Analysis of Flatness Measurement and Form Stability of a Granite Surface Plate

ir. W. de Bruin (2) and ir. J. Meijer, Twente University of Technology, Enschede/the Netherlands — Submitted by prof. ir. R. van Hasselt (1)

1. INTRODUCTION

Since 1974 experience has been gained with methods of flatness measurement and evaluation of the measuring results by means of a specially developed computer program. The merits of the applied method are described in more detail in [12].

In short: every straightness measuring method, applied on a grid pattern of intersecting straight lines on the surface, may be used. In a fair competition two measuring methods have been investigated further: a member of the "direct" method, making use of well calibrated and lapped straightedges in combination with an also straightedge method, making use of an electronic level indicator in combination with an electronic displacement transducer, measuring height differences of pairs of gridpoints.

Both methods have their specific pros and cons. The systematic errors, especially of the straightedge method, are reduced so that at the moment the results of both measuring methods are comparable within the regions of uncertainty. Nevertheless the preference is going to the second method because it is easier to handle, faster and more accurate.

The results of straightness deviations on the surfaces of a grid pattern on the surface are used in the computer program, resulting in a measuring report mentioning:

- height deviations in the measured grid points with respect to a reference plane,
- the standard deviation of the flatness deviations which can be used to calculate the confidence limits,
- a map of the surface with contour lines of certain height steps, found by spline-interpolation of heights within a finer grid pattern.

2. THE NEED OF QUANTITATIVE COMPARISON OF SHAPE ALTERATIONS

The term long-term investigation of the form stability of a granite surface plate just after its "birth", which has already been announced in [2] was only possible in the beginning by comparison of the contour plane and the plotting of changes of height values of (certain) gridpoints. Rather big changes could be detected in this way, certainly in the beginning of lifetime of the surface plate (fig.7 and 8 in [2]).

Now we believe to have found a solution for this problem by separating the shape of the surface into a geometrical part and a random part, so that characteristic parameters to the surface shape can be adjudged.

3. CURVATURE DUE TO SPHERICITY

The height difference \( s_{ij} \) of a point \( P_{ij} \) of a real sphere \( S \) with respect to a reference plane \( Q \) (fig.1) can be expressed as:

\[
s_{ij} = k( x_i^2 + y_i^2) + k_{R_{ij}} \]

in which \( k \) is the curvature in rad/m (\( k \) is thus the reciprocal of the radius of curvature in m/rad).

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\( k_{R_{ij}} \) is a factor which is only dependent on the co-ordinates of the point \( P_{ij} \). In fig.1 the impression may be raised that the centre of curvature must be on the Z-axis underneath \( Q \) (\( z = 0 \)). However, by shifting the reference plane \( Q \) any position with respect to the measured surface \( S \) is possible.

4. CURVATURE DUE TO REAL TORSION

Torsion can be expressed by means of a free vector \( T \) which is supposed to lie in \( Q \) and which has been placed in fig.2 in \( O \) for convenience sake. Not only the magnitude of \( T \), but also its direction with respect to the Z-axis is of importance in the next. Although \( T \) may also have a component along the Z-axis this cannot give rise to height differences with respect to the XOY-plane and it can be ignored furthermore.

If the torsion per unit of length is constant \( T \) (rad/m) a point \( P_{ij} \) encounters a height deviation \( t_{ij} \) due to torsion only, which obeys:

\[
t_{ij} = \frac{x_i T \cos \alpha - y_i T \sin \alpha}{z_j} = \frac{|x_j T \cos \alpha - y_j T \sin \alpha|}{z_i}
\]

The factor \( T \) is not only dependent on the co-ordinates of \( P_{ij} \), but also as could be expected, on the direction of \( T \).

5. SUPERPOSITION OF SPHERICITY AND TORSION

In [2] and [3] it has been explained that the measuring procedure results in finding a flat reference plane \( R \) so that from that reference the height \( h_{ij} \) of a point \( P_{ij} \) in the measured surface could be calculated (fig.3).

