Nonlinearity and Hysteresis of Resonant Strain Gauges

Chengqun Gui, Rob Legtenberg, Harrie A. C. Tilmans, Jan H. J. Fluitman, and Miko Elwenspoek

***Abstract*—The nonlinearity and hysteresis effects of the electrostatically activated voltage-driven resonant microbridges have been studied theoretically and experimentally. It is shown that in order to avoid vibration instability and hysteresis to occur, the choices of the ac and dc driving voltages and of the quality factor of a resonator, with a given geometry and choice of materials, are limited by a hysteresis criterion. The limiting conditions are also formulated as the hysteresis-free design rules. Expressions for the maximum allowable quality factor and maximum attainable figure of merit are given. Experimental results, as obtained from electrostatically driven vacuum-encapsulated low-pressure chemical-vapor deposition (LPCVD) polysilicon microbridges, are presented and show good agreement with the theory. [191]**

***Index Terms*—Figure of merit, hysteresis, nonlinearity, quality factor, resonant strain gauges.**

I. Introduction

NONLINEARITY is easily encountered in the resonant microbridges, leading to additional stiffening known as the "hard-spring effect" and, moreover, hysteresis and instability occurring. In particular, the latter two phenomena seriously limit the freedoms of the geometric design, operating conditions, maximum allowable quality factor, and, subsequently, figure of merit.

The nonlinearity of the micromachined resonant microbridges has been observed and studied by several authors [1]–[9]. In 1987, Andres *et al.* have experimentally observed the nonlinear vibration and hysteresis of the micromachined silicon resonators [1]. Ikeda *et al.* have discussed the influence of the nonlinear vibration of the Si resonant beams on the sensor characteristics and introduced a method to eliminate this influence in 1989 [2]. Zook *et al.* have also found in 1991 that the microbeam resonators are sensitive to the operating conditions: as the drive voltages increase, the resonant frequency shifts to a higher value, leading to hysteresis depending on the direction of frequency scan [3]. Pratt *et al.* have reported in 1991 that the nonlinear response curves of the lateral vibrating micromechanical structure are well modeled by Duffing's equation for a stiffening spring [4]. Detection-related frequency-pulling mechanism in amplitude-stiffened resonators have been investigated by Shirley *et al.* in 1993. An automatic level control (ALC) loop has been used to minimize the frequency shift of the resonators [5]. Tilmans *et al.* have discussed the nonlinear large amplitude effects and derived the expressions for the stiffening effect and for the critical amplitude of the resonant microbridges [6]–[9]. The formulations are based on the theory of a discrete stiffening spring as introduced by Landau and Lifshitz [10].

In this paper, we demonstrate a hysteresis criterion, a necessary condition for the hysteresis-free operation of the resonant microbridges. This hysteresis criterion depends on the quality factor, operating conditions, geometric properties, and material properties of the microresonator. The influence of the hysteresis on the performance of the microbridges, such as the quality factor, figure of merit, sensitivity, and noise, are discussed. Hysteresis-free design rules associated with the quality factors, figure of merit, geometry, and operating of the microresonator are addressed. Experimental results that show the validity of the hysteresis criterion are given.

II. Hysteresis Criterion

The resonant microbridge considered here consists of a prismatic wide beam with a rectangular cross section, having clamped-end conditions (see Fig. 1). Axial stress effects, due to either a built-in strain field or to externally applied loads, are not taken into account. This is justified to some extent, recalling that axial stress effects are generally tensile in nature, which leads to a stiffer structure. This means that for such structures, the criteria as derived in this section are the worst case conditions and as such define a safe operating region.

The harmonic-forced nonlinear vibration equation of the clamped–clamped beam has the same form as Duffing's equation {see [11, eq. (21)]}, which means that the microbridge can be approximated as a simple spring-mass system with a restoring force having a cubic dependence on the amplitude [12], [13]. Thus, the fundamental resonant frequency ω, which depends on deflection, can be written as [6]

$$\omega^2 = \omega_0^2[1 + 0.53(1 - \nu^2)\,(W_{\max}/h)^2] \quad (1)$$

with ω_0 the resonant frequency in the linear limit, ν Poisson's ratio, and $W_{\max}/h$ the ratio of the maximum vibration amplitude to the thickness of the microbridge.

