Scientific and Technical Record

Spatial filtering in multichannel magnetoencephalography

P. Nicolas and M. Kouwenhoven*

INSERM U3, Hôpital de la Salpêtrière, 47 Bd. de l'Hôpital, 75651 Paris Cedex 13, France and *Twente University of Technology, Department of Technical Physics, 7500 AE Enschede, The Netherlands

Received October 1987; accepted July 1988

ABSTRACT

Partial differential equations in boundary-value problems have been studied in order to estimate the influence of several geometrical and physical parameters involved in the outward transmission of the brain's magnetic field. Explicit Green kernels are used to obtain integral forms of generalized solutions which can be deduced from each other, as expressed over concentric spherical surfaces. That leads to numerical applications dealing with the radial component of the magnetic field. From this study, a new spatial filtering is proposed as a possible improvement in two-dimensional magnetoencephalographic mapping using large multisensors.

Keywords: Magnetoencephalography, spatial filtering, partial differential equations, boundary-value problems

INTRODUCTION

This paper proceeds with two main purposes. The first is an attempt to conduct an extensive study of the general phenomena involved in the outward transmission of the magnetic activities of the brain. The second, a consequence of the first, is to establish a new data processing method.

Studies on biomagnetic activities of the brain (MEG) using monochannel SQUID magnetometers started in the seventies[1–5]. Since that time, following a growing interest in the related findings, considerable improvements have been made in the instrumentation. The most important of them is probably the advent of multichannel sensors, which allowed several simultaneous measurements to be taken from different brain locations[6–8]. Existing systems work with a few channels (between 3 and 8), but it is already feasible to construct wider systems[9], and it seems likely that up to 100 or more channels should be available in the future. Such a technological development would require parallel progress in signal processing to improve our knowledge of the brain sources.

Yet, in this domain we are limited by a well-known fundamental hindrance. This is the inverse source problem itself, which cannot be solved in general. Therefore, other approaches should be tried, for instance by considering some problems of 'reverse', or 'inward' transmission for the magnetic field. Such problems could be posed in terms of transmitting two-dimensional 'images', or field distributions, from one surface to another. Indeed, when many points are observed at the same time, an image processing problem is necessarily posed.

Correspondence and reprint requests to: Dr P. Nicolas

© 1989 Butterworth & Co (Publishers) Ltd
0141–5425/89/010079–08 $03.00

In the specific case of the MEG, the wide multichannel systems would replace the usual 'scalp mapping' by real-time two-dimensional images obtained over a large measurement surface area over the head. One major difficulty would be caused by the superimposition of several signals coming from different brain sources[10,11], because the resolving power of the sensors would be inadequate. As shown by Duret and Karp[12], to distinguish equivalent dipolar current sources situated at different respective locations with the same depth d, the minimal separation S of the dipoles should be about $S = 2\sqrt{d}$, which is very far on the brain scale. A convenient image processing method must be found in order to improve that resolution. This paper is an attempt to develop such a method as a result of a mathematical study based on an abstract formulation of the electromagnetic phenomena in their most general form.

Taking into account as many parameters as possible to obtain a well defined boundary-value problem[13,14], the field transmission is analysed in both the outward and inward directions. The reader should be familiar with generalized functions (or distributions[15]) for best understanding of the full text. The theoretical study starts from fundamental equations (Maxwell), which allow one to establish an explicit correspondence between fields over spherical surfaces. More precisely, the corresponding field values are calculated over the surface of the 'smallest' spherical domain surrounding the sources from the field values as detected over one spherical measurement surface. Theoretically, the signals calculated in this way are less affected than the raw data by the superimposition of fields coming simultaneously from different sources (including noise). In this way, computed spherical maps may be obtained whose spatial resolution should be better than those of the original mapping (on the measurement surface), because they are 'closer' to the sources.

Briefly, summarized, the method is a spatial filter using spatial averaging. One of its prominent advantages is that it requires no strong hypotheses concerning the sources. Nevertheless, in practice, the relevance of the results would closely depend on the applicability of the spherical model, and on the adequacy of the sensor design.

GENERAL EQUATIONS

Let **E**, **H** be respectively the electric and the magnetic vector fields, let us denote ε, μ, respectively the permittivity and the magnetic permeability, and let us assume the existence of a current distribution **J** and of a charge distribution ρ in some related part of the tridimensional space. We start from the system:

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\text{curl } \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \tag{2}$$

$$\text{div } \mathbf{B} = 0 \tag{3}$$

$$\text{div } \mathbf{D} = \rho \tag{4}$$

$$\mathbf{B} = \mu \mathbf{H} \tag{5}$$

$$\mathbf{D} = \varepsilon \mathbf{E} \tag{6}$$

The last two equations, (5) and (6), do not belong to the general theory. Their only meaning is as a linear approximation for the fields, when the coefficients ε and μ, are assumed to be scalar. The relations (1), . . . , (4) are the Maxwell system for any media where the above approximations are valid. From this background, if we assume that ε and μ are constant and uniform everywhere, then we obtain the so-called inhomogeneous wave equations for the fields whose sources are ρ, **J** (Δ denotes the laplacian ∇^2):

$$-\Delta \mathbf{E} + \varepsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu \frac{\partial \mathbf{J}}{\partial t} - \text{grad}\left[\frac{\rho}{\varepsilon}\right] \tag{7}$$

$$-\Delta \mathbf{H} + \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2} = \text{curl } \mathbf{J} \tag{8}$$

Remark 1. Let us recall that the only meaning of the laplacian of a vector is as a mnemonic for three coupled equations. From div $\mathbf{B} = \mu$ div $\mathbf{H} = 0$ (equations 3 and 5), it can be shown that equation (8) uncouples and reduces in rectangular coordinates to three separate equations, one for each rectangular component of **H** (reference 16). Such a reduction does not hold in spherical coordinates.

