

## Thermally activated conductance of a silicon inversion layer by electrons excited above the mobility edge

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**Abstract.** The thermally activated conductivity  $\sigma$  of an n-type inversion layer on a (100) oriented silicon surface and its derivative  $d\sigma/dT$  were measured in the temperature range 1.4 K–4.2 K. Above  $T \approx 2.5$  K both the temperature dependence of  $(T/\sigma)$  ( $d\sigma/dT$ ) and the relation between this quantity and  $\sigma$  cannot be reconciled with a universal pre-exponential factor, i.e. the minimum metallic conductivity, but are shown to be satisfactorily described by a prefactor which is proportional to the temperature. The experimental results presented are consistent with activation of the number of mobile electrons above a mobility edge in the lowest sub-band, and indicate a mobility which is independent of both temperature and electron density.

### 1. Introduction

During recent years there has been continuous interest in the properties of carrier transport in inversion layers showing, at low temperatures, a transition from an activated conductance to a temperature-independent metallic conductance (see review article by Adkins 1978a). This occurs when the Fermi level  $E_F$  is raised above a characteristic threshold denoted as the mobility edge  $E_c$ . In spite of many efforts there is still little unanimity about the details of the temperature dependence of the activated conductivity reported by several groups and the theoretical considerations which interpret the varying results. Some experiments at temperatures sufficiently high for carriers to be excited above the mobility edge are claimed to be in fair agreement with Mott's mobility edge model (Adkins *et al* 1976, Pollitt 1976) and seem to affirm the concept of the universal minimum metallic conductivity  $\sigma_{mm}$  and its value which is predicted to be close to  $0.12 e^2/\hbar \approx 3 \times 10^{-5}$  mho (Licciardello and Thouless 1975). Owing to much higher values of  $\sigma_{mm}$  obtained from other devices which were sometimes found to even increase with electron density, the universality of  $\sigma_{mm}$  is open to question (e.g. Allen *et al* 1975, Sjöstrand and Stiles 1975, Hartstein and Fowler 1975). Substrate bias experiments, in favour of this model, have been explained in terms of a close connection between the

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observed value of the minimum metallic conductivity and the range of disorder experienced effectively by the electrons within the inversion layer (Pepper 1977, 1978a, Fowler 1975). According to this view the estimated value  $0.12 e^2/h$  is typical of a short-range disorder which can be made dominant by forcing the carriers to move close to the interface, whereas higher values correspond to long-range behaviour of the disorder.

When conduction is by excitation to a mobility edge, however, the number activation of carriers can be obtained from the Hall effect only in exceptional cases (Pepper 1978b). A thermally activated Hall mobility is usually observed (Arnold 1974, Thompson 1978). Because of these contradictory results the validity of the independent particle mobility-edge model itself was questioned and a microscopic description in terms of an electron fluid with strong electron correlation as well as a macroscopic effective-medium theory was recently proposed (Adkins 1978b, 1979a, b).

In this paper we shall examine the conductivity of a device which, judging from the rough features of the usual  $\ln \sigma$  versus  $1/T$  plots, presents a typical example of Mott-Anderson transition and seems to yield a common intercept  $\sigma_{\text{mm}} \approx 3 \times 10^{-5}$  mho at  $1/T = 0 \text{ K}^{-1}$ , if a small increase in the conductivity above the predicted  $1/T$  plots, as is observed at higher temperatures, may be disregarded. The inadequacy of these plots in proving  $\sigma_{\text{mm}}$  to be the pre-exponential factor in the activated conductivity will be illustrated by two methods both employing the dimensionless quantity  $(T/\sigma) (d\sigma/dT)$  which is more sensitive to a feasible temperature dependence of the pre-exponential factor. Allowing for a temperature-dependent pre-exponential factor instead of  $\sigma_{\text{mm}}$ , an accurate description of the experimental results can be obtained by a simple independent-particle mobility-edge model.