A consequence of (1) is that beyond a critical amplitude W_c the resonant curve becomes triple valued (see Fig. 2). The

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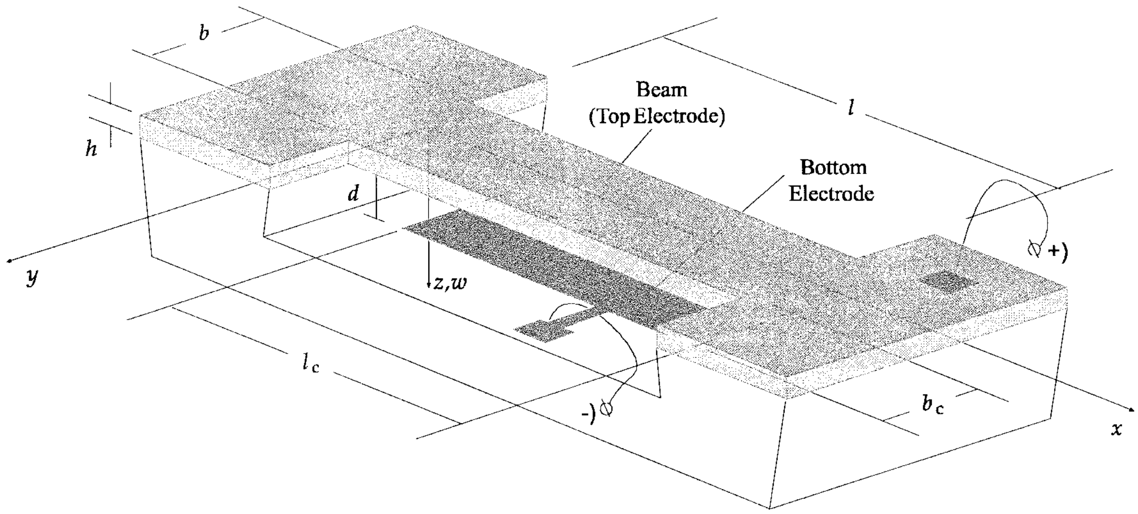

Fig. 1. Schematic drawing of a beam (length l, width b, and thickness h) configured as a one-port resonator. The beam itself is homogeneous and is composed of a conducting material. The lower electrode considered here has the same shape as the beam (length $l_c = l$ and width $b_c = b$) [9].

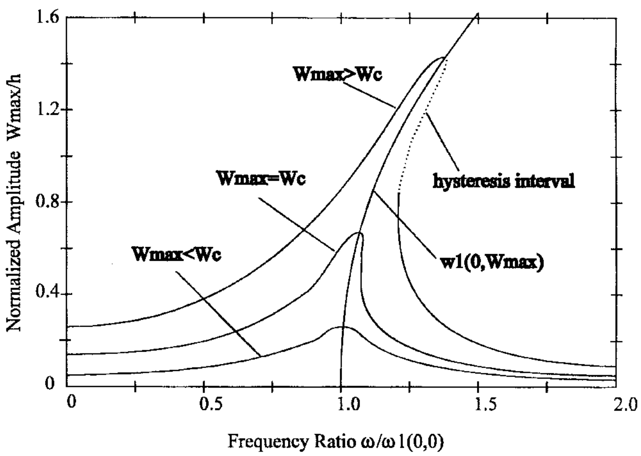

Fig. 2. Normalized amplitude versus the normalized frequency for a clamped–clamped beam with a rectangular cross section and with zero built-in axial strain [6].