Within the head, the sources ρ and **J** are contained inside a compact domain Ω_0. Moreover, we must take into account the influence of the so-called volume-conduction phenomenon over some region Ω which contains Ω_0. More precisely, this region is conductive, with σ the conductivity. **J** has to be broken into one ohmic component, $\mathbf{J}_H$, which obeys the Ohm's law:

$$\mathbf{J}_H = \sigma \mathbf{E}$$

and one ionic component $\mathbf{J}_I$ which does not obey this law. Inside neural tissues, $\mathbf{J}_I$ denotes current distributions resulting from the addition of ionic flows at the site of neurological activities. They are most often called impressed currents, or primary currents, in contrast with $\mathbf{J}_H$, which denotes volume current distributions induced outside the sources[17,18]. As a realistic approximation, one may assume that $\mathbf{J}_H$ is negligible in comparison with $\mathbf{J}_I$ within the source domain Ω_0. This implies that the electrical conductivity of the extra-cellular medium does not play a major role in the brain's activities, which is obvious.

Conversely, the peri-encephalic media (brain's coverings: scalp, skull and meninges), do not contain any powerful generator. Their mean charge density is zero, but their electrical conductivity might be no longer negligible with regard to weak fields which could be affected by volume conduction at a distance from the sources. Then, inside the periencephalic domain $\Omega - \Omega_0$, the current distribution must be written:

$$\mathbf{J} \cong \mathbf{J}_H$$
$$= \sigma \mathbf{E}$$

This means only that **J** varies linearly with respect to **E**, inside the different layers surrounding the brain. The corresponding media having different conductivities, σ is discontinuous on every separating surface between any two adjacent layers inside $\Omega - \Omega_0$. This implies that secondary sources are induced on each of these surfaces. For this reason, the partial derivatives in equations (7) and (8) must be taken in the generalized sense. This allows us to represent the secondary sources in the form of Dirac functions[15].

In that sense:

$$\text{curl } \mathbf{J} = \text{curl}(\sigma \mathbf{E})$$
$$= \sigma \text{ curl } \mathbf{E} - \mathbf{E} \times \text{grad } \sigma$$

Hence then from equation (1), we obtain:

$$\text{curl } \mathbf{J} = -\sigma\mu \frac{\partial \mathbf{H}}{\partial t} - \mathbf{E} \times \text{grad } \sigma$$

and equation (8) turns on:

$$-\Delta \mathbf{H} + \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2} + \sigma\mu \frac{\partial \mathbf{H}}{\partial t} = \mathbf{E} \times \text{grad } \sigma \tag{8a}$$

Assuming that σ is uniform inside each layer L_k, $k = 1, \ldots, p$, separated by $p - 1$ surfaces S_k, this last equation may be broken formally into p homogeneous equations, with p boundary conditions as follows:

$$-\Delta \mathbf{H} + \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2} + \sigma\mu \frac{\partial \mathbf{H}}{\partial t} = 0 \text{ on } L_k$$

$$\mathbf{H}|_{S_{k-1}} = f_k \qquad k = 1, \ldots, p. \tag{8b}$$

where S_0 is the boundary of the source domain Ω_0.

The physical meaning of these formal boundary conditions is not obvious. It will be discussed and explained further, within the framework of a spherical model.

Remark 2. The wave equations (7), (8), (8a) and (8b) must be taken only as a consequence of the Maxwell system and of the linear approximations equations (5), (6) for the fields. However, there is no equivalence between these two systems: if equations (7), . . . , (8b) have one unique solution in the sense of some well-posed problem, then it also satisfies equations (1), . . . , (6) in a certain related sense, but without uniqueness.

HELMHOLTZ EQUATION

Equation (8b) is one special case of the telegraph equation. Following Remark 1 and for greatest clarity, it will be studied first for an abstract scalar function, which is most often denoted u. The possible significance of u will be explained later.

In some cases, time and space coordinates can be directly separated in equation (8b), which reduces to an eigenvalue. It implies that the solution takes the form $u(\mathbf{x}, t) = X(\mathbf{x})\,T(t)$. The field varies everywhere with the same time function T (standing wave).