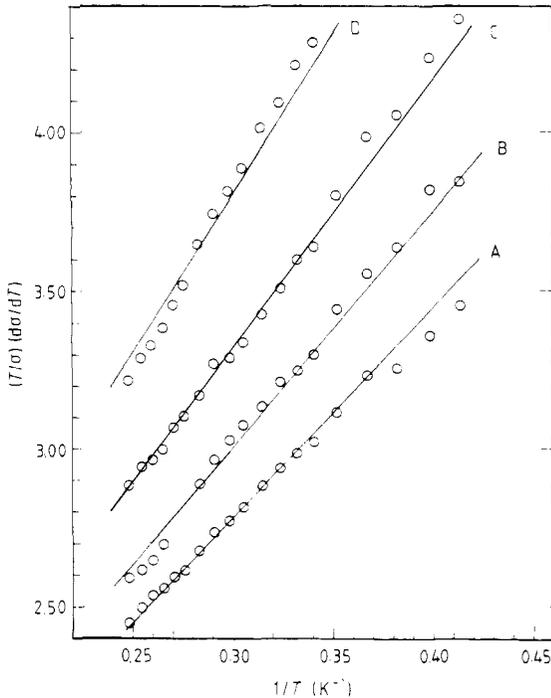
## 2. The pre-exponential factor

For the activated conductivity due to carrier excitation to the mobility edge Mott proposed the following expression (see Mott and Davis 1979):

$$\sigma = \sigma_{\text{mm}} \exp(-E_{\text{act}}/kT), \quad (1)$$

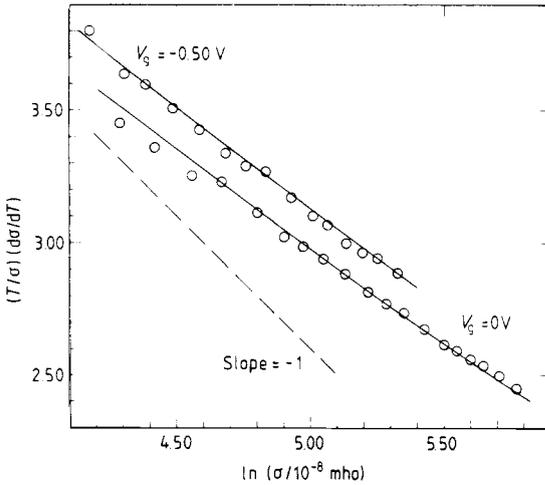
where  $E_{\text{act}} = E_c - E_F$ . Despite small systematic deviations in the linearity as observed in some  $\ln \sigma$  versus  $1/T$  plots, even within one decade of the reciprocal temperature and especially when  $E_F$  approaches  $E_c$ , the overwhelming confidence in such plots seems to be inspired by the occurrence of an apparent common intercept of the  $\ln \sigma$  curves extrapolated to  $1/T = 0 \text{ K}^{-1}$  which is found in some special devices to agree with the predicted estimate of  $\sigma_{\text{mm}}$  (Pollitt 1976, Pollitt *et al* 1976). Unless arguments can be adduced in support of a temperature-independent pre-exponential factor, this method is less suitable for gaining a good understanding of the applicability of equation (1) and particularly the concept  $\sigma_{\text{mm}}$ . This prerequisite in applying equation (1) can be conveniently proved from plots of  $(T/\sigma) (d\sigma/dT)$  versus  $1/T$ , which must be straight and must intercept  $(T/\sigma) (d\sigma/dT) = 0$  at  $1/T = 0 \text{ K}^{-1}$ . For that purpose, at several gate voltages, the channel conductivity  $\sigma$  of the MOS device which has been described in a preceding paper together with experimental details (Niederer *et al* 1981) was measured as a function of the temperature at a frequency of 9.3 Hz or 0.5 Hz. Utilizing the offset facility of the lock-in technique as was previously used, the appropriate derivative  $d\sigma/dT$  could easily be determined from the increment of  $\sigma$  corresponding to a small temperature increase which was taken to be about 0.10 K. In order to achieve an accuracy of the derivative

that was better than 5 per cent, provision was made to measure both the temperature and the corresponding difference  $\Delta T$  within 3 mK. For the present results the applied source-drain field varied between  $4 \text{ mV cm}^{-1}$  and  $40 \text{ mV cm}^{-1}$ , and did not cause any non-ohmic behaviour. The results obtained for several applied gate voltages are presented in figure 1 which shows for temperatures above 2.5 K straight plots with extrapolated intercepts of  $(T/\sigma)(d\sigma/dT)$  at  $1/T = 0 \text{ K}^{-1}$  varying between 0.3 and 1.0 which disagrees with the behaviour expected from equation (1).