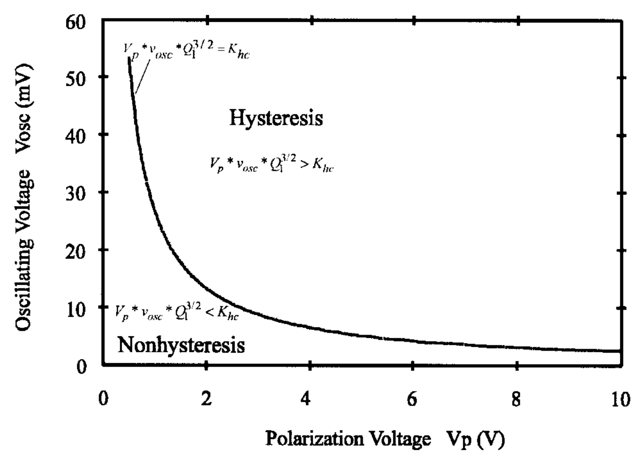

Fig. 3. Hysteresis criterion of a typical encapsulated LPCVD polysilicon resonant microbridge. The resonator size is 210 × 100 × 1.5 μm. The gap spacing is 1.0 μm, and the pull-in voltage is 28.0 V. $Q_1 = 2700$. The plate modulus $E' = 166$ GPa, and a Poisson's ratio of 0.22 for the LPCVD polysilicon was used.

critical amplitude for the first mode is given by [6], [10]

$$W_c = \frac{h}{[0.53Q_1(1-\nu^2)]^{1/2}} \tag{2}$$

with Q_1 the quality factor for the first mode.

The maximum vibration amplitude W_{max} for the first mode of a resonator electrostatically driven by a dc polarization voltage V_p plus an ac oscillating voltage V_{osc} is [9]

$$W_{max} = 0.37dQ_1\frac{V_pV_{osc}}{V_{pio}^2} \tag{3}$$

with d the gap spacing and V_{pio} the pull-in voltage of a microbridge with a uniform electrode. An approximated expression of V_{pio} is given by

$$V_{pio} = 3.48\,\frac{1}{l^2}\,[E'\,h^3\,d^3/\varepsilon_0]^{1/2} \tag{4}$$

where $E' = E/(1-\nu^2)$ is the plate modulus, E is Young's modulus, l denotes the beam length, ε_0 is the dielectric constant in vacuum [8], [9].

Obviously, for hysteresis-free operation, we must have $W_{max} < W_c$, which leads to the hysteresis criterion

$$V_pV_{osc}Q_1^{3/2} < K_{hc} \tag{5}$$

where

$$K_{hc} = 45.0\left[\frac{E'}{\varepsilon_0(1-\nu^2)^{1/2}}\right]\left(\frac{h}{l}\right)^4 d^2 \tag{6}$$

is a critical hysteresis constant that depends on the geometric properties and the material parameters of the microbridges.

Equation (5) represents a necessary condition for the hysteresis-free operation of a resonant microbridge. The hysteresis criterion for the driving voltages and the quality factor of a typical low-pressure chemical-vapor deposition (LPCVD) polysilicon microbridge is graphically shown in Fig. 3, where a plate modulus of 166 GPa [8] and a Poisson's ratio of 0.22 for the LPCVD polysilicon material [15] were used.

In general, the chance for the vibration instability can be lowered, by choosing a smaller beam aspect ratio l/h or by increasing the gap spacing. Also, recall, as indicated at the beginning of this section, that an axial tensile stress will make the beam stiffer, which leads to an increase of the pull-in voltage [9] and, thus, a higher value for K_{hc}.

III. Hysteresis-Free Design Rules

The hysteresis criterion expressed by (5) and (6) leads to a complex interplay between the desire for a stable hysteresis-free design and the best attainable performance of the microbridge expressed in terms of the figure of merit and of the (axial) strain sensitivity and resolution. In this section, (5) will be used to show that the quality factor may not exceed a certain maximum value in order to avoid instability and hysteresis to occur. Also, the expression for a maximum attainable figure of merit will be derived.