Next step up to now was to define height difference \( d_{ij} \) between a flat regression plane \( Q \) and the measured surface in \( P_{ij} \) by:

\[
d_{ij} = h_{ij} - \left( a + b x_j + c y_j \right)
\]

By minimising \( d_{ij}^2 \) it was possible to find the position of that regression plane \( Q (a, b, c) \) from that the positive and

Fig.1 Height difference due to the shape of a sphere S

* A hint was given in correspondence by Mr. Kilpi, former CIRP-member of Finland.
In any real twisted surface two directions of extreme curvature can be indicated:
- maximum curvature (symbol $c_1$) in the direction $\gamma_1 = \phi + 45^\circ$
- minimum curvature (symbol $m$) in the direction $\gamma_2 = \phi - 45^\circ$

These so-called main directions are under directions of $\pm 45^\circ$ with the $r$-vector.
Moreover there are two directions where straight lines in the surface may be expected ($b_{t, r} = 0$).
In fig. 6 these lines are $a$ and $b$ at $\gamma_1 = 0$ and $\gamma_2 = 0$. They are indicated with the symbol $-$. 

Fig. 3 Defining form deviations with respect to a flat regression plane
From now on the height difference $d_{ij}$ may be defined with respect to a non-flat regression plane $Q$, which may be the superposition of $z$-flat, a real sphere and a real twisted component:
$$d_{ij} = b_{ij} - [a + j\lambda + i\delta + k\theta + \tau T_{ij}]$$

In order $d_{ij}$ is minimum:
$$\frac{\partial^2}{\partial i^2} + \frac{\partial^2}{\partial j^2} + \frac{\partial^2}{\partial k^2} = 0.$$

These 5 equations make it possible to solve $a$, $\lambda$, $\delta$, $k$, and $\tau$.

If flatness has been measured in $p$ points of the gridplane the form noise $R$ may be defined as:
$$R = \frac{1}{p} \sum_{ij} a_{ij}.\text{sign}.$$

One differentiation $\frac{\partial^2}{\partial i^2} + \frac{\partial^2}{\partial j^2} + \frac{\partial^2}{\partial k^2}$ is missing. This is rather a cumbersome procedure so that in the program it has been replaced by a calculation of $R = f(s)$ and finding that $s$ where $R$ is a minimum.

Fig. 4 At the definition of the dimensionless bowrise $b$
Although real sphericity $k$ and real torsion $t$ are expressed in the unit rad/m so that they are independent of the magnitude of the measured surface, preference is going to dimensionless parameters as a measure for the curvature due to sphericity and torsion.
In fig. 6 a dimensionless bowrise $b$ may be defined by $b = \frac{t}{k}.$

The dimensionless bowrise $b^*_t$ due to true sphericity is thus:
$$b^*_t = \frac{b}{k}.\lambda.$$

Finding $k$ in the computer program in $\text{rad/m}$ the unit of $b^*_t$ is thus $\mu \text{m/m}.$

Fig. 5 A surface with true torsion
True torsion results in a hyperbolic paraboloid or a "saddle"-surface, containing two families of straight lines (fig. 5).

The extreme bowrise $t = \frac{\pi}{2}$, where $t = \pi, r^2 = \frac{\pi^2}{4}$.

The dimensionless bowrise $b^*_t$, due to true torsion is thus:
$$b^*_t = \frac{b}{k}.\lambda.$$

Here, also, finding $\lambda$ from the computer program in $\text{rad/m}$, the unit of $b^*_t$ is $\mu \text{m/m}.$

A simple example of a map of a surface with true torsion is shown in fig. 6.