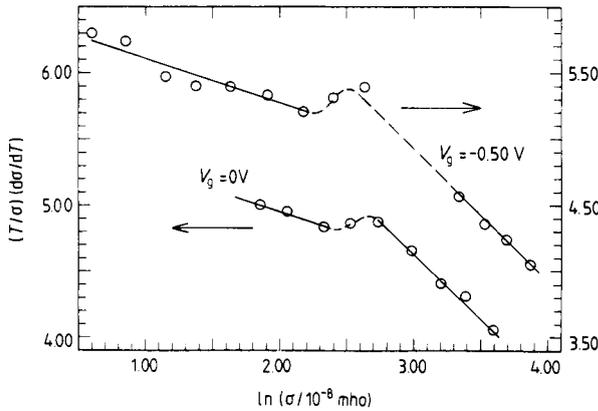


**Figure 1.**  $(T/\sigma)(d\sigma/dT)$  as a function of the reciprocal temperature for  $V_g = 0 \text{ V}$  (A),  $-0.25 \text{ V}$  (B),  $-0.50 \text{ V}$  (C) and  $-0.75 \text{ V}$  (D). The full curves represent equation (4) with  $p = 0$ ; the corresponding activation energies are 0.55, 0.61, 0.70 and 0.83 meV, respectively.

The second test starts from the slope of the plot of  $(T/\sigma)(d\sigma/dT)$  versus  $\ln \sigma$  which has to be  $-1$  in order to ensure the applicability of equation (1) to the activated conductivity. In figure 2 the disagreement with equation (1) is illustrated once more for  $V_g = 0 \text{ V}$  and  $V_g = -0.50 \text{ V}$  at temperatures above 2.5 K where carrier excitation to the mobility edge is beyond doubt. In figure 3 one may also notice an intermediate temperature region roughly between  $T = 2.5 \text{ K}$  and  $T = 2.0 \text{ K}$  with slope  $-1$ , and an adjacent region below about 2.0 K with slope  $-\frac{1}{2}$ . This behaviour is presumably due to respectively fixed-range and variable-range hopping, which are not further discussed in this paper. We note here that the gradual transition to the slope  $-1$  at decreasing temperature, cannot be seen from just a plot of  $\ln \sigma$  versus  $1/T$ .



**Figure 2.**  $(T/\sigma)(d\sigma/dT)$  versus  $\ln \sigma$  in the region of number-activated conduction in extended states, i.e. for  $T > 2.5$  K. The full curves have been calculated from equations (3) and (4) with  $p = 0$ , by taking  $e\mu_0 D(E_c)k = 4.22 \times 10^{-6}$  mho  $K^{-1}$  at  $V_g = 0$  V and  $4.05 \times 10^{-6}$  mho  $K^{-1}$  at  $-0.50$  V.



**Figure 3.** Plots of  $(T/\sigma)(d\sigma/dT)$  versus  $\ln \sigma$  for  $T$  below 2.5 K, showing a change of the slope from  $-1$  into  $-0.33$  when the temperature is about 2 K.

**3. Conductivity due to an activated number of electrons**

The density of states at the mobility edge  $D(E_c)$ , has been argued to be well below the value  $D_0$  of the unperturbed lowest sub-band (see Mott and Davis 1979). In his early work Mott already estimated  $D(E_c)$  to be of the order of  $0.3 D_0$ , but recently Thouless and Elzain (1978) obtained a substantially higher value of about  $0.85 D_0$ . As mainly electrons in a narrow energy range above the mobility edge contribute to the conductance, a weak increase of the density of states compared with  $D(E_c)$  may be disregarded.