Most often a reduction in Fourier components is necessary to separate time and space dependences. In this case, for each time harmonic component u_ω of u, dependent on the angular frequency ω, equation (8b) reduces to the well-known Helmholtz equation, which is also called the reduced wave equation:

$$-\Delta u_\omega + \lambda^2 u_\omega = 0 \text{ where } \lambda^2 = -\omega^2\varepsilon\mu - i\omega\mu\sigma,\quad i = [-1]^{1/2} \tag{9}$$

Under certain sufficient 'regular' conditions of λ and the related domain, it has been shown that equation (9) has one, and only one, regular solution (cf. in particular references 18 and 19). In the following text, such a solution will be represented outside a spherical domain containing the sources.

The variables x, y, z will belong to the tridimensional euclidean space $\mathscr{R}^3$, i.e.: $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3)$, $z = (z_1, z_2, z_3)$.

EXTERIOR HELMHOLTZ PROBLEM FOR A SPHERE

Assuming that the values of the solution u of equation (9) are given everywhere on a spherical surface S which surrounds the sources, the exterior Helmholtz problem deals with a possible representation of u, outside the domain with boundary S (*Figure 1*).

Hypotheses: Let us assume that:

1. Ω_0 = a sphere with boundary $\partial\Omega_0 = S$.
2. $\Omega = \mathscr{R}^3 - (\Omega_0 \cup S)$, exterior of the sphere Ω_0.
3. f_0 is a real-value continuous function, defined on S.

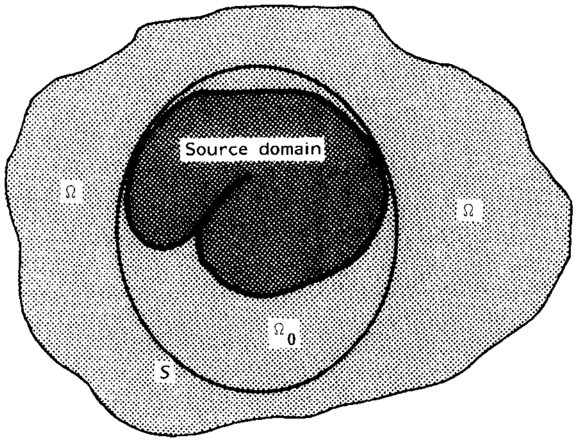

Figure 1. Geometrical model of the abstract Helmholtz exterior problem P1 (forward transmission problem). The source domain (unknown) is enclosed within a spherical surface S where the physical quantity of interest is known. The problem is calculating the same quantity of interest over any other larger concentric sphere. The exterior domain $\boldsymbol{\Omega}$ is unbounded.

Problem (P1)

To find a scalar function u, twice continuously differentiable on $\boldsymbol{\Omega}$, which is zero at infinity, and whose derivatives are zero at infinity, such as:

$$-\Delta u(x) + \lambda^2 u(x) = 0,\ x \in \Omega \tag{10}$$

$$u(x) = f_0(x),\ x \in S \tag{11}$$

$$\begin{cases} u(x) = O(|x|^{-1}) \\ \dfrac{\partial}{\partial |x|}\, u(x) - i\lambda u(x) = o(|x|^{-1}) \end{cases} \tag{12}$$

The radiation conditions (12) are well-known (Sommerfeld). Their physical meaning involves waves fading away to infinity. Problem (P1) is the so-called exterior Helmholtz problem for a sphere, with a Dirichlet boundary condition (11). Under the hypotheses 1, 2 and 3, such a problem has one and only one solution on $\Omega \cup S$ (References 20, 21, 22). That solution may be represented in using the Green kernel method as follows.

GREEN KERNELS

By definition, the Green kernel of problem (P1), also named Green function, G, depends on two variables y and x, and it obeys the system:

$$-\Delta_x G(y, x) + \lambda^2 G(y, x) = \delta_y(x),\ (y, x) \in (\Omega \times \Omega) \tag{13}$$

$$G(y, x) = 0,\ x \in S,\ y \in \Omega \cup S \tag{14}$$

$$\begin{cases} G(y, x) = O(|x|^{-1}) \\ \dfrac{\partial}{\partial |x|}\, G(y, x) - i\lambda G(y, x) = o(|x|^{-1}) \end{cases} \tag{15}$$

where Δ_x denotes the laplacian related to x. In equation (13), δ_y denotes the δ function (or Dirac distribution) concentrated at the point y in the sense of generalized functions, for fixed $y \in \Omega \cup S$.

It is clear from the above definition that the Green kernel depends only on the geometry of the problem and on the media whose characteristics are given in λ. It does not depend on the particular boundary condition (equation 11) of the Helmholtz problem (P1).

Furthermore, G can be written $G(x, y) = E(x, y) - h(x, y)$, where E is the elementary (standard) solution of equation (13). h obeys equation (10), with $h_y|_S = E_y|_S$ in such a way that G obeys the boundary condition in equation (14). This condition is necessary for solving the Helmholtz problem (P1) in using the Green's integral theorem[15] as will be shown later. In the particular case of the spherical problem (P1), E is well known and an analytic form of h was found[22], which allows us to write G in the the form of an elementary function:

$$G(x, y) = \frac{1}{4\pi}\left[\frac{e^{-i\lambda|x-y|}}{|x-y|} - \frac{R}{|y|}\,\frac{e^{-i\lambda\frac{|y|}{R}|x-y'|}}{|x-y'|}\right]$$

for every $(x, y) \in \Omega \times \Omega$, $x \neq y$, where y' is given by Kelvin inversion from an exterior to an interior surface:

$$y' = y\,\frac{R^2}{|y|^2},\ y \in \Omega \cup S$$

with

$$R = |x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

Hence, it is clear that the Green kernel is symmetric: $G(x, y) = G(y, x)$. Nevertheless, it must be emphasized that G is a generalized function, which explains why the above analytic form is not valid everywhere.