A. The Maximum Allowable Quality Factor

From (5) and (6), it follows, that the maximum allowable quality factor $Q_{1\,\max}$ is given by

$$Q_{1\,\max} = 12.7 \left[\frac{E'}{\varepsilon_0 (1-\nu^2)^{1/2}}\right]^{2/3} \left[\frac{d^2}{V_{\mathrm{p}} V_{\mathrm{osc}}} \left(\frac{h}{l}\right)^4\right]^{2/3}. \tag{7}$$

In the design of the microresonant strain gauges, a high-quality factor is preferred to attain a low-energy dissipation, high resolution, and high-efficient oscillator. In order to increase the freedom of the hysteresis-free quality factor, it is suggested in (7) to decrease the drive voltages and the aspect ratio, or increase the gap spacing. However, decreasing the drive voltages is limited by noise. The penalty of decreasing the aspect ratio of the beam has to be paid by a decreased sensitivity. The sensitivity of the resonant strain gauges is characterized by its gauge factor (in case of a zero axial force) [6]

$$G_{\varepsilon 1} = 0.5\,\gamma_1\,(1-\nu^2)\,(l/h)^2 \tag{8}$$

with $G_{\varepsilon 1}$ the gauge factor and $\gamma_1 = 0.295$ the coefficient for the first mode. Obviously, as decreasing the aspect ratio of the resonant beam, the gauge factor will be decreased on the order of $(l/h)^2$.

B. The Maximum Attainable Figure of Merit

The figure of merit is another relevant parameter that is much influenced by the hysteresis criterion. In an electromechanical microresonator, the electrostatically driven microbridge is coupled to a control circuit to obtain self-oscillation in the required mode. To achieve this, it is important to have a large figure of merit. The figure of merit, defined as the ratio of the motional and the static admittance, provides a measure that combines the quality factor and the coupling factor of the microresonator into one parameter [8]

$$M \equiv \frac{|Y_{\mathrm{mot}}(\omega_1^s)|}{|Y_{\mathrm{stat}}(\omega_1^s)|} \approx Q_1 K_{\mathrm{eff}}^2 \tag{9}$$

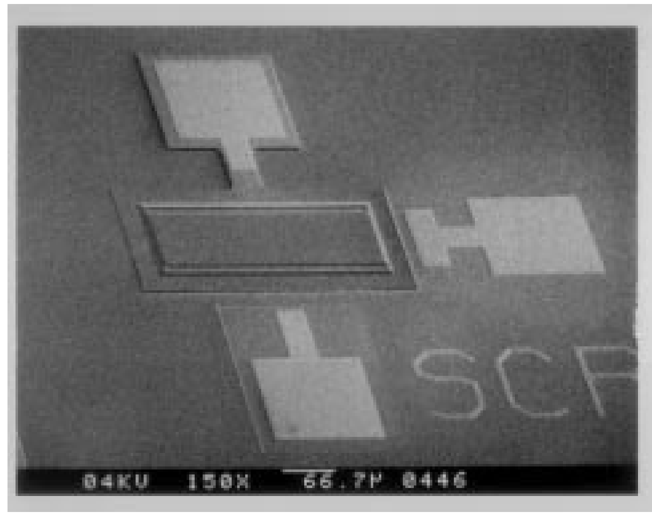

Fig. 4. An SEM photograph of cap and bond pads of the microbridge.

where M is the figure of merit, $|Y_{\mathrm{mot}}(\omega_1^s)|\,/\,|Y_{\mathrm{stat}}(\omega_1^s)|$ is the ratio of the motional, and the static admittance, respectively, K_{eff} is the coupling factor of the microresonator. For the definitions of the motional and the static admittance and the coupling factor, readers are referred to [8, Ch. 3].

