1 \frac{\partial^2 A}{\partial \xi^2} + \alpha_2 |A|^2 A \]  

(A2)

\[ \tau = \varepsilon^2 \frac{A}{T} \]
\[ T = \frac{\varepsilon}{\frac{i \varepsilon h_s}{C_d} \left( \frac{\pi y}{h_s} \right)^2} \]
\[ \xi = \varepsilon \left( \frac{x}{h_s} + \nu_k t \right) \]
\[ \alpha_0 = R_c (\tau_r + iv) \]

\[ R_c = \frac{\pi \delta^2}{C_d} (1 + 2\sqrt{\delta} + 4\delta) \]
\[ \delta = \frac{\gamma C_d}{(b - 1)} \]
\[ \tau_r = \frac{2\delta(b - 1)^2}{\gamma} \left( 1 - 4\delta + O(\delta) \right) \]
\[ \nu_k = \frac{2\sqrt{2}(b - 1)^2 \delta}{\gamma} \left( 1 - 3\delta + O(\delta) \right) \]
\[ \alpha_1 = -\frac{1}{2} \left( \gamma_{1e1} + iv_{1e1} \right) \]
\[ \tau_{21} = -\frac{8(b - 1)\delta}{\pi} \left( 1 + 2\delta + O(\delta) \right) \]
\[ \nu_{21} = -\frac{5\sqrt{2}(b - 1)\delta}{\pi} \left( 1 - 6\delta + O(\delta) \right) \]
\[ \alpha_2 = c_r + ic_i \]
\[ c_r = -\frac{2\pi}{3(b - 1)} + \frac{(62 - 78b + 19b^2 - 9b^3)\pi}{12(b - 1)} \delta + O(\delta) \]
\[ c_i = \frac{(20 - 30b + 9b^2)\sqrt{2\pi}}{24(b - 1)} \]
\[ + \frac{(-162 + 294b - 133b^2 + 15b^3)\sqrt{2\pi}}{24(b - 1)} \delta + O(\delta) \]
\[ \epsilon = \sqrt{\frac{R}{R_c}} - 1 \]
\[ k_c = \sqrt{2\pi} \delta \left( 1 + \frac{19}{4} \delta + O(\delta) \right) \]
\[ \omega_c = -ik_c \left( 1 + (b - 1)\delta - 5(b - 1)b + O(\delta) \right) \]
\[ \nu_k = \left( 1 + 7(b - 1)\delta - 18(b - 1)b + O(\delta) \right) \]
\[ z_{202} = -\frac{b}{2(b - 1)} \left( 1 + (1 + b)\delta + O(\delta) \right) \]
\[ z_{222} = \frac{i\sqrt{2}}{3(b - 1)} \delta^2 - \frac{b - 2}{3(b - 1)} + \frac{i\sqrt{2}(20 - 22b - b^2)\delta}{12(b - 1)} \]
\[ + \frac{38 - 59b + b^2}{24(b - 1)} \delta^2 + O(\delta) \]
\[ z_{222} = -\frac{i\sqrt{2}}{3(b - 1)} \delta^2 + \frac{b - 2}{3(b - 1)} + \frac{i\sqrt{2}(20 - 22b - b^2)\delta}{12(b - 1)} \]
\[ + \frac{38 - 59b + b^2}{24(b - 1)} \delta^2 + O(\delta) \]

Notation

- \( A \) scaled alternate bar amplitude, dimensionless.
- \( b \) nonlinearity of sediment transport, dimensionless.
- \( c.c. \) complex conjugates, dimensionless.
- \( c_b \) bed wave celerity, m/s.
- \( c_i \) complex part of \( \alpha_2 \).
- \( c_r \) real part of \( \alpha_2 \).
- \( c_{1e} \), \( c_{2e} \) boundary layer effect parameters, dimensionless.
- \( C_d \) drag coefficient, dimensionless.
- \( C_z \) Chezy coefficient, \( \sqrt{\text{m/s}} \).
- \( d_{50} \) mean grain size, \( \mu \m \).
- \( d_{90} \) 90% grain size limit, \( \mu \m \).
\( E_{rms} \) root-mean-square error of bed level prediction, dimensionless.
\( g \) gravity constant, m/s².
\( G \) nonlinear amplitude correction, dimensionless.
\( h_s \) mean water depth, m.
\( i_b \) average longitudinal bed slope, dimensionless.
\( k \) the wave number of the alternate bars, dimensionless.
\( k_c \) the critical wave number of alternate bars, dimensionless.
\( k_{nl} \) the nonlinear part of wave number, dimensionless.
\( K \) nonlinear wave number correction of bed waves, dimensionless.

Nonlinear terms, dimensionless.
\( Q \) sediment discharge m³/s.
\( R \) width-to-depth ratio, dimensionless.
\( R_c \) critical width-to-depth ratio, dimensionless.
\( S \) sediment transport vector, m³/s.
\( S_c \) longitudinal sediment transport, m²/s.
\( t \) time, s.
\( U \) flow velocity vector, m/s.
\( u \) longitudinal flow velocity, m/s.
\( u_\ast \) the flow velocity averaged over time and place, m/s.
\( W \) nonlinear frequency correction of the alternate bars, dimensionless.
\( x \) longitudinal coordinate, m.
\( y \) transverse coordinate, m.
\( y_s \) channel width, m.
\( z_b \) bed level relative to reference point, m.
\( z_{2b} \) disturbed bed level relative to mean longitudinal slope, m.
\( z_{22s} \) nonlinear bed wave amplitude, dimensionless.
\( z_{22c} \) nonlinear bed wave amplitude, dimensionless.
\( z_{02} \) nonlinear bed wave amplitude, dimensionless.
\( \alpha_0 \) exponential amplitude growth coefficient, dimensionless.
\( \alpha_1 \) horizontal amplitude variation coefficient, dimensionless.
\( \alpha_2 \) nonlinear amplitude decay coefficient, dimensionless.
\( \Delta \) relative density of the sediment in water, dimensionless.
\( \delta \) ratio between frictional and bed slope effects of sediment transport, dimensionless.
\( \epsilon \) small dimensionless parameter, dimensionless.
\( \eta \) boundary layer effect, dimensionless.
\( \gamma \) downhil preference of sediment transport, dimensionless.
\( \mu \) bed roughness parameter, dimensionless.
\( \nu_k \) group speed of the alternate bars, dimensionless.
\( \nu_{r2} \) real part of \( \alpha_1 \).
\( \nu_r \) real part of \( \alpha_0 \).
\( \phi_0 \) phase of the alternate bars, dimensionless.
\( \rho_s \) sediment density, g/cm³.
\( \sigma \) sediment transport proportionality, dimensionless.
\( \tau \) morphological time, dimensionless.
\( \tau_{12} \) complex part of \( \alpha_1 \).
\( \tau_r \) complex part of \( \alpha_0 \).
\( \theta \) shields parameter, dimensionless.
\( \theta_c \) critical shields parameter, dimensionless.
\( \xi \) morphological longitudinal coordinate, dimensionless.
\( \zeta \) elevation disturbed free surface, m.
\( \omega \) the frequency of the alternate bars, dimensionless.
\( \omega_c \) the critical frequency of the alternate bars, dimensionless.
\( \omega_{nl} \) the nonlinear part of the bed wave frequency, dimensionless.

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S. J. M. H. Hulscher and M. A. F. Knaapen, Water Engineering and Management, Department of Engineering Technology, University of Twente, P.O. Box 217, 7500 AE Enschede, Netherlands. (s.j.m.h.hulscher@ctw.utwente.nl; m.a.f.knaapen@ctw.utwente.nl)