As y will be somehow fixed in the following text, G will be denoted G_y in such a way that $G_y(x) = G(y, x)$.

In that way, the following result gives an integral representation of the solution u of the Helmholtz problem (P1), deduced from its Green kernel G_y.

Result I: If u obeys the problem (P1), then it is given by:

$$u(y) = \int_S f_0(x)\,\frac{\partial}{\partial n_x}\,G_y(x)\mathrm{d}S(x),\ \forall y \in \Omega$$

where $\frac{\partial}{\partial n_x}$ is the normal outward derivative with respect to x at the surface S.

The verification follows from the definition of G and from the second Green identity (double partial integration, or Green's integral theorem in the generalized sense):

$$\int_\Omega (u\Delta G_y - \Delta u G_y)\mathrm{d}x = \int_S \left(u\,\frac{\partial}{\partial n_x}\,G_y - G_y\,\frac{\partial}{\partial n_x}\right)\mathrm{d}S(x)$$

taking into account that equation (10) for u is homogeneous.

$(\partial/\partial n_x)G_y$ is a generalized function whose support is $\Omega \cup S$, and $(\partial/\partial n_x)G_y|_S = \partial_y$, $y \in S$. This generalized function is often called the Poisson kernel. In using the above result I on the surface S, the result II was obtained[22]

Result II: The above representation of u may be continuously extended to S by:

$$\lim_{|y|\to R} u(y) = f_0(y)$$

which implies that u is continuous everywhere on $\Omega \cup S$, under the hypotheses of the problem (P1).

To apply Results I and II, we need the Poisson kernel, which may be obtained from the analytic form of the Green kernel as follows. The normal outward derivative at a spherical surface S is a partial derivative operator which may be written:

$$\frac{\partial}{\partial n} = -\frac{1}{R}\sum_{j=1}^{3} x_j\,\frac{\partial}{\partial x_j},\quad x \in S$$

As stated earlier, the analytic form of the Green kernel G takes form:

$$G_y(x) = E_y(x) - h_y(x)$$

with:

$$E_y(x) = \frac{e^{-i\lambda|x-y|}}{4\pi|x-y|}$$

$$h_y(x) = \frac{R}{|y|}\,\frac{e^{-i\lambda\frac{R}{|y|}|x-y'|}}{4\pi|x-y'|}$$

for every $(x, y) \in S \times (\Omega \cup S)$, $x \neq y$

From this we obtain

$$\begin{aligned}\frac{\partial G}{\partial n}(y, x) &= \frac{\partial}{\partial n_x}\,G(y, x)\\ &= \frac{1}{4\pi R}\left(\frac{|y|^2}{R^2} - 1\right)\\ &\quad\times\left[\frac{R^2}{|x-y|^3} - i\lambda\,\frac{R^2}{|x-y|^2}\right]e^{-i\lambda|x-y|}\\ &= \frac{|y|^2 - R^2}{4\pi R}\left(\frac{1}{|x-y|^3} - \frac{i\lambda}{|x-y|^2}\right)e^{-i\lambda|x-y|}\\ &= \frac{|y|^2 - |x|^2}{4\pi|x|}\left(\frac{1}{|x-y|^3} - \frac{i\lambda}{|x-y|^2}\right)e^{-i\lambda|x-y|}\end{aligned}$$

for every $(x, y) \in S \times (\Omega \cup S)$, $x \neq y$, $(|y| \geq |x| = R)$.

Let us now define a generalized function denoted

K, such as:

$$K(z, x) = \begin{cases} \dfrac{\partial G}{\partial n}(z, x), & \text{if } |x| < |z| \\ -\dfrac{|x|}{|z|}\dfrac{\partial G}{\partial n}(x, z), & \text{if } |x| > |z| \\ \delta_z(x), & \text{if } |x| = |z| \end{cases}$$

K is often called the Poisson kernel of the operator $(-\Delta + \lambda^2)$. It can be seen that $K(z, x) = K(x, z)$ for every $(x, z) \in (\Omega \times \Omega)$.

Because u is the unique solution of the exterior problem (sources inside S), its representation given by Results I and II, realizes a forward transmission which allows numerical calculations over any sphere containing S, concentric with S.

REVERSE REPRESENTATION (INWARD TRANSMISSION)

Let us recall that overall we are in search of a reverse representation of u. More precisely, this representation should realize an inward transmission from a spherical surface (where data are given) towards the smallest concentric sphere that contains the sources. For this purpose, let us consider the restriction of u, solution of the external problem (P1), to a ring domain D, surrounding the sources. This restriction u_D, whose support is D, obeys the Helmholtz equation inside D, with convenient boundary conditions. Thus it is a solution of the following problem.