Based on the theory presented in [8] and [9], the figure of merit can be conveniently expressed as a function of the drive voltages

$$M \approx 0.20\,Q_1 \left(\frac{V_{\mathrm{p}}}{V_{\mathrm{pio}}}\right)^2. \tag{10}$$

Introducing (5) into (10), we have

$$M < M_{\max} \approx \frac{2.76}{(1-\nu^2)} \left(\frac{h}{d}\right)^2 \left(\frac{V_{\mathrm{pio}}}{V_{\mathrm{osc}} Q_1}\right)^2. \tag{11}$$

Interestingly, on the one hand, it is seen from (10) that the value of the figure of merit increases with increasing the quality factor and with decreasing the pull-in voltage. On the other hand, from a stability point of view [i.e., (11)], the allowed upper bound of the figure of merit will be smaller for higher values of the quality factor and for smaller values of the pull-in voltage. Also note that low driving levels are beneficial for obtaining a high value for $M_{\max}$.

IV. Experiment and Results

In order to verify the foregoing theory with experimental data, vacuum-encapsulated LPCVD polysilicon microbridge resonators have been fabricated using the process flow as described in [8] and [14]. A scanning electron microscope (SEM) photograph of a completed device is shown in Fig. 4. The resonators having microbridges of two different lengths (210 and 310 μm) with a width of 100 μm, a thickness of 1.5 μm, and a gap spacing of 1.0 μm have been tested. The quality factor was set by controlling the cavity pressure during the reactive sealing step with LPCVD silicon nitride [16]. This way, the quality factors could be varied between 100–3000.

The distortion of the resonance curve and the hysteresis beyond W_{c} due to the nonlinearity can be obtained by measuring the admittance and the phase as a function of the frequency using a HP4194 gain-phase analyzer. While increasing the

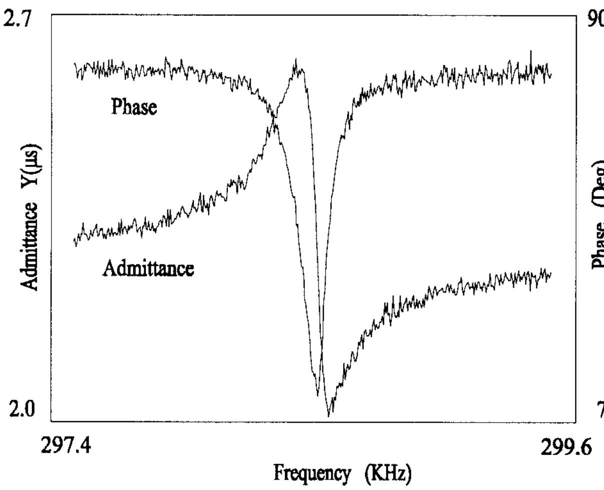

Fig. 5. Admittance, phase versus frequency plot (nonhysteresis). Resonator number is 3209, size is 210 × 100 × 1.5 μm, gap spacing is 1.0 μm, $V_p = 1$ V, $V_{osc} = 10$ mV, and $Q_1 = 2871$.

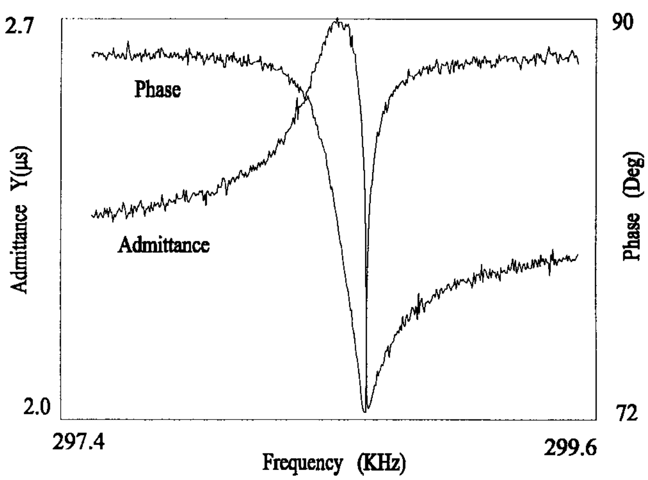

Fig. 6. Admittance, phase versus frequency plot (at critical load). Resonator number is 3209, size is 210 × 100 × 1.5 μm, gap spacing is 1.0 μm, $V_p = 1$ V, $V_{osc} = 15$ mV, and $Q_1 = 2871$.