The torsion-bowrise $b^*_t$ in any direction with the horizontal axis may be derived from:
$$b^*_t = b_t\sin 2(\phi - \delta).$$

In any real twisted surface two directions of extreme curvature can be indicated:
- maximum curvature (symbol $c_1$) in the direction $\gamma_1 = \phi + 45^\circ$
- minimum curvature (symbol $m$) in the direction $\gamma_2 = \phi - 45^\circ$
These so-called main directions are under directions of $\pm 45^\circ$ with the $r$-vector.
Moreover there are two directions where straight lines in the surface may be expected ($b_{t, r} = 0$).
In fig. 6 these lines are $a$ and $b$ at $\gamma_1 = 0$ and $\gamma_2 = 0$. They are indicated with the symbol $-$. 

Fig. 6 Map of a surface with true torsion
If true sphericity and true torsion are superimposed (fig. 7) the total dimensionless bowrise $b^*_r$ in any direction $\gamma$ with the horizontal axis equals:
$$b^*_r = b_t + b_t\sin 2(\phi - \delta).$$

Fig. 7 Map of a surface with true sphericity and torsion
Now the maximum curvature may be expected in the direction $\gamma_1 = \phi + 45^\circ$. sign $b_t$ and the minimum curvature in the direction $\gamma_2 = \phi - 45^\circ$. sign $b_t$.

Here sign $b_t$ has been added in the formulae, because in the computer program the direction of the $r$-vector is always between $0^\circ$ and $90^\circ$. But due to the kind of torsion in the surface $b_t$ may be positive or negative, making $b_t$ positive or negative.

If $|\lambda| < 1$ the surface has two directions of straight lines, because the saddle-shape overrules the sphericity. They fulfill:
$$i_1 = i + j [\arcsin(-b_t/b^*_t)]$$
$$i_2 = i + j [180^\circ - \arcsin(-b_t/b^*_t)]$$

If $|b_t| > b^*_t$ the surface has a cylindrical shape with one axis $x = i$.

If $|b_t| > b^*_t$ the surface is a part of:
- an ellipsoid ($b_t = 0$),
- or a sphere ($b_t = 0$).

So at the end of the calculation by the computer not only the height deviations $d_{ij}$ with respect to a flat regression plane are mentioned, but also:
- the dimensionless bowise parameters $b_s$ and $b_t$,
- the direction $\phi$ of the torsionvector $z$ (if any),
- the restnoise-value $R$,
- depending on the values of $b_s$ and $b_t$ an indication whether the surface is mainly a saddle-surface, a cylinder, an ellipsoid, a sphere or random.

In fact the parameters $b_s$, $b_t$, $\phi$ and $R$ are sufficient to characterize the shape of the measured surface. The above-mentioned flat regression plane can be considered as a sphere with curvature $b_s = 0$.

6.1 The "childhood"-period
After more than two months of measuring every 3 day (measurements nr.1-15) and once a week (nr.16-20) it was thought on the quasi-stability of the heightvalues and the contournaps that the most spectacular changes of form were behind us. Re-calcualting the data of measurements on punch cards with the aid of the extended computerprogram it is now possible to plot the $b_s$-parameter in a sequence of measurement (fig.8), whereas fig.9 shows the $b_t$-parameter and the angle $\phi$ of the torsion-vector in that same sequence.

Fig.8 Alterations of the $b_s$-parameter in course of time.

6. LONG-TERM INVESTIGATION OF A GRANITE SURFACE PLATE
The investigation into the form stability of a "just born" granite surface plate ($630 \times 400 \times 80$ mm$^3$) started at the end of 1976. It was inspired on some vague doubts about the stability of this kind of material, caused by:
- measurements on several plates of different origins of material,
- experience of form changes of optical glass material, which is being processed in about the same way and the chemical composition of which has similarity with that of granite.

Measurements nr.19 and 20 were based on the steady state at illumination of the upper surface with a normal 40 W-filament lamp at 1 m height above the centre of the plate. Between 19 and 20 the bulb was turned over $180^\circ$ along its vertical axis in order to eliminate the unsymmetrical Lux-profile on the surface.

Surprising are:
- the decreasing trend of $b_s$ which was not ended at all after measurement nr.18,
- the more or less "constant" level of $b_t$-values and $\phi$-values in that period.