Problem (P2): Let $\mathbf{\Omega}_1$ be a sphere concentric with $\mathbf{\Omega}_0$, such that $\mathbf{\Omega}_1 \supset \mathbf{\Omega}_0$ (*Figure 2*). The surfaces $\partial\mathbf{\Omega}_0$, $\partial\mathbf{\Omega}_1$ will be denoted S_0, S_1 respectively. The intermediate region situated between S_0, S_1 will be called D. To find u_D, twice continuously differentiable on D, such that

$$-\Delta u_D(x) + \lambda^2 u_D(x) = 0,\ x \in D \quad (16)$$

$$u_D(x) = f_0(x),\ x \in S_0 \quad (17)$$

$$u_D(x) = f_1(x),\ x \in S_1 \quad (18)$$

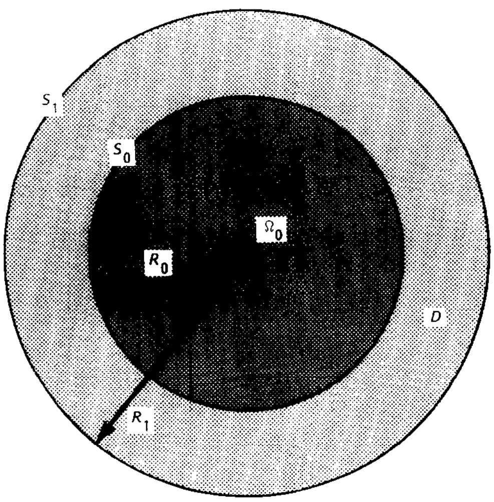

Figure 2 Geometrical model of the inward transmission (P2). The quantity of interest is 'measured' over an external spherical surface S_1. It must be calculated over the smallest concentric spherical surface S_0 which contains the source domain.

where f_0 and f_1 are continuous bounded functions, one of them 'given' on the boundary of D. The function u_D is unique because it is a restriction of the solution of the problem P1.

In practice, f_1 is known by means of a finite number of measurements. We must represent the unknown f_0, which is the subject of the following result.

Result III: Under the hypotheses of the problem (P2), $f_0(x)$ is given by:

$$f_0(x) = \int_{S_1} K(z, x) f_1(z) \mathrm{d}S_1(z) \quad (19)$$

Verification

Let r be such as $R_0 \leqslant r \leqslant R_1$ and let us consider the spherical surface S_r, concentric with S_1 and S_0. It is easy to see that K obeys the Chasles identity:

$$K(x, z) = \int_{S_r} K(y, z) K(x, y) \mathrm{d}S_r(y) \quad (20)$$

for every $x \in S_0$, $z \in D \cup S_0 (R_0 = |x| \leqslant |y| \leqslant R_1)$.

From this identity and the definition of K it follows:

$$\partial_z(x) = K(x, z) = \int_{S_r} K(y, z) K(x, y) \mathrm{d}S_r(y)$$

for every $r \leqslant R_1$.

Thus, applying the Fubini's theorem:

$$u_0(x) = \int_{S_1} K(z, x) u_1(z) \mathrm{d}S_1(z)$$

This solves the problem (P2) with $u_0 = f_0$, $u_1 = f_1$.

Remark 3. Result III gives a representation of the 'signal' u_0 on the inner surface S_0 as a function of the 'signal', u_1 which is measured on the outer surface S_1. It must be emphasized that u_0 is not the solution of some inverse problem for the sources, because no source parameters are known.

The model in problem (P2) is related with only one homogeneous medium (inside the domain D) between two spherical surfaces. Thus, only one value of the parameter λ (constant and uniform everywhere on D) is used. As already pointed out, this is not true within the layers surrounding the brain, because the conductivity σ is discontinuous.

A more realistic model should assume that D is divided into p different concentric ring shaped layers D_j, with parameters λ_j. Nevertheless in such a model, in the general case, the above theory should be no longer valid, because (in particular) Dirac functions should appear at the right-hand side of the Helmholtz equation. In the next paragraph, it will be shown under which conditions and for which physical variable Result III may be applied.

PHYSICAL INTERPRETATION

It will be verified further that λ may be neglected within the low-frequency range of interest (< 1000

Hz), which involves that the Poisson kernel K, as an approximation in the reverse problem (P2) would reduce to:

$$K_a(x, y) = \frac{R_1^2 - R_0^2}{4\pi R_0 |x - y|^3}$$

This last kernel is well-known, because it corresponds with the Laplace equation. Such an approximation implies that the quasi-static approximation may be applied. Within this framework, let us assume now that the total magnetic field $\mathbf{H}$ obeys the vector equation:

$$-\Delta \mathbf{H} = \sum_j \mathbf{c}_j \delta_j \quad (21)$$

where δ_j is the Dirac function extended over the spherical surface S_j, and $\mathbf{c}_j$ is a vector associated with the jth discontinuity of σ. As already stated (Remark 1), the vector equation (21) is a mnemonic for three coupled equations in arbitrary coordinates. In particular, it must be emphasized that in spherical coordinates (r, θ, ϕ), we unfortunately always have:

$$[\Delta \mathbf{H}]_r \neq \Delta H_r$$

where the subscript r denotes the radial component of the vectors.