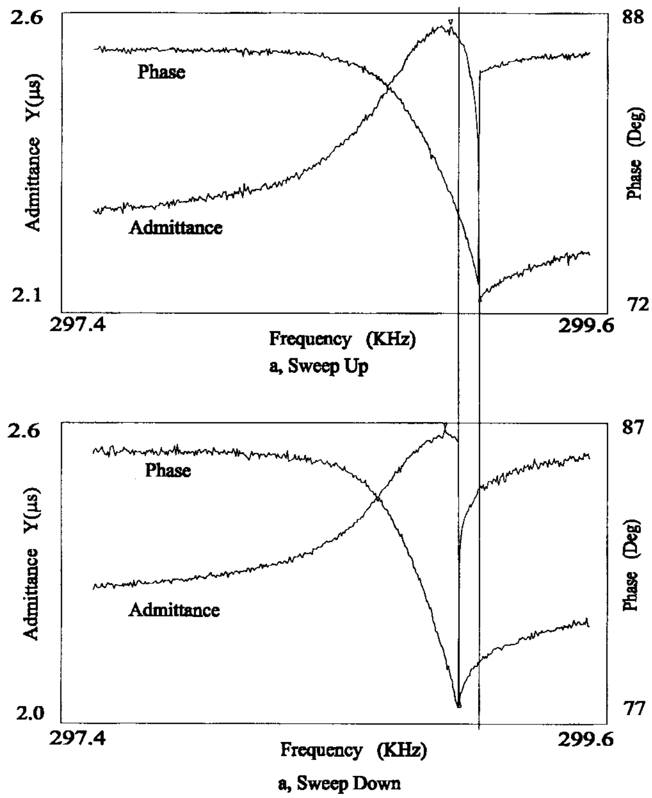

Fig. 7. Admittance, phase versus frequency plots (hysteresis). Top shows admittance and phase versus frequency plot for frequency sweep up. Bottom shows admittance and phase versus frequency plot for frequency sweep down. The hysteresis is clearly seen. Resonator number is 3209, size is 210 × 100 × 1.5 μm, gap spacing is 1.0 μm, $V_p = 1$ V, $V_{osc} = 20$ mV, and $Q_1 = 2871$.

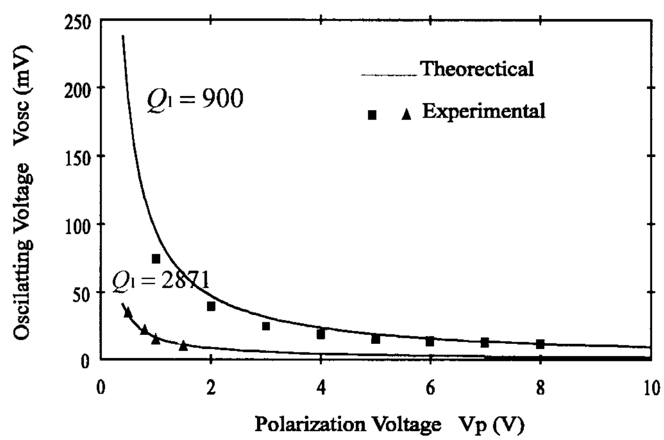

Fig. 8. Hysteresis criterion for resonator with a length of 210 μm. Resonator size is 210 × 100 × 1.5 μm. Resonator number is 3209, $Q_1 = 2871$, resonator number is 3220, and $Q_1 = 900$. The solid lines are the theoretical results calculated from (5). A plate modulus of 166 GPa and a Poisson's ratio of 0.22 were used in the calculations. Higher quality factor means less freedom in choosing the driving voltages.

oscillating voltage, the resonant frequency shifts to a higher value (see Figs. 5–7). The resonator will experience three conditions: a nonhysteresis region (Fig. 5), a critical point (Fig. 6), and a hysteresis region [Fig. 7].