Assuming a temperature-dependent conductivity mobility for the electrons excited above the edge of the form:

$$\mu = \mu_0(T/T_0)^p \quad (2)$$

where  $\mu_0$  may depend on the electron density and  $T_0$  and  $p$  are as yet undetermined constants, the conductivity is then expressed by:

$$\sigma = e\mu_0 D(E_c) kT(T/T_0)^p \ln[1 + \exp(-\zeta)] \quad (3)$$

where  $\zeta = E_{\text{act}}/kT$ . It should be noticed that by taking  $p = -1$  the conductivity mobility assumes the form proposed by Cohen (1970) for the possibly diffusive nature of motion of electrons just above the mobility edge, which is inherent in the concept of  $\sigma_{\text{mm}}$ . So in this case equation (3) is approximately reduced to expression (1). Examination of the temperature dependence of the logarithmic temperature derivative of  $\sigma$  provides the value for  $p$ , and will thus clarify whether this concept is applicable to a specific device.

The quantity  $(T/\sigma) (d\sigma/dT)$  is easily obtained from equation (3), giving:

$$(T/\sigma) (d\sigma/dT) = (1 + p) + \zeta(1 + \exp \zeta) \ln[1 + \exp(-\zeta)]. \quad (4)$$

The assumed temperature-independent activation energy  $E_{\text{act}}$  can be anticipated from the temperature dependence of  $E_F - E_c$  which was found to decrease as  $E_F$  approaches  $E_c$  from the extended states to within about 0.10 meV above the mobility edge (Niederer *et al* 1981) and which finally vanished when  $E_F$  was more than 0.40 meV below  $E_c$ . Depending on the actual value of  $\zeta$  on the right-hand side of equation (4) the following approximations proved to be convenient in comparing expression (4) with the experimentally determined behaviour:

$$(T/\sigma) (d\sigma/dT) \approx (1 + p) + \zeta/2 \ln 2 \quad \text{when } 0 < \zeta \ll 2.3 \quad (4a)$$

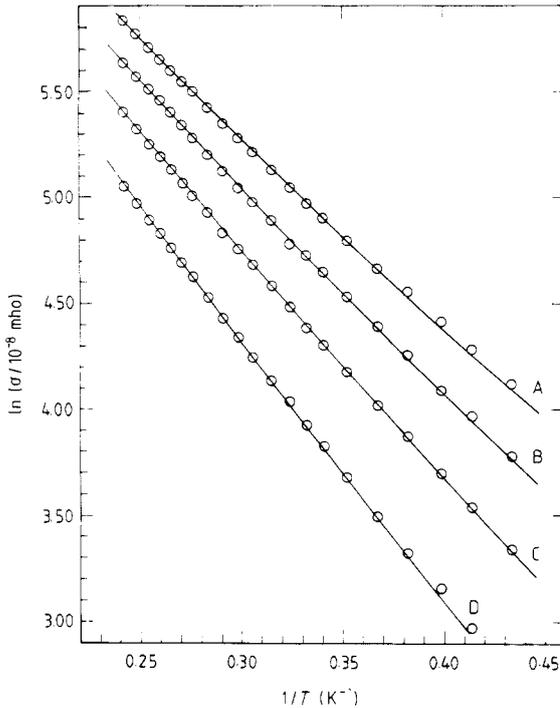
$$\approx (1 + p) + \zeta \quad \text{when } \zeta \gg 2.3 \quad (4b)$$

$$\approx (0.774 + p) + 1.052 \zeta \quad \text{when } \zeta \approx 2.3. \quad (4c)$$

Expression (4c) is the equation of the inflectional tangent at  $\zeta = E_{\text{act}}/kT = 2.342$ . It provides an approximation which is accurate within 0.5 per cent when  $E_{\text{act}}$  ranges from 0.50 meV to 0.90 meV at temperatures between 2.5 K and 4.2 K. This range of  $E_{\text{act}}$  is typical for the results presented.

The intercepts at  $1/T = 0 \text{ K}^{-1}$  of the extrapolated experimental plots of  $(T/\sigma) (d\sigma/dT)$  versus  $1/T$  as shown in figure 1 and discussed in the preceding section, can be made to agree with equation (4c) by taking  $p = 0$ . These plots also affirm the temperature-independent value of  $E_{\text{act}}$  which then only varies with the gate voltage applied. Having taken  $p = 0$ , at a fixed gate voltage,  $E_{\text{act}}$  can be determined as the average value obtained by applying equation (4) to all the values of  $(T/\sigma) (d\sigma/dT)$  measured between 2.5 K and 4.2 K. This procedure is preferable to the unnecessarily approximate result which is obtained from the slope of equation (4c). The spread between the individual results amounted to about 1 per cent. In figure 1, equation (4) is plotted as a function of the reciprocal temperature making use of the values of  $E_{\text{act}}$  belonging to the gate voltages applied.