Nevertheless, if we use the identity (for any vector $\mathbf{U}$):

$\nabla \times (\nabla \times \mathbf{U}) = \nabla\nabla . \mathbf{U} - \Delta \mathbf{U}$ where $\Delta = \nabla^2$ as above,

and if we take into account that div $\mathbf{H} = 0$ then it can be shown that (the full verification is tedious):

$$[\Delta \mathbf{H}]_r = \frac{1}{r} \Delta[rH_r] \quad (22)$$

Let us now consider the scalar function $w = rH_r$. It is obvious that, within the spherical layers model, w is twice continuously differentiable everywhere outside the source domain (which contains the origin). Moreover, because of the behaviour of H_r at infinity (dropping off at least as fast as r^{-2}), w satisfies the Sommerfeld condition and, from equations (21) and (22), the Laplace equation $\Delta w = 0$, does hold outside the source domain Ω_0, whereas w does exist on S_0. Thus Results I, II and III, apply to w in choosing $\lambda = 0$, and its restriction $w_D = w|_D$ satisfies:

$$\Delta w_D(x) = 0,\ x \in D \quad (23)$$

$$w_D(x) = f_0(x),\ x \in S_0 \quad (24)$$

$$w_D(x) = f_1(x),\ x \in S_1 \quad (25)$$

where f_1 is given on the external boundary of D, and f_0 can be calculated by:

$$f_0(x) = \int_{S_1} K_a(z, x) f_1(z) \mathrm{d}S_1(z)$$

Finally, one may easily obtain true values of H_r, because R_1, R_0 are known, by computing:

$$H_{R_0}(x) = \frac{R_1}{R_0} \int_{S_1} K_a(z, x) H_{R_1}(z) \mathrm{d}S_1(z)$$

Practically the data $w_1 = f_1$ come from radial mean field measurements over S_1. The choice of R_0 will depend on the anatomy of the subject (skull shape), whereas R_1 (radius of the sphere measurement) is fixed in every particular multichannel system. Most often the range of $R_1 - R_0$ will be about 2.5–3.5 cm.

NUMERICAL APPLICATIONS

In the next paragraphs, the following notations will be used: $w = ur$; $w_0 = u_0 R_0$; $w_1 = u_1 R_1$.

STABILITY WITH RESPECT TO THE MEASUREMENT ERRORS

In spite of the noise which is always added to the data, the computing algorithm always has to be stable. In other words, the final error on the calculated solution u_0 must be linearly bounded[16,23] in function of the measurement errors. For that purpose, let us assume that the measured signal $\hat{u}_1$ does contain an additive noise n which is superimposed to the signal of interest u_1:

$$\hat{u}_1 = u_1 + n$$

The inward transmission turns on:

$$\hat{u}_0(x) = \frac{R_1}{R_0} \int_{S_1} K_a(z, x) \hat{u}_1(z) \mathrm{d}S_1(z).$$

If we define the essential norm $\| \times \|$ by $\| \mathrm{f}(x) \| = \sup_x |\mathrm{f}(x)|$ for every function f continuous on $\mathbf{\Omega}$, then:

$$\| u_0 - \hat{u}_0 \| \leqslant 4\pi R_1^2 \frac{R_1}{R_0} \| K_a \| \| n \|$$

where $4\pi R_1^2$ is the surface area of S_1.

Since the norm of the fields is decreasing when the distance increases, we also have $\| u_0 \| > \| u_1 \|$. Then:

$$\frac{\| u_0 - \hat{u}_0 \|}{\| u_0 \|} \leqslant 4\pi R_1^2 \| K_a \| \frac{\| n \|}{\| u_1 \|}$$

This means that the global relative error in the signal u_0 is bounded by a constant which is proportional to the inverse of the signal-to-noise ratio of the measured data.

NUMERICAL ESTIMATES

Let us now study the influence of the parameter $\lambda = [-\omega^2 \varepsilon \mu - i\omega\mu\sigma]^{1/2}$.

The following estimates are at our disposal:

$\varepsilon \leqslant 84 \varepsilon_0 = 7.5 \times 10^{-10}$ (value in water)

$\mu \approx \mu_0 = 4\pi 10^{-7}$ (value in vacuum)

$\sigma \leqslant 0.33$ S m^{-1} (brain/scalp)

$\sigma \approx 0.005$ S m^{-1} (skull)

$\omega \leqslant 200\pi$ (frequencies $\leqslant$ 100 Hz)

It is obvious that the term $i\omega\mu\sigma$ is prominent in λ.

Moreover, we have always $|\lambda| \leqslant 0.016$, so that if

$$|\lambda| \ll |x-y|^{-1} \qquad (26)$$

then λ may be neglected in $K(x, y)$, which becomes:

$$K_a(x, y) = \frac{R_1^2 - R_0^2}{4\pi R_0 |x-y|^3}$$

for each, $x \in S_0$, $y \in S_1$. K_a is an approximated kernel, which coincides with the Poisson spherical kernel for the Laplace equation (quasi-static approximation).