The experimental results of the hysteresis criterion for resonators with a length of 210 and 310 μm are illustrated in Figs. 8 and 9, respectively. The calculated hysteresis criterion from (5) for both resonators are also shown in two plots. Again, a plate modulus of 166 GPa and a Poisson's ratio of 0.22 for the LPCVD polysilicon material were used in the theoretical calculations. Both plots have shown the validity of (5).

V. Discussion and Conclusion

The hysteresis and instability due to the hard-spring effect of the resonant microbridges have been studied. A hysteresis criterion, which depends on the quality factor, operating, and geometry of the microresonator, has been obtained. This hysteresis criterion represents a necessary condition for the hysteresis-free operation of the resonant microbridge. It was found that the freedom of the hysteresis-free microresonator may be increased by decreasing the quality factor, the drive voltages, and the aspect ratio of the beam, or increasing the pull-in voltage of the microbridge. All these measures have their shortcomings: decreasing the quality factor will be accompanied with a loss of resolution and frequency stability, decreasing the drive voltages is limited by noise, and decreasing the aspect ratio of the beam will cause the

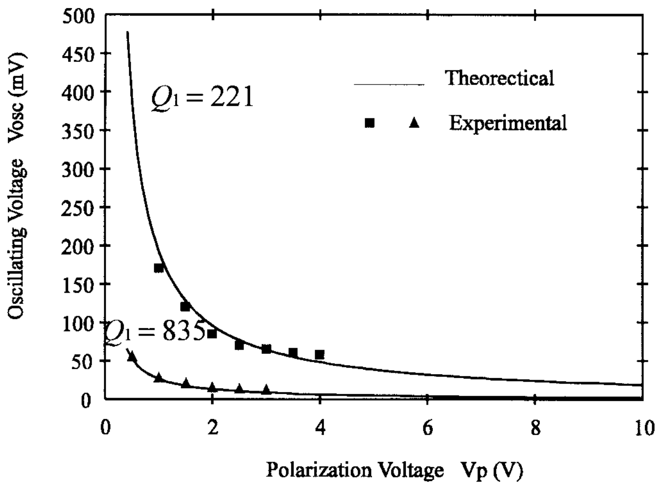

Fig. 9. Hysteresis criterion for resonator with a length of 310 μm. Resonator size is 310 $\times$ 100 $\times$ 1.5 μm, resonator number is 3304, $Q_1 = 835$, resonator number is 5307, and $Q_1 = 221$. The solid lines are the theoretical results calculated from (5). A plate modulus of 166 GPa and a Poisson's ratio of 0.22 were used in the calculations. Lower quality factor implies larger freedom in choosing the driving voltages.

undesirable loss of sensitivity, which is decreased on the order of $(l/h)^2$.

The figure of merit is limited by the hysteresis criterion. On the one hand, the value of the figure of merit will be increased by increasing the quality factor, on the other hand the nonhysteresis freedom of the figure of merit will be decreased by increasing the quality factor. For a given Q factor, the figure of merit can be increased by decreasing $V_{\rm osc}$ up to a maximum value bounded by hysteresis. Hysteresis-free design rules associated with quality factor, figure of merit, operating, and geometry of the microresonator sufficiently demonstrate the complex interplay between stability and the performance of the resonant microbridges.

Hysteresis and instability are successfully detected by using a gain-phase analyzer. Experimental results of resonant microbridges with a length of 210 and 310 μm have shown the validity of the hysteresis criterion.

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REFERENCES

[1] M. V. Andres, K. W. H. Foulds, and M. J. Tudor, "Nonlinear vibration and hysteresis of micromachined silicon resonators designed as frequency-out sensors," *Electron. Lett.*, vol. 23, pp. 952–954, 1987.