6.2 Experiments with illumination
Measurements nr.19 and 20 were based on the steady state at illumination of the upper surface with a normal 40 W-filament lamp at 1 m height above the centre of the plate.
In measurement nr.21 and 23 a 40 W-fluorescent lamp was mounted at 1 m height and parallel to one diagonal NK—SE, whereas measurement nr.22 belonged to the position of that lamp along the other diagonal direction SW—NE. These measurements only result in a change of sphericity (fig.8). The rise of about 2 μm/m can fully be derived from the thickness of the plate (50 mm), the coefficient of linear expansion (γ = 5.6·10⁻⁶ K⁻¹) and the temperature difference between upper and lower surface of only ±0.25 K. The same experience as to sign and magnitude of the effect is being confirmed in [3;4].

5.3 Experiments with humidity

For some reason it was decided to increase the relative humidity within the aluminium tent around the surface plate up to 80 & 85%. Although there were changes in the "character" of the countourmaps of those measurements (nr.24—31) the new parameter b₄, shows in fig.8 that these phenomena were more on account of the original "trend" than on account of humidity!

Nevertheless it was decided to go to the utmost: putting wet towels on the upper surface during 24 hours and then to measure immediately after removal.

Fig.10 shows the result of this measurement nr.32 as a contourmap. Effects of curvature can better be valued from fig.8 and fig.9. The temperature of the upper surface was just before the measurement 19.8°C and afterwards 19.4°C: a normal rise due to the presence of the observer in the tent and some indirect illumination. The general temperature in the tent rose from 17.8°C till 19.0°C. Relative humidity was in the beginning 83%, at last 78%.

From these data it can be derived that during the measurement the dewpoint was probably at 15 ± 2°C. Evaporation of a waterfilm on the surface would make the upper surface cooler than the lower surface. This would tend to a concave shape. However water in the micro-cracks of the finishing operation of the upper surface and in between crystal grains at the surface causes internal compressive stresses which decrease at lower levels under the upper surface. This effect (also demonstrated in optical glass material) causes thus a convex upper surface and is to our opinion mainly responsible for the shape of contourmap (fig.8). The same experience as to sign and magnitude of the effect is being confirmed in [3;4].

5.4 The second drying period

Because it was thought that no further measurements of interest measurement nr.45 was done in unloaded state of the plate after measurement nr.44 followed immediately afterwards putting a constant "point"-weight of 18.2 kg (a cylindrical square, 2 x 50 mm, height 400 mm, supporting B = 30—70 mm) in the centre of the plate in order to demonstrate the formdeviation due to this normal attribute on a surface plate. This weight was only for one hour on the table. Because this measurement nr.46 is not of interest in the scope of this paper the values of parameters have been omitted in the graphs.

In the beginning of 1979 a constant weight of the same order, but on a larger supportarea was allowed on the plate during a week or two.

In spite of waiting more than 4 month (the plate strictly in unloaded condition) measurement nr.47 showed a quite strange contourmap notwithstanding the normal measuring conditions. Both curvatures due to real sphericity b₃ and due to real torsion b₄ gave quite unnormal values. The angle ϕ changed too.

Nevertheless measurement nr.48, after another 35 weeks of unloaded state of the plate, promises a recovering to the original values b₃, b₄ and ϕ.

Some measurements after composing this paper will confirm this supposition or upset it.

7. THE REMAINING PARAMETERS CHARACTERIZING THE FLATNESSMEASUREMENT

Up to now nothing has been mentioned about the last characterizing parameter of a flatness measurement: the restnoise R. This parameter has been plotted as a function of the sequence of measurements in fig.11.

Up to and including measurement nr.43 this value is surprisingly constant at R = 0.12 ± 0.03 mm. In between that concave shape (nr.38) there is more scatter in the R-values. The reason that cannot be explained.