Obviously, the distance $|x-y|$ is smaller than $2R_1$, the diameter of the actual measurement surface, which is at most 0.3 m. It implies that the condition (26) is always satisfied. Thus as long as we are only interested in very low frequencies, the above approximation K_a is valid.

DISCRETIZATION

Because the signal u_1 is known at only a few points, we need a discrete numerical version of the inward transmission. Let us suppose that n measurement locations are available, each of them indexed by j. Then each location j corresponds with a certain surface A_j, since each signal which is actually measured is the mean field detected by the gradiometer (the flux divided by the effective surface area of the gradiometer sensing coils). Thus we obtain an approximation $\bar{u}_0$ of u_0 as follows (standard method):

$$\bar{u}_0(x) = \sum_{j=1}^{n} g_j(x) u_{1j}$$

where $\bar{u}_0(x)$ is the field to be determined at the point $x \in S_0$, and u_{1j} is the mean field (normalized flux) in the area A_j:

$$u_{1j} = \frac{1}{A_j} \int_{A_j} u_1(y) \mathrm{d}A_j(y)$$

and g_j is the corresponding coefficient, or weight coming from the Poisson kernel:

$$g_j(x) = \int_{A_j} K_a(x, y) \mathrm{d}A_j(y)$$

As a rough approximation, one often takes g_j such that: $g_j(x) = K_a(x, y_j)A_j$. Theoretically, the approximation $\bar{u}_0$ obtained in that way is valid only if the surfaces A_j are disjoint, such as $\cap A_j$ is empty and $\cup A_j = S_1$. (The full sphere S_1 should be overlapped). In practice these conditions cannot be satisfied because the gradiometer coils are circular, and the grid measurement usually contains many empty areas (*Figure 3*). This does not critically affect the actual values u_{1j}, because they are normalized flux (mean field). Only the weight g_j has to be overestimated, in taking A_j larger than the sensing coils area, such that $\cup A_j = S_1$ for the computation.

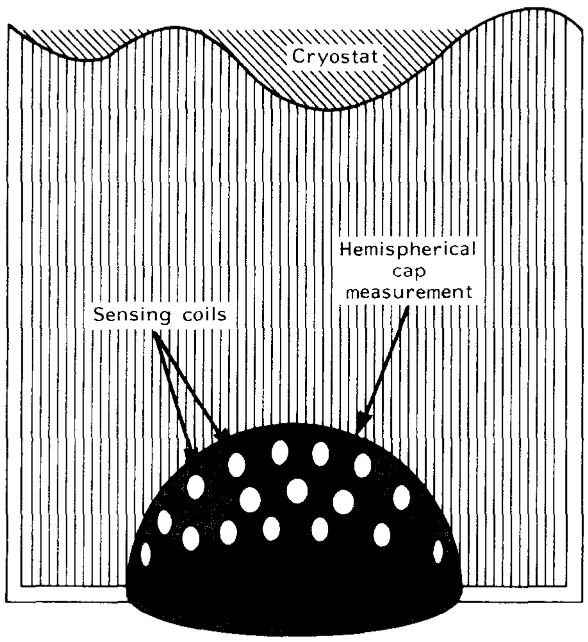

Figure 3 Diagram of a MEG multisensosr system tail with a hemispherical cap measurement. The pick-up coils of the flux transformers are tangentially distributed over this cap.

Another obvious theoretical difficulty comes from the fact that the full actual surface measurement itself is (at most) a hemispherical cap (*Figure 3*). In other words, the mean field values $\{u_{1j}\}$ cannot be measured over a full sphere surrounding the head. However, in practice, it is known that the signals of interest are most often sharply focused, decreasing rapidly at a distance from their extrema. This means that their amplitudes should be negligible outside the hemispherical cap measurement. This hypothesis may be easily verified. It is sufficient to justify the application of the above approximated formulae.

CONCLUSION

In order to get a general overview of the phenomena that are involved in the field transmission, our theoretical model was based on the reduced wave equation in its most complete form: $-\Delta v + \lambda^2 v = f$ with $\lambda = (-\omega^2 \varepsilon\mu - i\omega\mu\sigma)^{1/2}$

Using the Poisson kernel deduced from the corresponding Green function, surface integrals are solutions of both the forward and the inward problems for any sphere surrounding the sources of the fields. Because of the spherical geometry of these problems, there is an explicit form (using elementary functions) of the Green kernel, which is not the general case. Indeed, such an explicit form is unknown in non-spherical domains, which most often prevents its application.

Nevertheless we must recall that the actual geometry of the different layers which surround the brain is not spherical. It implies that the secondary sources that are induced by the discontinuities in conductivity over the separating surfaces might have an influence on the radial component of the field as measured over a spherical surface. This influence was not estimated numerically, but it could be non-negligible for large deviations of the actual layer surfaces from the spherical geometry.

The only specific relevance of the Helmholtz Poisson kernel is as a full theoretical representation of the behaviour of the field with respect to different physical and geometrical parameters. Yet, for the low-frequency phenomena under consideration, the Laplace Poisson kernel which corresponds to the quasi-static approximation ($\lambda = 0$) is sufficient in practice. Within the framework of a spherical model and within the frequency range of interest, this means that there is no significant contribution of volume currents on the radial component of the magnetic field (no measurable attenuation or distortion from conductivity outside the sources).