Now that the activation energy at a fixed gate voltage is determined, the prefactor  $e\mu_0 D(E_c)k$  corresponding to  $p = 0$ , can be obtained in the same straightforward way by fitting equation (3) to the conductivity. It should be emphasised that in the range of  $V_g$  from  $-0.75 \text{ V}$  to  $0 \text{ V}$  no significant change of the prefactor was observed. On average it



**Figure 4.**  $\ln \sigma$  versus  $1/T$  for  $V_g = 0$  V (A),  $-0.25$  V (B),  $-0.50$  V (C) and  $-0.75$  V (D). The full curves have been calculated from equation (3) with  $p = 0$ ; the corresponding values of the activation energy and the prefactor are: (A)  $0.55$  meV,  $4.2 \times 10^{-6}$  mho  $K^{-1}$ ; (B)  $0.61$  meV,  $4.1 \times 10^{-6}$  mho  $K^{-1}$ ; (C)  $0.70$  meV,  $4.1 \times 10^{-6}$  mho  $K^{-1}$ ; (D)  $0.83$  meV,  $4.1 \times 10^{-6}$  mho  $K^{-1}$ .

was found that  $e\mu_0 D(E_c)k = (4.1 \pm 0.1) \times 10^{-6}$  mho  $K^{-1}$ . The curves drawn in figure 4 for various values of  $V_g$  illustrate the excellent fit of equation (3) with  $p = 0$  to the measured activated conductivity.

#### 4. Discussion

From the fits of equation (4) to the  $(T/\sigma)(d\sigma/dT)$  versus  $1/T$  plots, the activation energy is a known function of the gate voltage which yields an almost constant rate of change  $dE_{act}/dV_g = -0.4 \pm 0.1$  meV  $V^{-1}$  between  $V_g = -0.75$  V and  $V_g = 0$  V. Compared with  $d(E_F - E_c)/dV_g = 1.45$  meV  $V^{-1}$  determined at strong inversion, this result indicates a density of states below  $E_c$  which is apparently much greater than the unperturbed band density  $D_0$ , provided that the position of the mobility edge is not affected by altering the electron density in the inversion layer. As only states at and just above  $E_c$  contribute to the conduction, a mobility edge which rises as more electrons are induced into the inversion layer is expected to cause an increase of the prefactor  $e\mu_0 D(E_c)k$ . If so, this result would be contrary to that of the preceding section, unless the edge is at an improbable position far within the unperturbed band. Consequently the absence of a noticeable change in the prefactor with the gate voltage is in favour of a mobility edge which is fixed within the sub-band. The relatively large density of states implied in the

region of electron densities where the conductance is activated is compatible with a band of bound states. This band was independently introduced by Niederer *et al* (1981) to explain the uniform temperature dependence of  $E_F - E_c$  emerging from this mobility-edge model when the Fermi level was at least 0.10 meV above the mobility edge. As already mentioned, when  $E_F$  was lowered through  $E_c$ , the temperature dependence of  $E_F - E_c$  gradually vanished, which indicates that the Fermi level enters this band of bound states from the top. Concerning the density of states at  $E_c$  it is interesting to compare the value of the prefactor  $e\mu_0 D(E_c)k$  obtained from the activated conductivity with the corresponding value gained from the temperature-dependent non-activated conductivity by utilising the rate of change of the Fermi level with the gate voltage at strong inversion. Within the experimental uncertainty no difference was observed. It therefore follows that  $D(E_c)$  is almost equal to  $D_0$ . Apart from the assumed validity of equation (1), extrapolation of the density of states within the exponential tail of the sub-band towards the mobility edge was also previously reported by Pollitt (1976) to yield  $D(E_c) \approx 1.0 D_0$ .

