

MULTIPLE REFLECTIONS IN AN OPTICAL RETARDER INVESTIGATED BY SPECTROSCOPIC TRANSMISSION ELLIPSOMETRY

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The response of the ellipsometric parameters as a function of wavelength has been derived for an optical retarder. Experimental results in the wavelength region 400–1000 nm are given, these are in agreement with the derived formalism.

The theory of multiple reflections in an isotropic medium and the optics of retarders have been described in the previous literature. Holmes [1] described multiple reflection in a quarter wave plate, an anisotropic medium. The theory is illustrated with model calculations, where the thickness of the retarder is the variable. Automatized spectroscopic transmission ellipsometry however gives the opportunity to perform measurements which show the multiple reflection in a retarder as a function of wavelength. The complex relative amplitude transmission coefficient of a wave plate can be defined as

$$\tau = \tan \psi \exp(i\Delta), \quad (1)$$

where $\tan \psi$ and Δ are the relative amplitude transmission coefficient and the relative phase shift at transmission respectively. For a single transmission Δ is given by

$$\Delta = (\delta_o - \delta_e) = 2\pi d (n_o - n_e) / \lambda, \quad (2)$$

where δ is the phase shift and n the refraction index, the subscripts o and e refer to the ordinary and extra ordinary beam, d is the thickness of the transmission element and λ the applied wavelength. The absorption along the two axes of the retarder is assumed to be equal, therefore ψ should be $\pi/4$. However as described by Holmes (1964) multiple reflection should be taken into account.

For the transmission of a parallel light beam, wavelength λ , at normal incidence on an isotropic transmission element with reflection coefficient r one can derive

$$t = [(1 - R) \exp(-i\delta)] / [1 - R \exp(-i2\delta)], \quad (3)$$

where $R = r^2$ and δ is the phase shift at a single transmission. The complex relative amplitude transmission coefficient of a retarder is

$$\tau = \frac{t_o}{t_e} = \frac{(1 - R_o) \exp(-i\delta_o)}{1 - R_o \exp(-i2\delta_o)} \frac{1 - R_e \exp(-i2\delta_e)}{(1 - R_e) \exp(-i\delta_e)}. \quad (4)$$

The subscripts o and e have been introduced in eq. (2). The real part of τ , $\tan \psi$ in eq. (1) can be calculated by

$$\tan \psi = (\tau \tau^*)^{1/2} = \left(\frac{1 + [4R_e / (1 - R_e)^2] \sin^2 \delta_e}{1 + [4R_o / (1 - R_o)^2] \sin^2 \delta_o} \right)^{1/2}, \quad (5)$$

while Δ can be derived by combining eqs. (1) and (4),

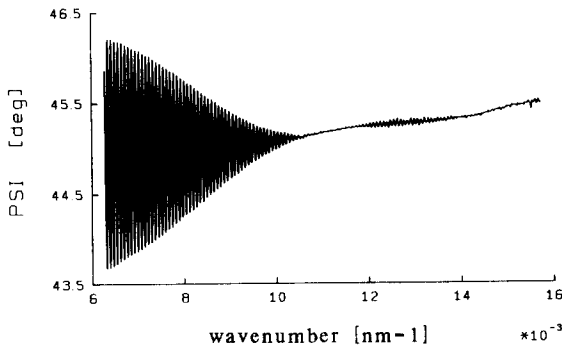


Fig. 1. The ellipsometric parameter ψ as a function of the wavenumber for a retarder.

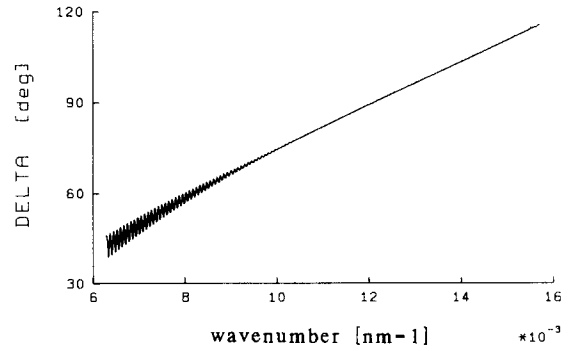


Fig. 2. The ellipsometric parameter Δ as a function of the wavenumber for a retarder.

$$\tau = \tan \psi \exp(i\Delta) = T_1 T_2 \exp[i(\delta_e - \delta_o)] , \tag{6}$$

where

$$T_1 = [1 - R_e \exp(-i2\delta_e)] / (1 - R_e) , \quad T_2 = (1 - R_o) / [1 - R_o \exp(-i2\delta_o)] . \tag{7}$$

By substituting the modules and argument of T_1 and T_2 in eq. (4), Δ can be calculated,

$$\Delta = (\delta_e - \delta_o) + \arctan \frac{R_e \sin 2\delta_e - R_o \sin 2\delta_o - R_e R_o \sin 2(\delta_e - \delta_o)}{(1 - R_e \cos 2\delta_e)(1 - R_o \cos 2\delta_o) + R_e R_o \sin 2\delta_e \sin 2\delta_o} . \tag{8}$$

In first order approximation, assuming that $R_e = R_o = R$, eqs. (5) and (8) are given by

$$\psi = \pi/4 + [R/(1 - R)^2] \sin 2(\delta_e - \delta_o) \sin 2(\delta_e + \delta_o) , \tag{9}$$

and

$$\Delta = (\delta_e - \delta_o) + R \sin(\delta_e - \delta_o) \sin(\delta_e + \delta_o) . \tag{10}$$

Because the phase shift δ can be written as a function of n ,

$$\delta = kdn , \tag{11}$$

where $k = 2\pi/\lambda_{\text{vacuum}}$ is the wavenumber, $\delta_e - \delta_o$ and $\delta_e + \delta_o$ can be replaced in eqs. (9) and (10) by

$$\delta_e - \delta_o = kd(n_e - n_o) , \tag{12}$$

and

$$\delta_e + \delta_o = kd(n_e + n_o) . \tag{13}$$

This means that a graphical representation of ψ and Δ as a function of k shows a fast and a slow variation. These variations are caused by the terms containing the sum and the difference in the refraction indices n_e and n_o .

The parameters Δ and ψ of a retarder (quarter wave plate at 546 nm) have been determined as a function of wavelength with a spectroscopic ellipsometer used in the transmission mode. The results are given in figs. 1 and 2. For a description of transmission ellipsometry one is referred to the review book of Azzam and Bashara [2]. The ellipsometer used is based on the principles described by Aspnes [3], while the automated instrument will be described elsewhere. The measurements have been performed in the wavelength region 400–1000 nm with an interval of 1 nm. The parameters Δ and ψ show a fast and a slow variation in accordance with eqs. (9) and (10).

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