Finally a measure for the accuracy of each flatness measurement is the standard deviation σ based on the mean value of flatness deviations per gridpoint over the whole gridpattern. Up to and including measurement nr.43 this σ-value is surprisingly constant at a level of σ = 0.12 ± 0.02 mm. In between that measurement and the next one 43 for some reason a pair of permanent magnets has been placed beside the Talyvel-unit of the measuring system and only for about half an hour. Since than we never have succeeded in getting the accuracy at the original level.

8. OTHER EXTENSION OF THE COMPUTERPROGRAM

A simple question of the Instituut voor Kernfysisch Onderzoek (I.K.O.) in Amsterdam to measure for them at the manufacturer's in Germany dipole-sets for an electron spectrometer to be built.
These pairs of magnets of special soft-iron material have to generate a very strong but homogeneous magnetic field within the gap between the parallel surfaces. On the drawing flatness was tolerated within 10 μm per surface. Last finishing operation was surface grinding.

First impulse was to find a suitable grid pattern that best fitted into the circumference of a surface. Especially at the first part of this is nearly impossible without ignoring relatively large areas of the surface, but which are functionally very important (side effects). At last a solution was found to put a larger grid over the surface that has a suitable number of measuring points in common with the magnet surface and ignoring the rest of the grid pattern. Of course, this procedure gives loss of the number of degrees of freedom in the approximation procedure of finding the reference plane Q, but this can be suffered to some extend.

Fig. 12 gives the contour map of such an item of the first pair of magnets. The machineshop Lorenz (Ettlingen, Germany) succeeded in keeping all pairs of surfaces within that 13 μm flatness tolerance. However, comparing all machined products of the same grinding machine these 5 surfaces (one surface was a prototype to show the possibilities of the workshop) have quite a lot of torsion with the vector practically always in the same direction.

We are convinced that this effect is due to a systematic way of movement of the machineslide, which can be corrected for in order to get (even) better products.

9. REFERENCES
10. APPENDIX
The black granite material of the surface plate concerned originates from a quarry in West Sweden. Arriving at the manufacturer, the firm Mytili at Apeldoorn, the Netherlands, it has been cut, ground and polished to become a surface plate (630x400x80 mm³) in about two weeks. After the last finishing process it has been carefully transported to the university and measured as to flatness.
On account of this measuring report a last hand-finishing and form-improving operation has been carried out in-situ. Just after that last finishing touch the plate has been installed on the same 3 supports of manufacturing in a tent-frame, covered with aluminium foil in an air-conditioned room. Moreover the plate was packed in isolation material on all sides with exception of the functional upper surface. 24 hours after finishing and installation the first measurement of the series started on 1976-12-24.

10.1 Chemical composition

A spectro-chemical analysis of material of that quarry resulted in 1976 in:

<table>
<thead>
<tr>
<th>Element</th>
<th>Percentage of weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon- oxide</td>
<td>56.1%</td>
</tr>
<tr>
<td>Titanium- oxide</td>
<td>13.3%</td>
</tr>
<tr>
<td>Aluminium-</td>
<td>19.1%</td>
</tr>
<tr>
<td>Calcium-</td>
<td>12.0%</td>
</tr>
<tr>
<td>Iron-</td>
<td>6.5%</td>
</tr>
<tr>
<td>Magnesium-</td>
<td>3.0%</td>
</tr>
<tr>
<td>Sodium-</td>
<td>1.5%</td>
</tr>
<tr>
<td>Potassium-</td>
<td>1.2%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
</tr>
</tbody>
</table>

Fig. 13 Crystalline structure of the granite concerned

10.2 Structural composition

In Fig. 13 the crystalline structure of the black granite material is shown. A sample of about 20x10x1 mm³ has been cut from a larger piece, ground and polished until a slice with thickness of less than 0.015 mm was left.

How the crystalline state can be studied in colour-interference microscopy and/or by macro-photography of which the picture is an example. This black kind of granite seems to have a rather big "density" of grains in comparison with other kinds of granite. The nature of the various crystals can better be explained with the aid of a coloured picture in interference contrast.