The absence of any change in the prefactor despite moving the Fermi level through 1 meV on both sides of the mobility edge indicates that the conductivity mobility does not exhibit a transition from a Bloch-like propagation of the electrons when  $E_F > E_c$  to a diffusive type of motion when  $E_F < E_c$ , which is generally assumed for the residual electrons just above the mobility edge. In spite of the loss of phase coherence it would appear that electrons excited above the edge tend to retain the scattering mechanism characteristic of the type of motion of electrons on the metallic side of the mobility edge. If  $D_0 = 1.60 \times 10^{15} \text{ m}^{-2} \text{ meV}^{-1}$  is taken for  $D(E_c)$ , then  $\mu = 0.19 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ , which agrees with the order of magnitude expected by the appearance of weak Shubnikov-de Haas oscillations in the metallic conductance for magnetic fields above 5 T.

It will be clear that the very accurate description of the experimental results, as well as the conclusions on  $\mu$ ,  $D(E_c)$  and  $\sigma_{\text{mm}}$ , are by no means invalidated by discarding the proposed model of bound states. Rejection of this model implies that the mobility edge is not fixed in the sub-band, but is hovering above the Fermi level, yet still far enough within the sub-band to meet the very likely constant value of  $D(E_c)$ .

In this context it is worth noting that at sufficiently low temperatures the 'number-activated' metal-like conductance was observed to disappear. This is illustrated by the plots of  $(T/\sigma)(d\sigma/dT)$  versus  $\ln \sigma$  which show a transition at a temperature  $T_{\text{tr}}$  of about 2.5 K being independent of the electron density. The implied limitation of the value of the prelogarithmic factor in equation (3) including the temperature, may be related to the proposed concept of the minimum metallic conductivity by assuming  $\sigma_{\text{mm}} \approx e\mu_0 D(E_c)kT_{\text{tr}} \approx \sigma_{\text{tr}} \approx 1 \times 10^{-5} \text{ mho}$ . If so, a drastic change in the transport mechanism of electrons just above  $E_c$  ought to play a critical role in the explanation of the conductivity at temperatures below 2.5 K. The mobility should then become either proportional to  $1/T$  ( $p = -1$ ) or thermally activated according to  $\mu \propto (1/T) \exp(-E/kT)$ .

The value obtained for the conductivity mobility of the device,  $\mu = 0.19 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ , may also be related to the value of the apparent  $\sigma_{\text{mm}}$ ,  $1.3 \times 10^{-5} \text{ mho}$ . As the effective mobility, which is smaller than the conductivity mobility, is usually found to vary between  $0.1 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  and  $2.0 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  for different devices, the apparent value of  $\sigma_{\text{mm}}$  is expected to range from approximately  $1 \times 10^{-5} \text{ mho}$  to  $3 \times 10^{-4} \text{ mho}$ . At present this view is speculative, but it is significant that an increase of the electron density which enhances the screening of the potential fluctuations at the  $\text{SiO}_2$  interface, often results in an increase of the intercepts of the  $\ln \sigma$  versus  $1/T$  plots at  $1/T = 0 \text{ K}^{-1}$ , which is similar to the behaviour of the mobility near the conduction threshold. The absence of a

noticeable rise of both the intercept and the conductivity mobility seems therefore to be characteristic for devices with a high oxide charge density close to the Si/SiO<sub>2</sub> interface.

## 5. Conclusions

The thermally activated transport in a silicon inversion layer was investigated by measuring the temperature dependence of both  $\sigma$  and  $(T/\sigma)(d\sigma/dT)$ . It was found that the experimental results obtained can be accurately described in terms of activation of the number of carriers in mobile states above a mobility edge and a conductivity mobility which is almost independent of both the temperature and electron density. The resulting temperature dependence of the prefactor is in disagreement with a universal pre-exponential factor  $\sigma_{\text{mm}} \approx 0.1 e^2/\hbar$  in two dimensions. A careful examination of the Hall effect and the Shubnikov–de Haas oscillations near the threshold of the conductance, on the basis of the ideas described above, would appear to be necessary to resolve this interesting difference and to gain a thorough understanding of the behaviour of electrons near the mobility edge.

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