From a strict theoretical point of view, the data processing method which is so proposed is valid only for the radial magnetic field. However, the actual measurements are averages of this field over the gradiometer coils, which underestimate the physical quantity of interest (i.e. the field at the centre of the pickup coil). Moreover, that underestimate depends on the position the gradiometer. This yields an aberration, or apparent displacement of both the field extrema and of the isofield lines, which results in computation errors. Nevertheless, in principle, it is possible to compensate for such errors[24].

In summary, the Poisson kernel method offers the following possibilities:

1. Giving an explicit representation of the role played by the leading parameters in the field transmission.
2. Improving the sharpness and signal-to-noise ratio of the raw maps, in computing maps 'closer' to the sources, which could be of major interest to increase the resolution of the instruments.

Lastly, and in spite of its limitations, the Poisson kernel method does offer two practical advantages. It is easy to apply on-line and it does not require large expensive and complicated computations. Further studies should be undertaken to find the best compromise between cost and efficiency in both the instrumentation and the data processing methods.

REFERENCES

1. Cohen D. Magnetoencephalography: detection of the brain's electrical activity with a superconducting magnetometer. *Science* 1972; **175**: 664–6.
2. Brenner D. Williamson SJ. Kaufman L. Visually evoked magnetic fields of the human brain. *Science* 1975; **190**: 480–2.
3. Hughes JR, Magnetoencephalography. *Am J EEG Technol* 1978; **18**: 9–18.
4. Zimmerman JE. SQUID instruments and shielding for low-level magnetic measurements. *J Appl Phys* 1977; **48**: 702.
5. Reite M, Zimmerman JE. Magnetic phenomena of the central nervous system. *Ann Rev Biophys Bioeng* 1978; **7**: 167–88.
6. Ilmoniemi RJ, Hari R, Reinikainen K. A four-channel SQUID magnetometer for brain research. *EEG clin Neurophysiol* 1984; **58**: 467–73.
7. Williamson SJ, Kaufman L. Frontiers in the new science of biomagnetism. In Weinberg H, Stroink G, Katila T, eds. *Biomagnetism: Application and Theory*, New York: Pergamon, 1985: 471–90.
8. Romani R, Leoni R. Multichannel instrumentation for biomagnetism. In Hahlbohm HD, Lubbig H, eds. *SQUID'85: Superconducting Interference Devices*. Berlin: Walter de Gruyter, 1985: 919.
9. Kaufmann L, Williamson SJ. Recent developments in neuromagnetism. In *Proc. 3rd Int Evoked Potentials Symp, West Berlin*. 1986, 1–13.
10. Nunez PL. The brain's magnetic field: some effects of multiple sources on localization methods. *EEG clin Neurophysiol* 1986; **63**: 75–82.
11. Okada YC. Discrimination of localized and distributed current dipole sources and localized single and multiple sources, In Weinberg H, Stroink G, Katila T, eds *Biomagnetism: Application and Theory*, New York: Pergamon, 1985: 266–72.
12. Duret D., Karp P., Figure of merit and spatial resolution of superconducting flux transformers. *J Appl Phys* 1984; **56**: 1762–8.
13. Angel E. Inverse boundary-value problems. Elliptic equations. *J. Math. Anal. Appl. (USA)* 1970; **30**:, 86–98.
14. Burton AJ. The application of integral equation methods to the numerical solution of some exterior boundary-value problems in discussion on numerical analysis of partial differential equations, *Proc. R. Soc. A (GB) (PRLAAZ)* 1971, **323** (1553): 201–10.
15. Yosida K. *Functional Analysis*. Berlin: Springer, 1966.
16. Eyges L. *The Classical Electromagnetic Field*, Reading MA. Addison-Wesley, 1972.
17. Plonsey R. *Bioelectric Phenomena*, New York: McGraw-Hill, 1969.
18. Hari R, Ilmonieni RJ. Cerebral magnetic fields, CRC Critical Rev. *Biomed Eng* 1986; **14**: 93–126.
19. Uspenskii AB, Fedorov VV. Experiment planning in inverse problems of mathematical physics. *Kibernetika*, 1974; **10**: (in Russian); *Trans Cybernetics* 1974; **10**: 700–7 (in English)
20. Vainberg BR. Principles of radiation, limiting absorption and amplitude in the general theory of partial differential equations. *Russian math surveys* 1966; **21**: 115–193.
21. Lax PD, Phillips RS. *Scattering Theory*, New York: Academic Press, 1967.
22. Nicolas P. Transmission problem applied to the electrical activities of the brain. *Rev Cethedec* 1979; **59**: 37–121 (in French).
23. Tikhonov AN. On the stability of inverse problems, *Proc Acad Sci USSR* 1943; **39**: 176–9.
24. Nicolas P, Kouwenhoven M, Compensation of the gradiometric effects in Magnetocephalography. *World Congress on Medical Physics and Biomedical Engineering (Works in Progress Session)*: San Antonio Texas, August 1988.