[2] K. Ikeda, H. Kuwayama, T. Kobayashi, T. Watanabe, T. Nishikawa, T. Yoshida, and K. Harada, "Study of nonlinear vibration silicon resonant beam strain gauges," in *Proc. 8th Sensor Symp.*, Tokyo, Japan, 1989, pp. 21–24.

[3] J. D. Zook, D. W. Burns, H. Guckel, J. J. Sniegowski, R. L. Engelstad, and Z. Feng, "Resonant micro beam strain transducers," in *Proc. 6th Int. Conf. Solid-State Sensors Actuators (Transducers'91),* San Francisco, CA, June 24–27, 1991, pp. 529–532.

[4] R. I. Pratt, G. C. Johnson, R. T. Howe, and J. C. Chang, "Micromechanical structures for thin film characterization," in *Proc. 6th Int. Conf. Solid-State Sensors Actuators (Transducers'91),* San Francisco, CA, June 24–27, 1991, pp. 205–208.

[5] T. E. Shirley and S. D. Senturia, "Frequency-pulling effects in amplitude-stiffened resonant sensors," in *Proc. 7th Int. Conf. Solid-State Sensors and Actuators (Transducers'93),* Yokohama, Japan, June 7–10, 1993, pp. 458–461.

[6] H. A. C. Tilmans, M. Elwenspoek, and J. H. J. Fluitman, "Micro resonant force gauges," *Sens. Actuators A,* vol. 30, pp. 35–53, 1992.

[7] M. Elwenspoek, "Micromechanical resonant sensors," *Journal A,* pp. 15–21, 1991.

[8] H. A. C. Tilmans, "Micro-mechanical sensors using encapsulated built-in resonant strain gauges," Ph.D. dissertation, Univ. Twente, The Netherlands, 1993.

[9] H. A. C. Tilmans and R. Legtenberg, "Electrostatically driven vacuum-encapsulated polysilicon resonators, Part II: Theory and performance," *Sens. Actuators A,* vol. 45, pp. 67–84, 1994.

[10] L. D. Landau and E. M. Lifschitz, *Lehrbuch der Theoretischen Physik-Mechanik,* 3rd ed. Berlin: Akademie-Verlag, 1964, pp. 101–104.

[11] J. G. Eisley, "Nonlinear vibration of beams and rectangular plates," *J. Appl. Math. Phys.,* vol. 15, pp. 9167–9175, 1964.

[12] S. Timoshenko, D. H. Young, and W. Weaver, Jr., *Vibration Problems in Engineering,* 4th ed. New York: Wiley, 1974, pp. 177–186.

[13] J. P. Den Hartog, *Mechanical Vibration,* 4th ed. New York: McGraw-Hill, 1956, pp. 370–373.

[14] R. Legtenberg and H. A. C. Tilmans, "Electrostatically driven vacuum-encapsulated polysilicon resonators, Part I: Design and fabrication," *Sens. Actuators A* vol. 45, pp. 57–66, 1994.

[15] W. N. Sharpe, Jr., B. Yuan, R. Vaidyanathan, and R. L. Edwards, "Measurements of Young's modulus, Poisson's ratio, and tensile strength of polysilicon," in *Proc. IEEE 10th Annu. Int. Workshop Micro Electro Mechanical Syst. (MEMS'97),* Nagoya, Japan, Jan. 26–30, 1997, pp. 424–429.

[16] C. Gui, R. Legtenberg, M. Elwenspoek, and J. H. J. Fluitman, "Q factor dependence of one port encapsulated polysilicon resonator on reactive sealing pressure," *J. Micromech. Microeng.,* vol. 5, pp. 183–185, 1995.

Chengqun Gui, for a photograph and biography, see this issue, p. 67.

Rob Legtenberg, for a photograph and biography, see this issue, p. 85.

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Dr. Fluitman is an Editor of the JOURNAL OF MICROELECTROMECHANICAL SYSTEMS.

Miko Elwenspoek, for a photograph and biography, see this issue, p. 67.